ECE 398GG – ELECTRICAL VEHICLES

2. Vehicle Basic Dynamics: Energy and Power Needs of Electric Vehicles

P. T. Krein

Department of Electrical and Computer Engineering

University of Illinois at Urbana–Champaign

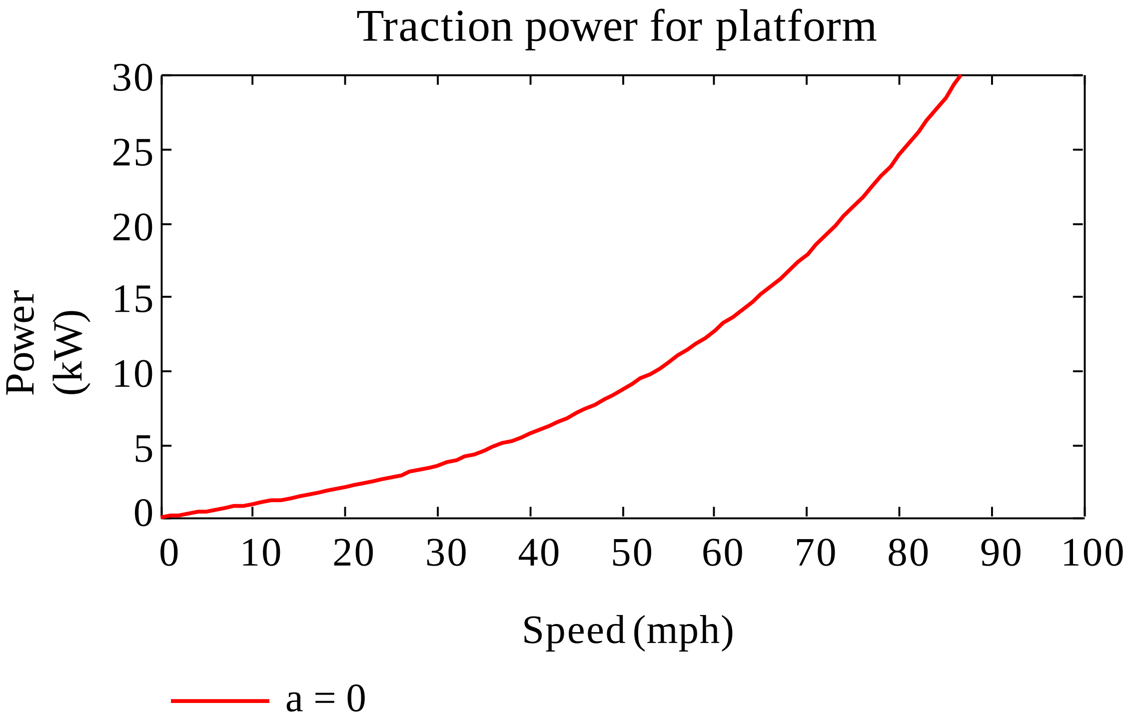
Vehicle Basic Dynamics: Energy and Power Needs of Electric Vehicles

P. T. Krein

Department of Electrical and Computer Engineering
University of Illinois at Urbana-Champaign, USA

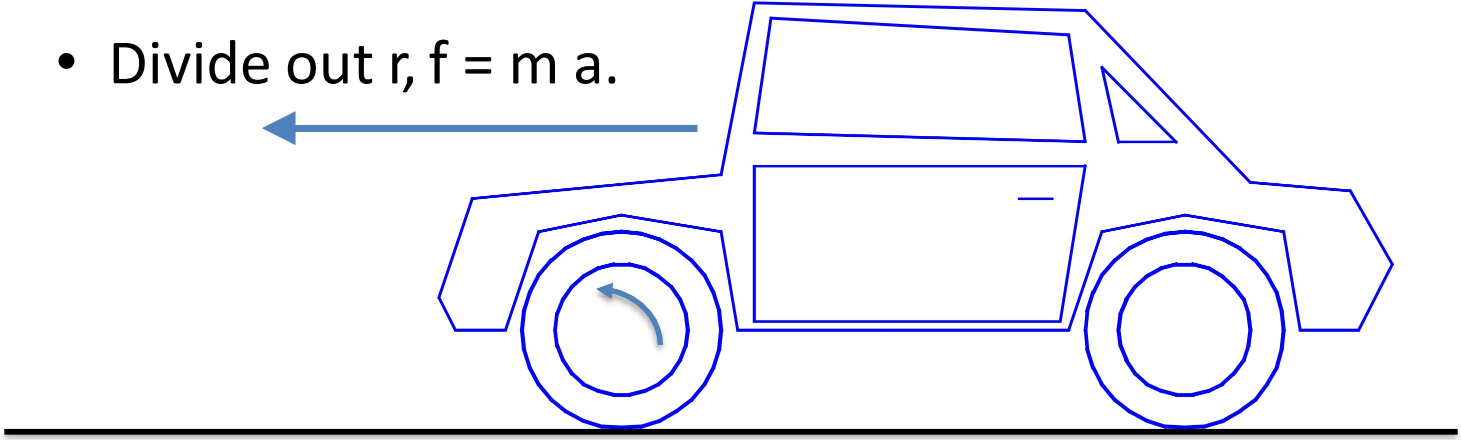
Must do work against external forces to move

- Main power needs:
  - Aerodynamic drag
  - Tire loss
  - Gravity
  - Acceleration
- Others:
  - “Hotel” loads
  - Controls
  - Overhead



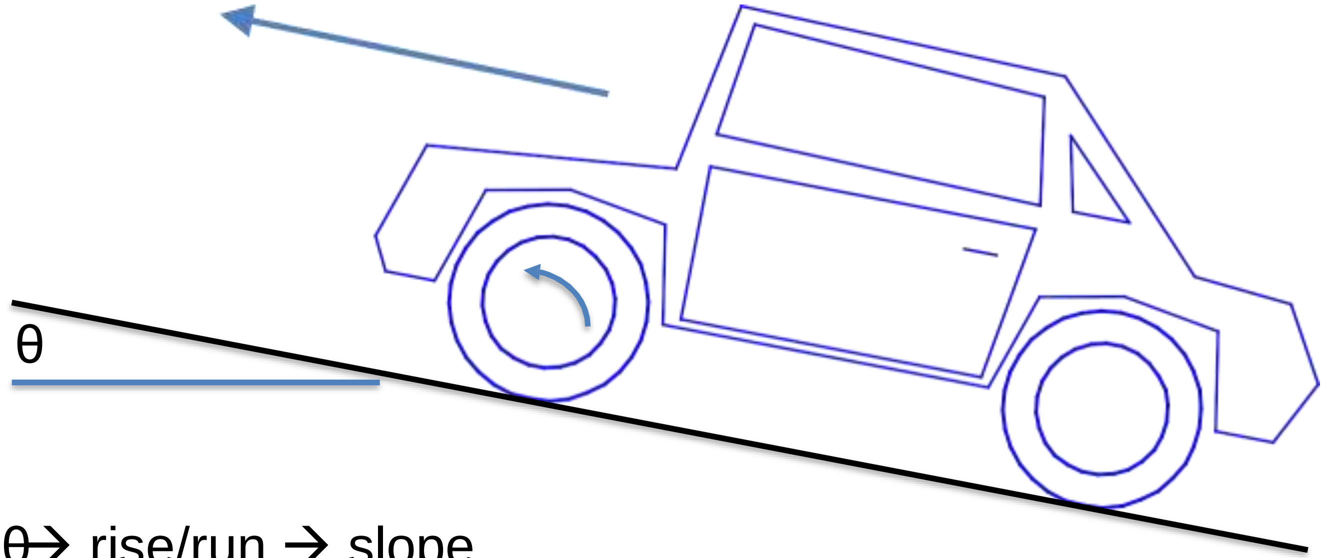
Vehicle dynamics

- The governing needs come from basic physics, $f = ma$ (force, mass, acceleration).
- There are also rotating components, for which $T = J\alpha$ (torque, moment of inertia, angular acceleration).
- Rotation follows, write it out: $T = f\,r$, $J = mr^2$, so $f\,r = m\,r^2\,\alpha$, with $r\,\alpha = a$.
- Divide out r, $f = m\,a$.



