ECE 398GG – ELECTRIC VEHICLES

5. Energy Conservation Principle

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ENERGY CONSERVATION PRINCIPLE

- ❑ Energy is, to some extent, an abstract term; for our purposes, we may view energy as *work*
- ❑ An unalterable characteristic of energy is its *invariance*: the total energy in the universe remains unchanged over time

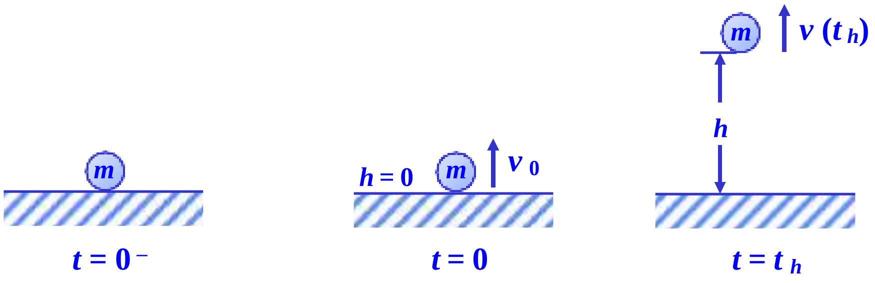
ENERGY CONSERVATION PRINCIPLE

- ☐ The *principle of energy conservation* underlies all of nature's *physical, chemical* and *biological processes*; the principle is, essentially, a *very general physical law* that is based on the work of the 19th century *British* physicist *Joule*
- ☐ Indeed, we derive for a purely *mechanical system* the *energy conservation principle* from the direct application of *Newton's laws of motion*

ENERGY CONSERVATION PRINCIPLE

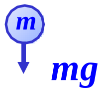
- ☐ We examine a very simple example in which a mass m, initially on the ground at time 0^-, is thrown vertically upwards at the speed v_0 at $t = 0$
- ☐ We determine the relationship between v_0 and the speed $v(t_h)$ at the time t_h, at which, the mass reaches the height h

ENERGY CONSERVATION PRINCIPLE



ENERGY CONSERVATION PRINCIPLE

- We start with *Newton's Second Law* and apply the relationship $F = ma$ to our simple system



- Clearly,

$$F = -mg$$

- Let z be the vertical distance traveled by mass m and so its velocity is $\frac{dz}{dt}$ and acceleration is $\frac{d^2z}{dt^2}$

ENERGY CONSERVATION PRINCIPLE

- At each instant *t*,

$$m\frac{d^2z}{dt^2} = F = -m\,g$$

so that

$$\frac{d^2z}{dt^2} = -\,g \qquad (*)$$

- The second order differential equation $(*)$ has initial conditions

$$z(0) = 0 \quad \textit{and} \quad \left.\frac{dz}{dt}\right|_{t=0} = v_0 \qquad (**)$$

ENERGY CONSERVATION PRINCIPLE

- Since

$$\frac{dz}{dt} = v(t)$$

then

$$\frac{dv}{dt} = \frac{d}{dt}\left(\frac{dz}{dt}\right) = \frac{d^2z}{dt^2} = -\;g \qquad (**)$$

- We obtain the solution for *v*(*t*) from the integration

$$\int_0^t\left(\frac{dv}{dt}\right)dt = \int_{v(0)}^{v(t)} dv = v(t) - v(0) = -\;g\,t \qquad (***)$$

ENERGY CONSERVATION PRINCIPLE

- ❑ **But**

$$v(t) = \frac{dz}{dt} = v_0 - g\,t$$

and upon integration

$$z(t) - z(0) = v_0\,t - \frac{1}{2}g\,t^2$$

- ❑ **At** $t = t_h, z(t_h) = h$ **and so**

$$h = v_0 t_h - \frac{1}{2} g \left[t_h \right]^2 \qquad (****)$$

$$v(t_h) = v_0 - g\,t_h \qquad (*****)$$

ENERGY CONSERVATION PRINCIPLE

- ❑ **From** $(*****)$

$$t_h = \frac{v_0 - v(t_h)}{g}$$

so that $(****)$ **obtains**

ENERGY CONSERVATION PRINCIPLE

$$h = v_0 \frac{v_0 - v(t_h)}{g} - \frac{1}{2} g \cdot \frac{\left[v_0 - v(t_h)\right]\left[v_0 - v(t_h)\right]}{g \cdot g}$$

$$= \frac{v_0 - v(t_h)}{2g} \left\{ 2v_0 - \left[v_0 - v(t_h) \right] \right\}$$

$$= \frac{v_0 - v(t_h)}{2g} \left[v_0 + v(t_h) \right]$$

$$= \frac{1}{2g} \left[v_0^2 - \left[v(t_h) \right]^2 \right]$$

ENERGY CONSERVATION PRINCIPLE

- **We rearrange the equation to obtain**

$$gh = \frac{1}{2} v_0^2 - \frac{1}{2} \left[v(t_h) \right]^2$$

and multiply by *m* to state the conservation effect

$$\frac{1}{2} m \left[v(t_h) \right]^2 = \frac{1}{2} m v_0^2 - mgh \qquad (\dagger)$$