Vehicle dynamics

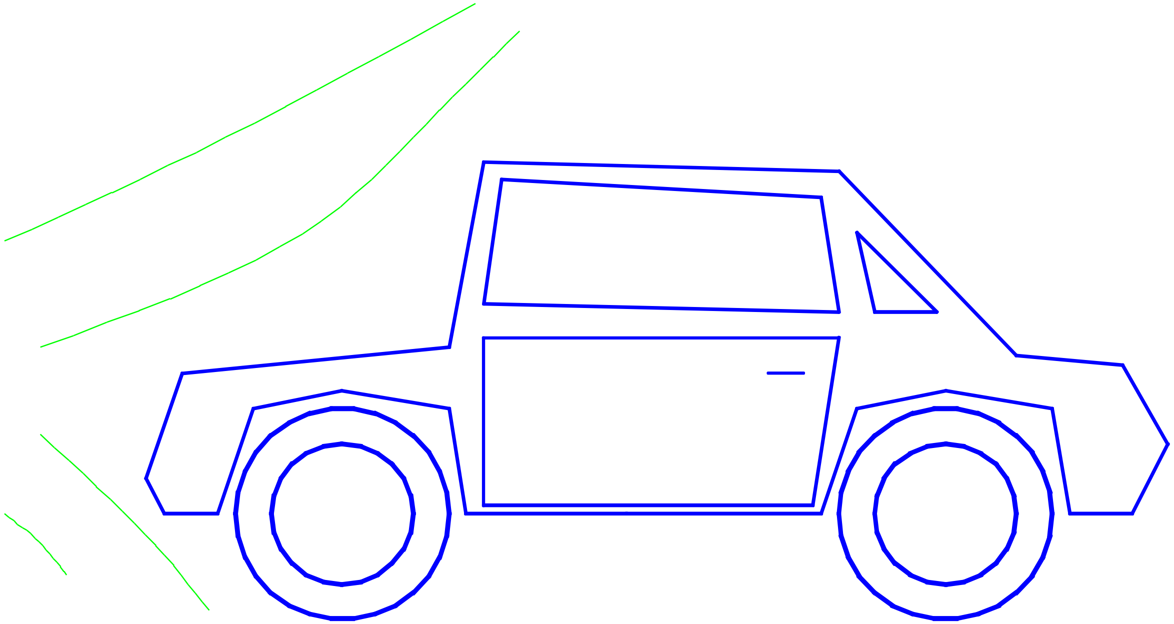
- Move up (or down) hill, adding potential energy m g h.
- Force is in the direction of motion, m g sin θ.



sin θ→ rise/run → slope
Vehicles: defined as rise/horizontal distance → tan θ
g = 9.807 m/s^2, call it 10.

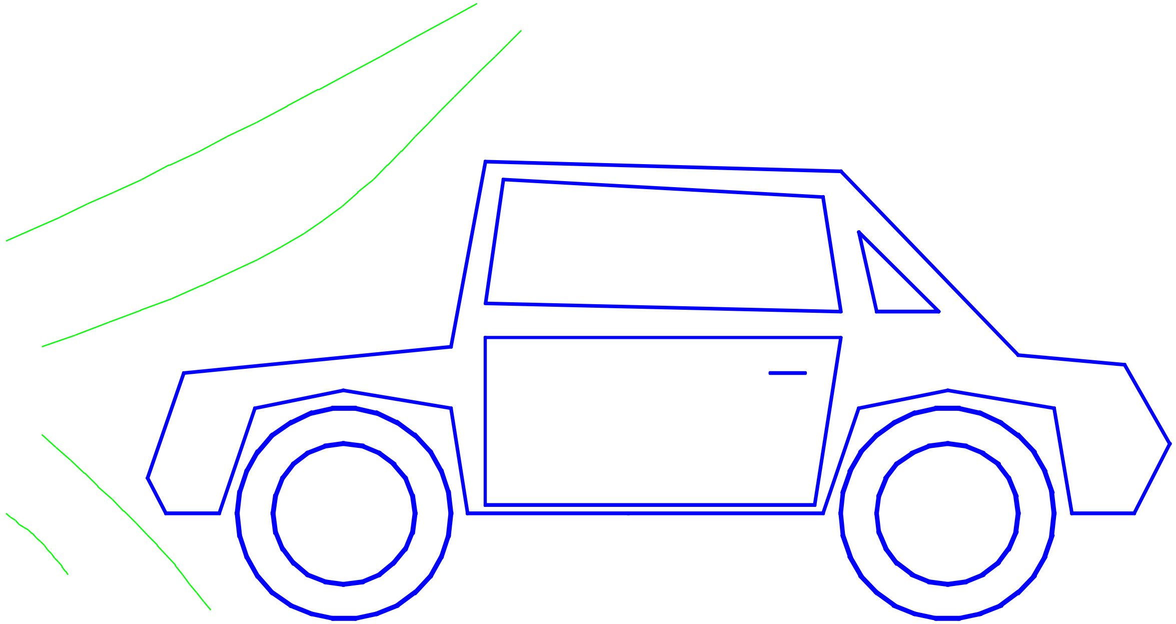
Forces

- A practical vehicle has both rotational and linear motion (we spin the tires and use them to translate to linear motion).



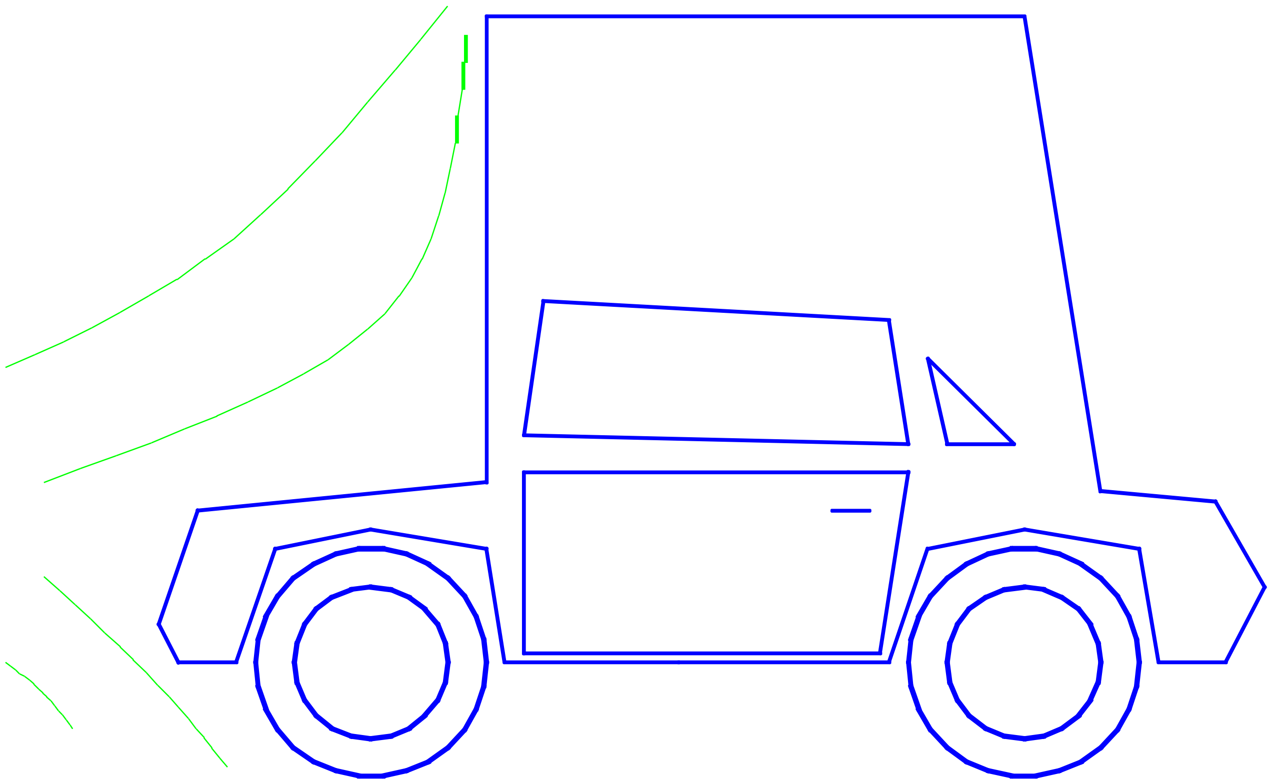
Aerodynamic drag

- Move air out of the way (*drag* force).
- We must inject enough kinetic energy into the displaced mass of air to move it away as fast as the car moves through.



Drag

- Bigger, less streamlined: more air, more drag, more change in the motion of the air.

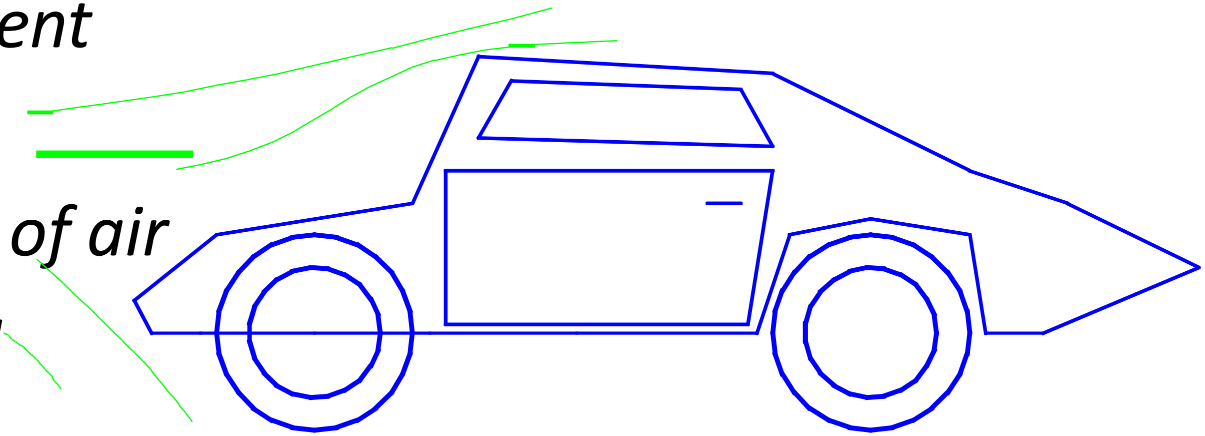


Drag

- Smaller, more like a wing: less air, less drag, less change in the motion of the air.
- Drag force:

$$f_{drag} = \frac{1}{2} C_d A_f \rho v^2$$

- C_d *drag coefficient*
- A_f *frontal area*
- ρ *mass density of air*
- v *vehicle speed*



Typical drag coefficients (various sources)

Vehicle	C_d
Typical truck	0.6 or more
Jeep Wrangler	0.45
Cadillac Escalade	0.37
Porsche Boxster	0.29
Ford Fusion	0.27
Tesla model 3	0.23
GM EV1	0.19

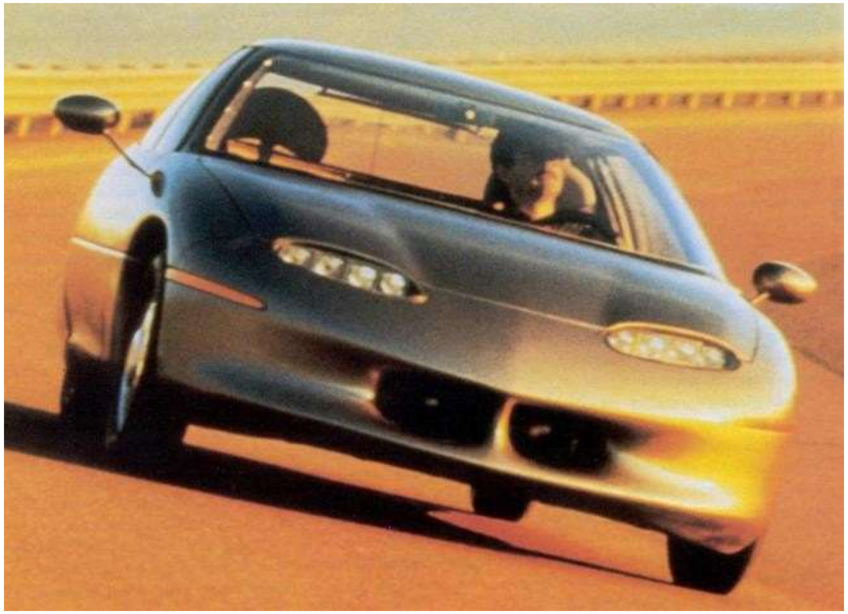
Below 0.20, we are usually beyond the realm of production cars.
Formula 1 and Indy cars tend to have $C_d \approx 1$, seeking to convert drag into downforce.

GM Precept concept car, C_d 0.16

Source: www.edmunds.com

Area, air density

- Frontal area is often measured from a careful front photograph. Older list at http://ecomodder.com/wiki/Vehicle_Coefficient_of_Drag_List
- Sometimes available from manufacturers.
- Air density falls off nearly linearly above sea level.
- Start with about 1.2 kg/m^3 (dropping to about 1.08 kg/m^3 at an elevation of 1000 m).
- Unless you are high in the mountains, the effect is small.



www.gmev.com

Tire force

- A tire flexes as the vehicle moves along.
- Flexing is not fully reversible, so it consumes energy and heats up the tire.
- Also, we need the tire to be in contact with pavement to provide force against the roadway and move the vehicle.

Driver61
Youtube channel

Tire force

- This means a certain energy is consumed in each tire during each revolution – energy per unit distance.
- Amount of flex increases with vehicle weight, increases with *contact patch* size.
- Decreases with pressure.

www.tirerack.com

Tire force

- Energy per unit distance?
- Energy or work is force integrated over a distance,

$$W = \int f \; dx$$

- This means that energy consumed per unit distance is the same as the force required to move something over that distance.

Tire diameter has limited effect: flex loss is about pavement contact and contact patch.

Tire force

- For our purposes, let's assume tire friction coefficients are high enough to stay "attached" to the road.
- If the tires just spin, no good. For vehicle motion,

$$f_{tire} = mgR_t$$

- Here m is the vehicle mass, g is gravity acceleration, R_t is the *coefficient of rolling resistance*.
- Typical value (cars) is about 0.01. Typical *low loss* tires 0.008.
- High-pressure racing bike tires perhaps 0.004.
- Steel train wheels 0.0004.

So far? Drive force needed at the tire edge:

- Need to add some force to accelerate the vehicle!

- One detail that might not be small: to accelerate a vehicle, we must provide both linear acceleration and angular acceleration to the wheels and other rotating parts.
- The extra moment of inertia is ~ mr^2
- This means the rotating parts count *twice* for acceleration.
- For acceleration, there is an *equivalent mass* given by the total vehicle mass **plus** the mass (again) of rotating parts.

That extra angular momentum

- Since rotating mass is double-counted for vehicle acceleration, this works strongly against in-wheel motors and hardware.
- Extra mass of rotating parts also gives a gyroscopic effect – tends to oppose changes in direction.
- Sometimes called “unsprung mass” since it attaches to wheels or axles rather than suspensions or springs.

techeblog.com

What else?

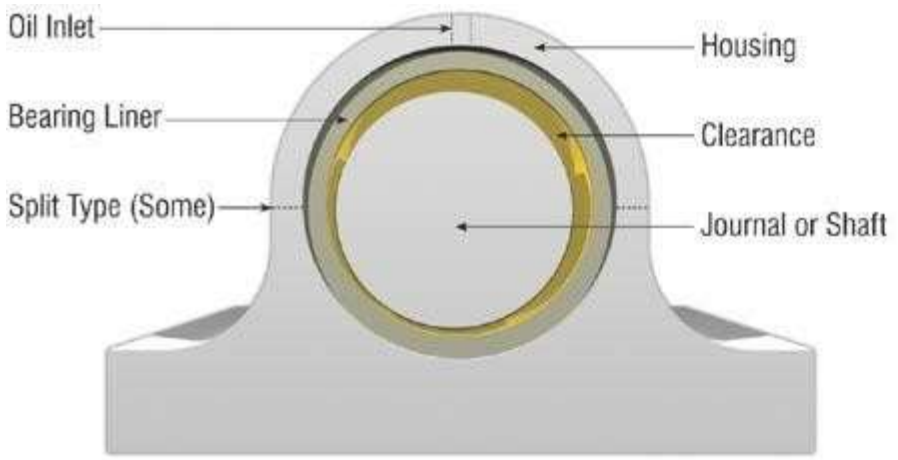
- "Hotel" loads: lights, heat, cooling, fans, electronics, sound system, …
- System: controls, sensors, coolant pumps, fuel pumps.
- Energy per unit time, rather than linked to distance.
- Friction?

motor1.com

Friction

Fraction matters when moving parts slide against stationary parts.

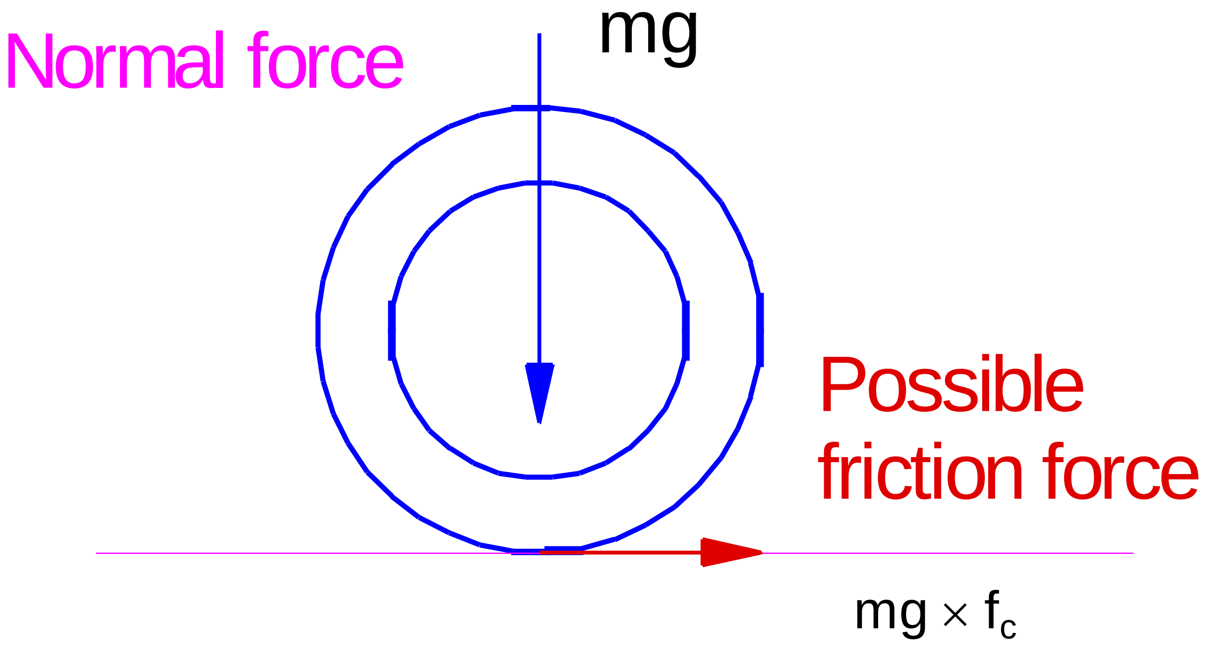
Bearings: Energy loss is approximately proportional to speed.



www.machinerylubrication.com

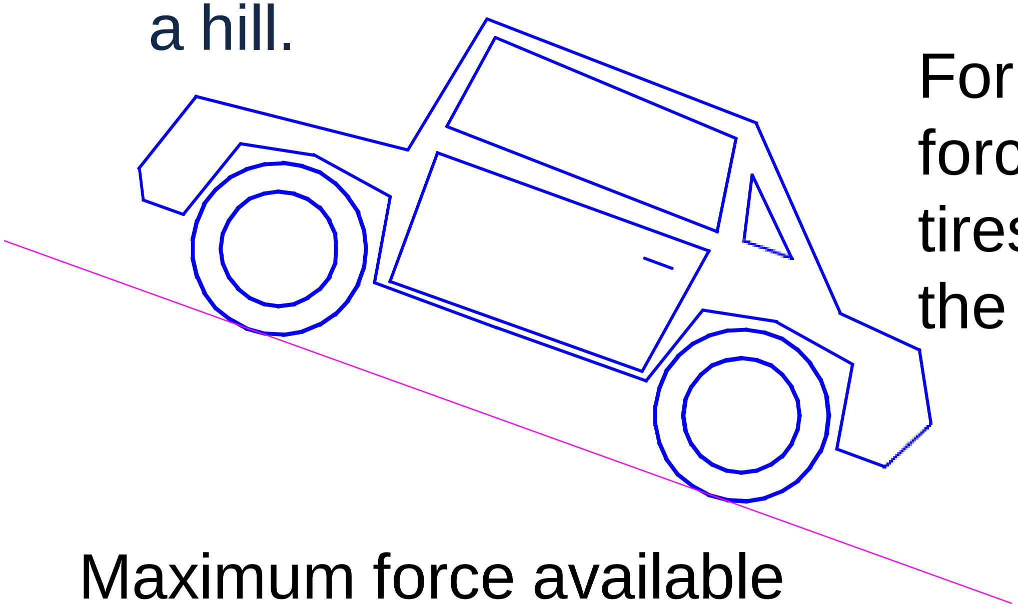
Tire friction – A much different issue

- Tires should attach to the road to transfer axle torque into linear driving force.
- For transportation, the friction force between tires and road is not linked to loss.
- It *is* a limit on force to be delivered.



Tire friction

- Tire friction is the only force that can hold a parked car on a hill.



For level motion, if the drive force exceeds the friction force, tires will slide rather than propel the vehicle.

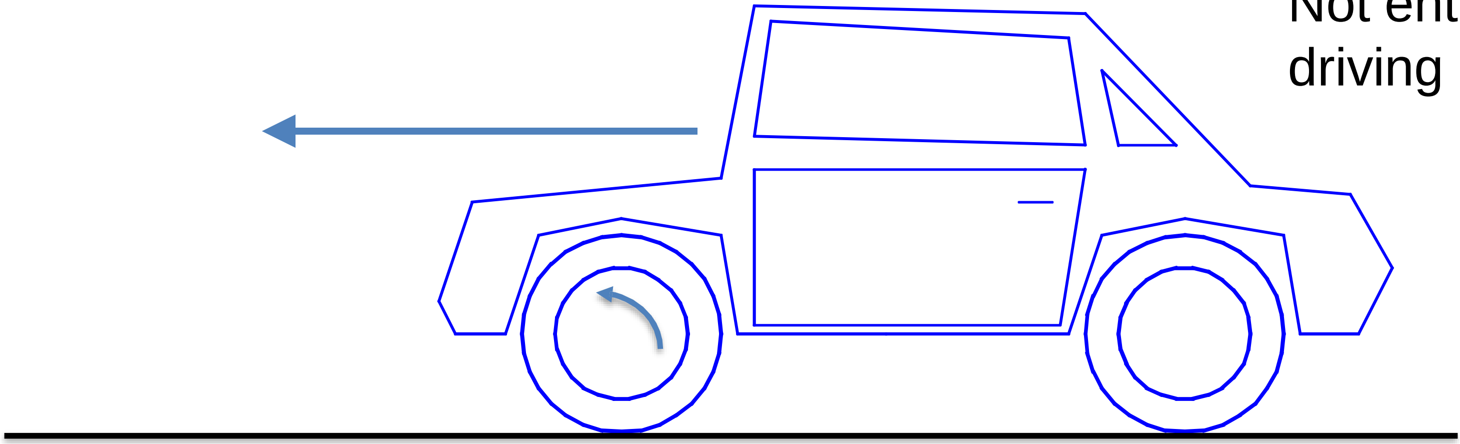
Maximum force available against sliding:
mg × (friction coefficient)

Tire: acceleration limit friction

- Vehicle acceleration force is m × a.
- Maximum tire friction force is mgf_c.
- Notice that $a < gf_c$ to avoid sliding.

We usually want f_c to be as high as possible for tires.

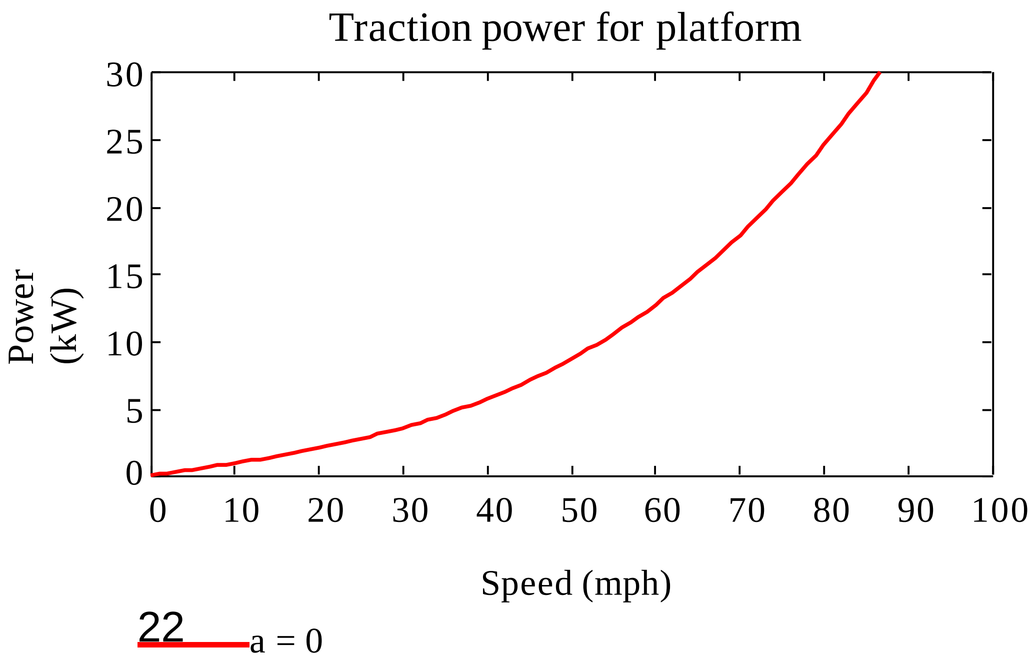
Not entirely a choice: driving on ice, …



- Does it matter? 60 mph → 26.8 m/s.
- 0 to 60 mph in 3 s requires $a = 8.94\ \text{m/s}^2$, 0.9g!!

Traction Power – DELIVERED TO THE AXLE

- Power is the rate at which work is done.
- For our vehicle, energy = force × distance (really $W = \int f \, dx$ but take the force outside the integral if independent of x).
- Force × distance/time → force × speed.
- Power = f × v,



Power and force: Things to notice

- Most terms depend on mass. Lower mass → lower power
- Aerodynamic power increases as the cube of the speed.
  - Example: Takes 50% more power to drive at 80 mph than at 70 mph.

Power and force: Things to notice

- How do terms stack up, typical ~2000 kg car, low-loss tires?
- Samples: mg = 20000 N, Cd = 0.27, Af=2.4 m^2, 2% slope.
- mg sin θ→ 400 N
- mgR_t → 160 N
- $\frac{1}{2}C_d A_f \rho v^2$ → 0.39 v^2 N
- It takes 600 N of force at 10 m/s (22 mph) on a 2% slope, 720 N at 20 m/s, …
- 720 N at 20 m/s is 14.4 kW.

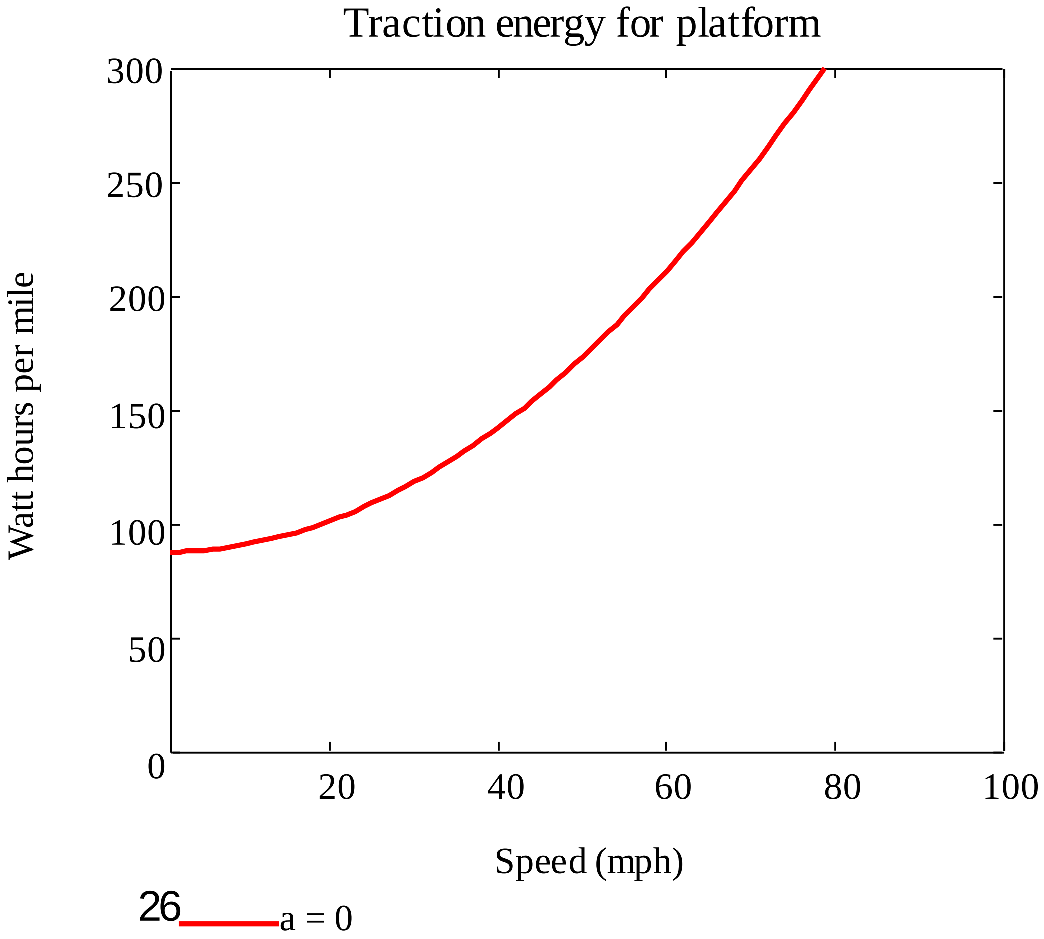
For slopes up to 10%, sin θ≈ tan θ

Traction power

- So far, this seems to apply to *any* vehicle.
- What is different if the vehicle is electric?
  - Traction power can be nearly all of the needed power. This is not the case for fuel-driven vehicles.
  - Batteries will tend to make the vehicle heavier (more about this soon).
  - Much less waste heat, more hotel power for cabin heat.
  - Fewer moving parts, less friction loss.
  - Electricity is an indirect energy source. How was it generated?
  - Braking (more shortly).

Energy

- Consumption: A good electric car uses less than 250 W-h per mile (with minimal hotel loads).
- Kinetic energy: 2000 kg car at 30 m/s has ½ mv^2 = 900 kJ.



Is that a lot? It could run a 10 W lamp for one day.
It is 0.25 kWh, a few cents worth.

Braking

$$f_{traction} = mg\sin\theta + mgR_t + \frac{1}{2}\rho C_d A_f v^2 + m_{eq}a + f_{brake}$$

- Brakes remove vehicle kinetic energy to slow or stop.
- For the typical car, we are trying to remove a few megajoules of kinetic energy in a few seconds.
- Power levels: 1 MJ/s = 1 MW.
- Disc and drum brakes do this with friction.
  - Energy is converted to heat and lost.
  - This can get extreme (see *Ford vs. Ferrari*).
- The expression shows brakes working against traction force.

Braking

$$m_{eq}a = -f_{brake} - mg\sin\theta - mgR_t - \frac{1}{2}\rho C_d A_f v^2$$

- Rewrite in term of acceleration. Seeking to make *a* < 0.
- We can slow by coasting down ($f_{brake} = 0$), let drag and tire resistance take the energy.
- Or backfeed into the drivetrain, let engine friction and rotating part friction (not shown) take some of the energy loss.
- If we are going downhill, sin θ is negative and works against brakes.

Regenerative braking

- In an electric drive train, the motor can deliver either positive torque or negative torque.
- Cannot do this with a fueled engine (except friction).
- We can enforce $f_{traction} < 0$ independent of friction brakes.
- Negative torque → negative power → energy recovered back from motion.
- This energy can be converted to heat (and loss) or returned to the source (storage).
- Diesel-electric trains use electric motors. Braking energy is sent to large air-cooled resistor banks.
- Also called *energy recovery braking.*

Regenerative braking

$$f_{traction} = mg\sin\theta + mgR_t + \frac{1}{2}\rho C_d A_f v^2 + m_{eq}a + f_{brake}$$

- In an electric vehicle, there is no problem to control acceleration *a* to a negative number – up to the tire friction limits.
- The same method can recover energy when going downhill.

$$P_{drive} = mgv\sin\theta + mgR_t v + \frac{1}{2}\rho C_d A_f v^3 + m_{eq}av + P_{hotel} + P_{cont} + \cdots$$

Caveats

- We have been examining *traction power*, the power needed into the drive axle to move a vehicle.
- In a fuel-driven car, the fraction of fuel energy that reaches the axle is low (10% to 20% is typical).
- In a motor-driven electric car, often 90% of stored battery energy can drive the axle.
- There is loss during battery charging, and so on.

Key Issue – Energy Storage

- A vehicle must carry along its own energy.
- It must be stored in a useful way that is easy to reverse.
- Exceptions:
  - Trains, busses, trollies with overhead catenaries.
  - Electrified rails (common for subways).
  - Wireless power transfer (still early).

Travelchinaguide.com

Efficiencies

- Efficiency of drive train – electric power input to mechanical power at axle.
- Battery cycle (in/out) efficiency – charger power input to inverter power output
- Charger efficiency – grid power input to battery storage output
- Efficiency of the electric power grid

Energy storage

- A vehicle needs to store energy (for range).
- Alternatives:
  - – Electrical storage: Capacitors or inductors
  - – Mechanical storage: Flywheels or springs
  - – Compressed air tanks
  - – Gravity potential energy *mgh*
  - – Electrochemical batteries
  - – Chemical storage: Fuel
- Figures of merit:
  - – Useful storage per unit mass
  - – Useful energy rate (power) per unit mass

Pv-magazine-usa.com

Specific Energy Density (J/kg)

Storage technology	Energy density	Comments
Lead-acid batteries	100 kJ/kg (30 W-h/kg)	
Lithium-ion batteries*	1000 kJ/kg	High efficiency
Compressed air, 10MPa	80 kJ/kg	Plus air tank
Conventional capacitors	0.2 kJ/kg	
Ultracapacitors	20 kJ/kg	
Flywheels	100 kJ/kg	
Gasoline	46400 kJ/kg	12900 W-h/kg
Hydrogen	142000 kJ/kg	

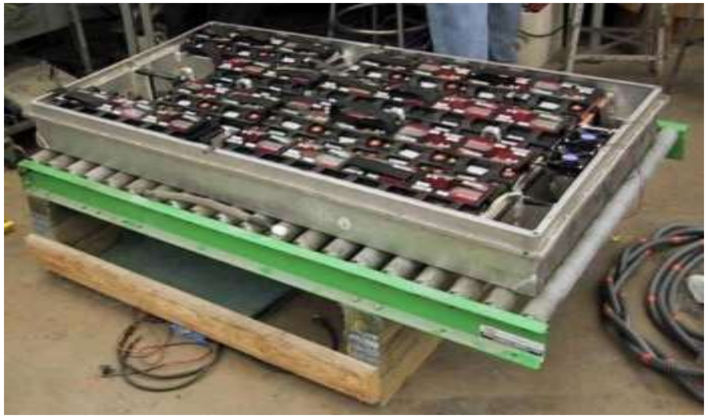
*Prototypes. Production units ~600 kJ/kg.

Compare to gravity potential energy

- Potential energy *mgh.* Raise a mass, and then recover energy as it is lowered.
- With g ≈ 10, a 1 kg mass stores 10 N-m per 1 m rise.
- For a 100 m rise, this would be 1 kJ/kg.
- Put a different way, a lead-acid battery stores enough energy to raise itself 10000 m.
- Storage methods have different scales.
  - Pumped hydro storage: Move millions of tonnes of water up ~100 m.
  - Compressed air: Need a big enough tank to make tank mass small.
  - Capacitors: Can be as small as needed.
  - Hydrogen: Very light, so how to carry enough mass for value?

Some perspectives

- Nobody likes batteries, but so far they are the only viable alternative to liquid (or gaseous) fuel.
- A combustion process can only extract a fraction of the energy in a chemical fuel.
- Lead-acid battery energy density is about 1% of the usable energy in gasoline, and lithium-ion batteries will get to 10%.
- Sample test car: 275 kg battery pack → equivalent to 4 L of gas!



Energy needs

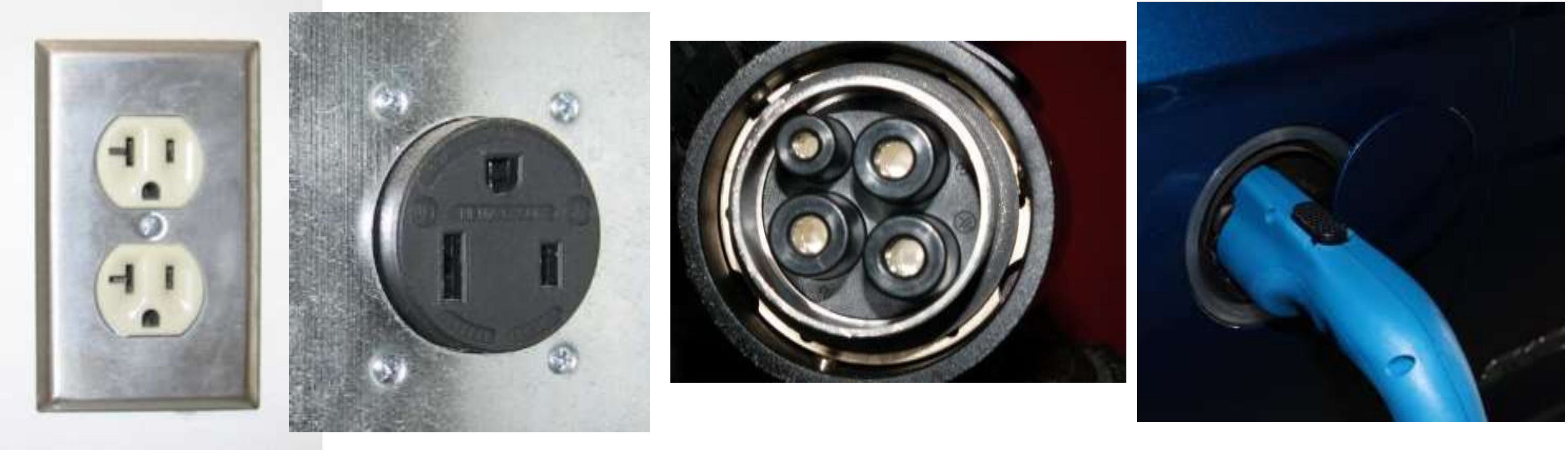
- The batteries will need to weigh a lot more than liquid fuel to provide useful range.
- We can talk about what constitutes “useful range,” but it would not necessarily match 1:1 against a fuel vehicle.
- Long uphill drives add a lot of energy need.
- Towing a trailer will increase drag and mass.

Rate and power needs

- Rate is a problem.
- Example: refill a gas tank with 15 gal in 5 min.
- The energy rate is that of a huge office building (6.7 MW).
- It is costly and problematic to fill batteries quickly.

Electric Charger Levels

- Level 1 – convenience outlet, 0 to 3.8 kW (typical 1.4 kW).
- Level 2 – dedicated charge point, 4 to 17 kW (typical 6 kW).
- Level 3 – fast charging, 25 kW and up (typical 50 kW).
  - Some are direct dc chargers.
  - Special charge points, not usual at homes or small businesses.



Electric Charger Levels

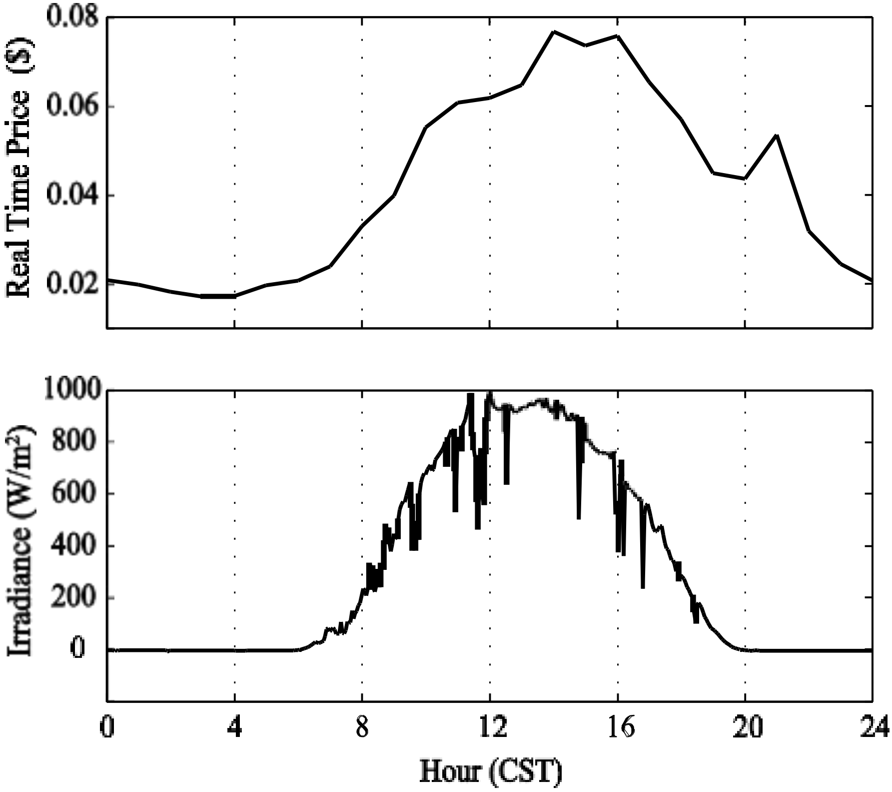
- Tesla Motors introduced the idea of thinking of these in terms of “miles per hour” (of useful charge).
- Level 1 – about 6 to 10 mph.
- Level 2 – about 25 to 60 mph.
- Level 3 – up to 500 mph.

Charge times

- This can result in a lot of confusion.
- Passenger cars are parked more than 80% of the time.
- You don’t park connected to a gas pump, but you *can* park connected to an electric outlet.
- 8 pm to 6 am: 10 hrs
  - Level 1 – 60 to 100 miles.
  - Level 2 – 250 to 600 miles.
  - Level 3 – not for overnight connection.
- I will not have this time in a long-distance trip, but likely for daily driving.

Incentive for long-term outlet connection

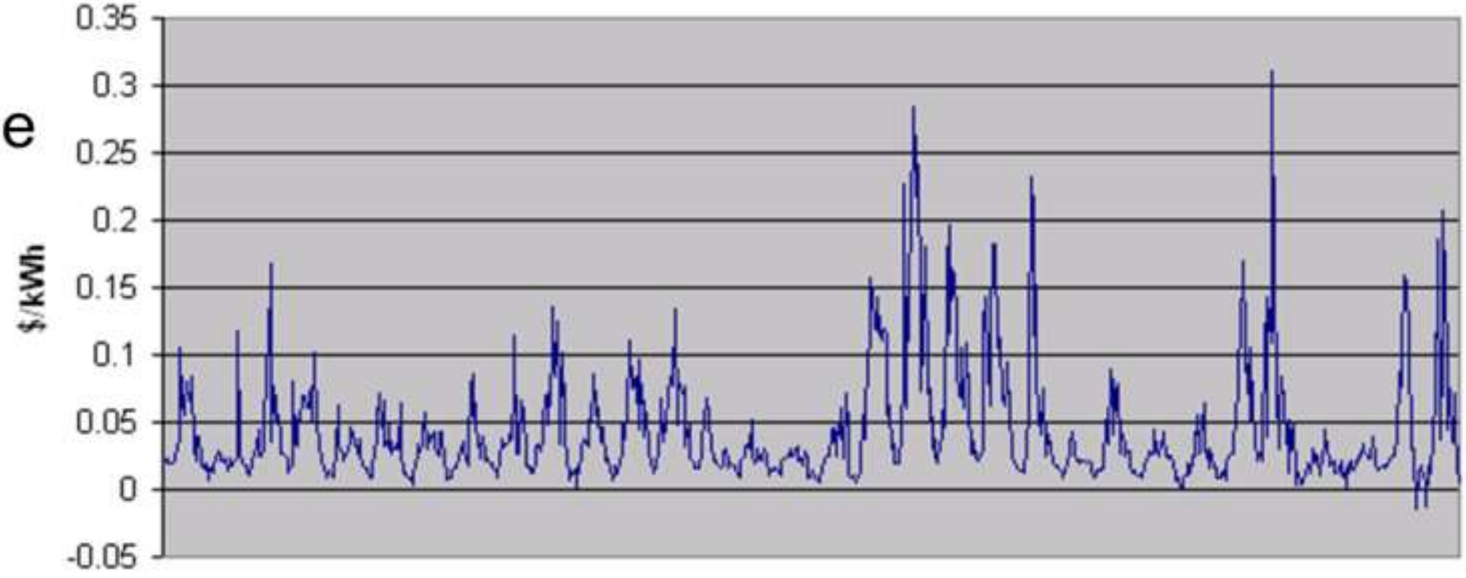
- Electricity is usually much cheaper at night than during the day. Likely 4x difference.
- If a vehicle can be programmed or controlled, it can choose to charge for the cheapest energy.
- Or sell timing flexibility back to the grid for a discount.



What does this mean?

- "It takes a long time to charge an electric vehicle."
  - – It takes a long *connection* time, but this is not the same as *driver* time.
- Rate matters in at least three major situations:
  1. Vehicles that park only a limited portion of time (trucking, taxi service, public transport, delivery, …).
  2. When I want to take a long trip.
  3. *When I have no access to an outlet.*

Retail real-time price, one month, Spring 2007



A quick economic check

- Take gasoline at $3.60/gallon, and a car that achieves 30 miles/gallon.
- Energy cost is $0.12/mile, and usage is about 1240 Wh/mi.
- Now take electricity at $0.12/kW-h, and a car that consumes 250 W-h/mile.
- Energy cost is $0.03/mile.
- Much cheaper with night charging.

This plug-in hybrid gives a choice.

Some points

- At a night rate of \$0.04/kW-h, cost becomes \$0.01/mile.
- Substantial incentive for the customer.
- But even at full retail electricity price, the energy is much cheaper than gasoline.
- Still the issue of range and long-range driving.

guardian.co.uk

Examples

- 2020 Nissan Leaf with 40 kWh battery pack.
- Data from the manufacturer, www.nissannews.com
- 6.6 kW charger on board – direct plug into 120 V or 240 V outlet.
- Drag and frontal area can be hard to find. Car and Driver reported 24.5 ft^2 (2.28 m^2) for the 2012 leaf in the June 6, 2014 issue. Nissan reports Cd = 0.28 for the 2020 model.
- Unloaded weight is 3540 lb (1606 kg), and fully loaded gross weight rating is 4750 lb (2156 kg).
- Let’s load it to 1800 kg and see what it needs!

Examples

- By the way, a more recent *Car and Driver* issue tells us the car can do 0 to 60 mph in 6.8 s.
- 60 mph → 26.8 m/s.
- 26.8 m/s in 6.8 s is 3.94 m/s^2, or 0.4g.
- The same test showed that the car could pull 0.76g in a tight turn before sliding. This gives us the tire friction coefficient, but only on their test pavement.

Examples

- 70 mph cruise, flat: Traction force 516 N, power 16.1 kW.
- 100 mph, flat: Traction force 905 N, power 40.5 kW.
- 0.3g acceleration, flat, 30 mph: Force 5850 N, power 78.5 kW.
- 0.4 g acceleration, flat, 60 mph: Force 7936 N, power 213 kW.
- Nissan reports a motor rating of 110 kW.
- 35 mph cruise, flat: 235 N, 3.67 kW.
- 5% grade at 72 mph: 1420 N, 45.7 kW.

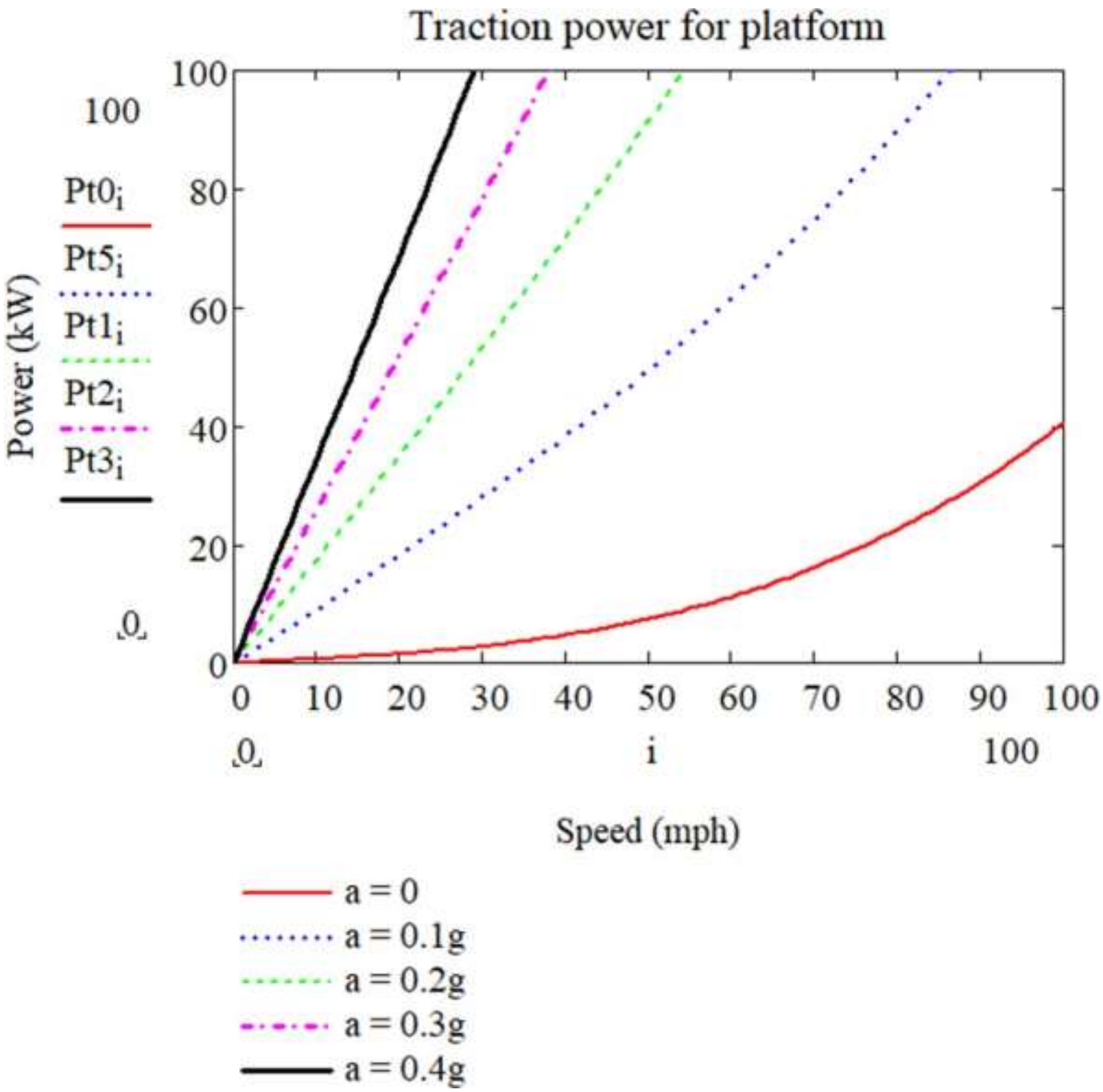
Examples

- What energy?
- 70 mph cruise, 90% drivetrain efficiency, plus 1 kW hotel load, for example:
- 16.1 kW/0.9 + 1 kW = 18.9 kW input, so 18.9 kWh/h, equal to 270 Wh/mile.
- Range based on using 80% of storage is 119 miles.
- “EPA range” is 149 miles, but this is for a combined city and highway test, not cruise at speed.
- 75 mph cruise, 19.1 kW traction, 22.3 kW input, 297 Wh/mile.
- Range based on using 80% of storage is 108 miles.
- 45 mph cruise, 170 Wh/mile, 189 miles of range.

Examples

- Move up a 30% grade: 5440 N (at zero speed)
- From experience, power to provide 2.5 m/s^2 at mph should be representative of top end power needs.
- Here that gives 112 kW (same as motor rated power!).
- Maximum speed, flat, 110 kW: 143.5 mph.
  - This defines “top speed” for a vehicle – continuous rating, maximum speed.
- Look like this vehicle runs at 200% rated power for a few seconds to achieve its 6.8 s time.

Examples


Traction power for platform
Power (kW)
100
Pt0i
Pt5i
Pt1i
Pt2i
Pt3i
0
100
80
60
40
20
0
0
10
20
30
40
50
60
70
80
90
100
0
i
100
Speed (mph)
a = 0
a = 0.1g
a = 0.2g
a = 0.3g
a = 0.4g

Examples

- Typical Class 8 tractor-trailer with cab fairing
- Data from NASA Ames and others.
- Frontal area 9.89 m^2, Cd 0.60, tire resistance 0.007.
- Load to 60000 lb.

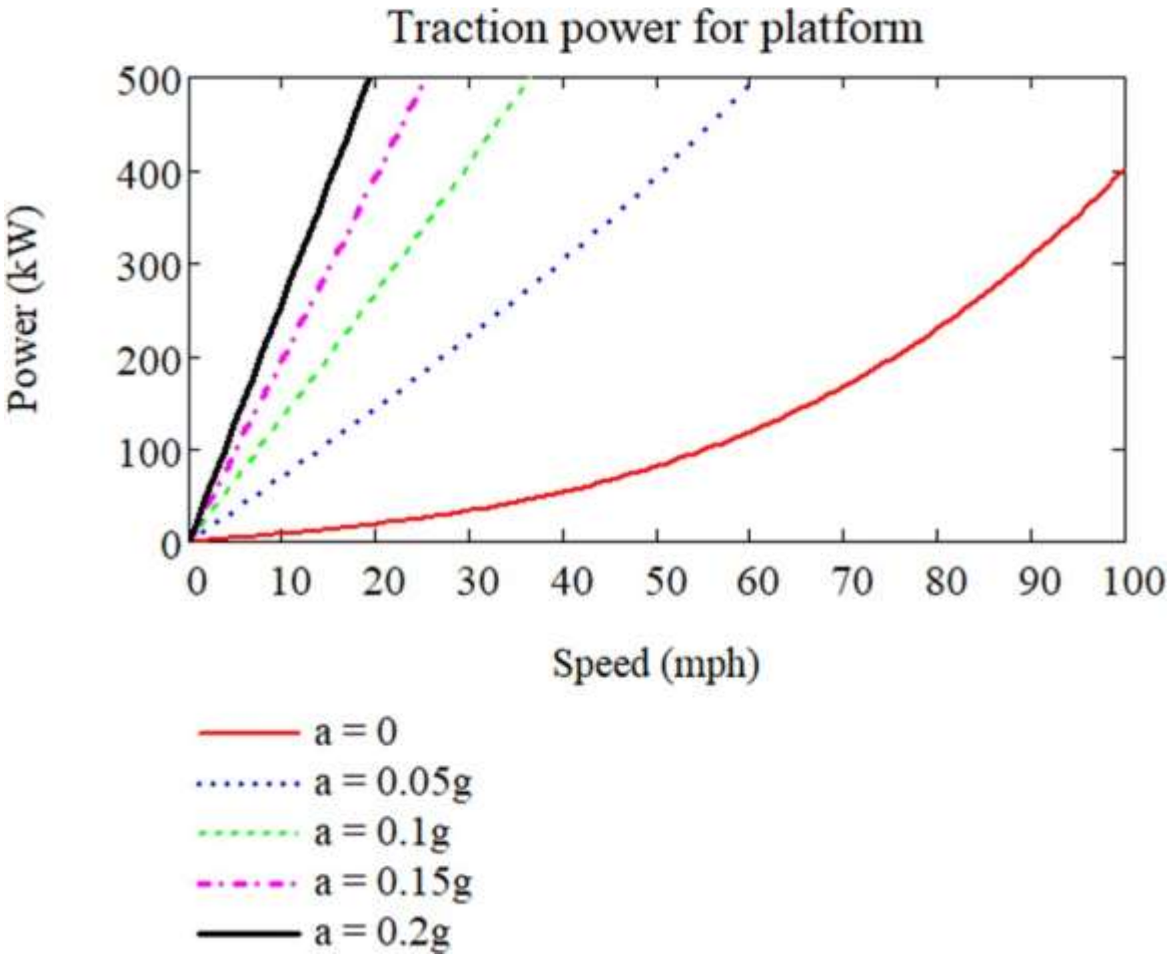
Examples

- 70 mph cruise, flat: Traction force 5356 N, power 168 kW.
- 100 mph, flat: Power 402 kW.
- 0.3g acceleration, flat, 30 mph: Force 86.3 kN, power 1.16 MW.
- 0.1 g acceleration, flat, 60 mph: Force 32.4 kN, power 868 kW.
- 5% grade at 72 mph: 18.9 kN, 608 kW.

Examples

- What energy?
- 70 mph cruise, 90% drivetrain efficiency, plus 1 kW hotel load, for example:
- 168 kW/0.9 + 1 kW = 187 kW input, so 18.9 kWh/h, equal to 2675 Wh/mile.
- 75 mph cruise, 197 kW traction, 220 kW input, 2930 Wh/mile.
- About 10x compared to passenger car.
- Implies 10x for storage needs!

Examples

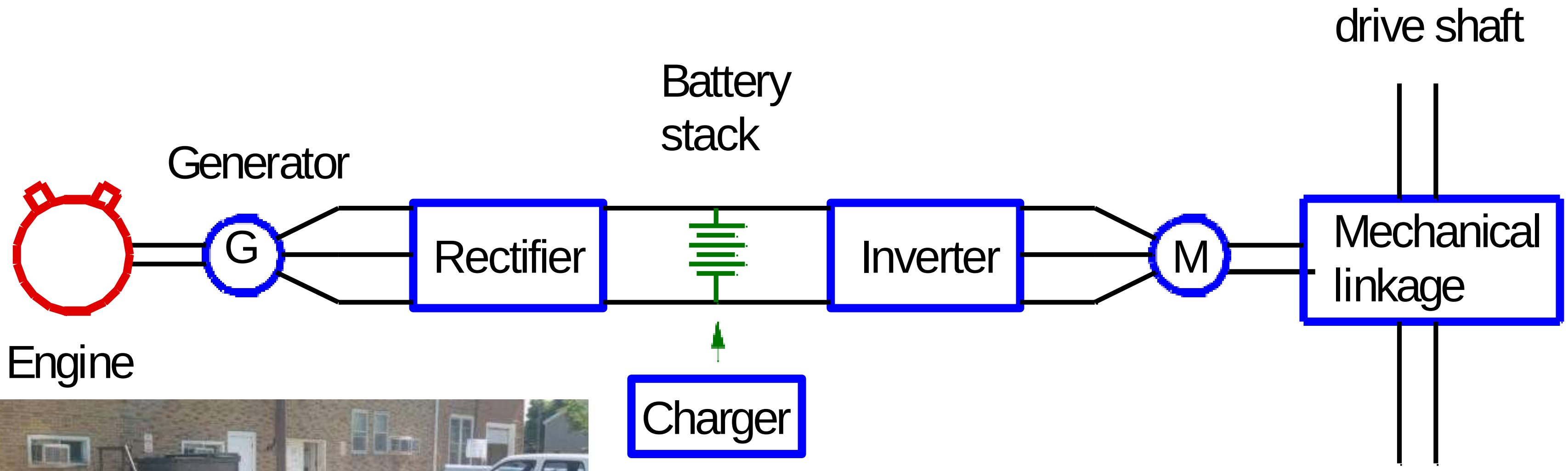

Traction power for platform
Power (kW)
0
100
200
300
400
500
Speed (mph)
0
10
20
30
40
50
60
70
80
90
100
a = 0
a = 0.05g
a = 0.1g
a = 0.15g
a = 0.2g

Energy

- Could we store 700 kWh on board a truck?
  - Diesel at 45000 kJ/kg. If we convert 30% of it, a truck can easily store 300 kg of fuel and has about 4 GJ, or 1125 kWh of useful energy stored up.
  - For Li-ion batteries, at a (future) 1000 kJ/kg, 700 kWh is 2.52 GJ, requiring 2520 kg of batteries (5500 lb).
  - This is not impossible (about 10% of the loaded mass), but it is a future development.

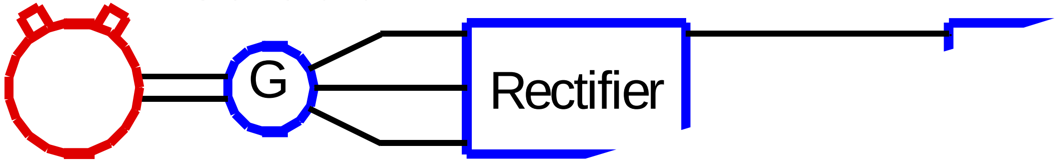
Baseline Architecture

- Generic architecture for a “series” hybrid electric vehicle.
- This means the energy comes together at an electrical bus.



Baseline Architecture

- Example: Diesel-electric train. Same (with ac motors) but no batteries.



drive sha

Battery stack

Generator

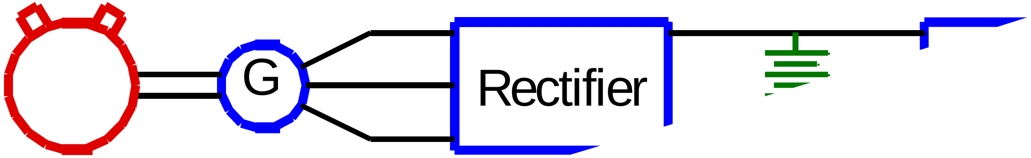
G

Rectifier

Engine

Baseline architecture

- Typical for plug-in hybrid.



drive sha

Battery
stack

Generator

G

Rectifier

Engine

Battery electric vehicle

- Pure EV, larger battery, typically an on-board charger.