ENERGY CONSERVATION PRINCIPLE

☐ We associate the relationship with energy since the *kinetic energy* of mass m with speed $v(t)$ at time t is $\frac{1}{2} m \left[v(t) \right]^2$; also, the potential energy of mass m at height h is mgh

☐ Therefore, we may restate $(\dagger)$ for height h as

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m \left[v\left(t_h\right) \right]^2 + mgh \qquad (*\dagger)$$

ENERGY CONSERVATION PRINCIPLE

☐ In words, at an arbitrary height h

$$\left. kinetic\ energy \right|_h + \left. potential\ energy \right|_h = \underbrace{\left. kinetic\ energy \right|_0}_{constant}$$

☐ At two arbitrary values of h, say h' and h''

$$\left. kinetic\ energy \right|_{h'} + \left. potential\ energy \right|_{h'} = \left. kinetic\ energy \right|_{h''} + \left. potential\ energy \right|_{h''}$$

☐ We derived in a straightforward way that the total energy of mass m is invariant over time

APPLICATION: ENERGY CONVERSION

- ❑ The *energy conservation principle* holds also when we convert energy from one form into another form
- ❑ We consider a hydroelectric system whose reservoir stores water behind a dam at the specified height *h*: the water flows through a penstock and drives a turbine, whose rotor is connected via a mechanical shaft to the electrical generator

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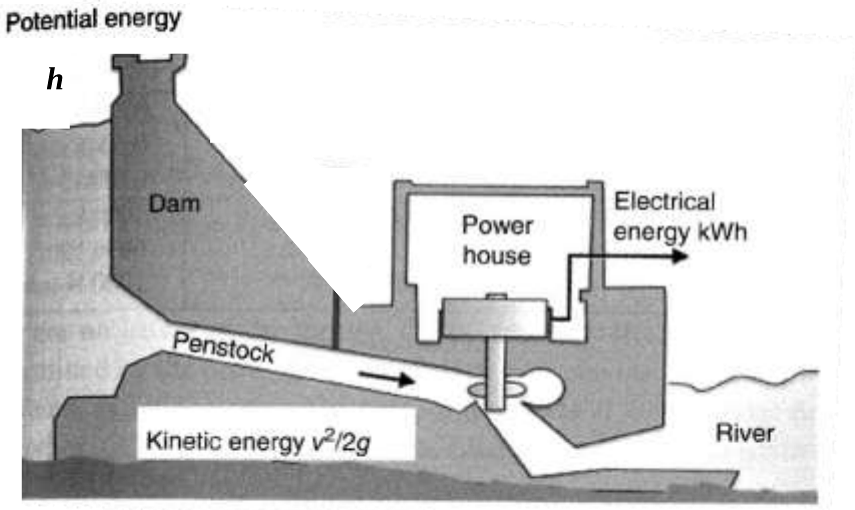
HYDROELECTRIC ENERGY GENERATION



HYDROELECTRIC ENERGY GENERATION

- The water at the height h – typically, called the *head h* – has *potential energy*, which gets converted into *mechanical energy* as the water flows through the penstock; the *mechanical energy* drives the turbine and is converted into *electric energy* by the generator
- Each unit volume of water in the reservoir has mass ρ, where ρ is the density of water, and so has *potential energy* $\rho g h$

HYDROELECTRIC ENERGY GENERATION

- As each unit traverses the penstock, its *potential energy* is converted into *kinetic energy* whose value depends on v; upon arrival at the turbine, each unit of volume of water has *kinetic energy* $\frac{1}{2}\rho v^{2}$
- We assume that the pressure energy is negligibly small and there are no losses in the system, *i.e.*, a frictionless penstock – so that the *energy conservation law* for the unit volume of fluid mass results in

HYDROELECTRIC ENERGY GENERATION

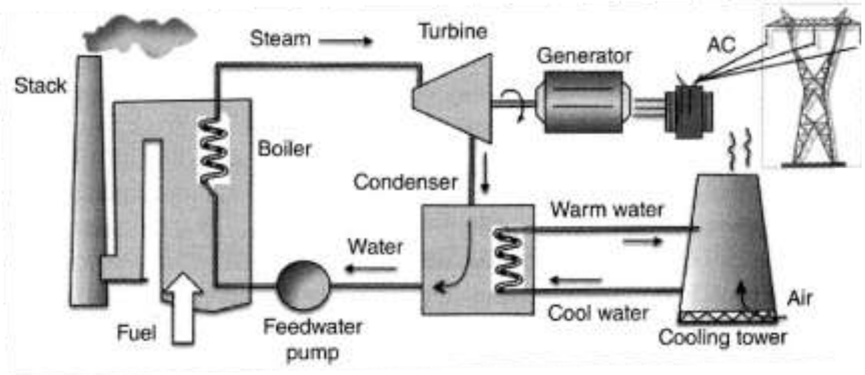
$$\rho g h = \frac{1}{2} \rho v^2$$

- ❑ The *energy conservation law* applies to each energy conversion process; as a particular process has losses due to inefficiencies of the process, some of the energy is converted into such losses and, as a result, the process efficiency is reduced

HYDROELECTRIC ENERGY GENERATION

- ❑ For wind turbines, the wind speed air mass has kinetic energy, which rotates the wind turbine that is connected to the rotor of an electric generator and converts that kinetic energy into electricity
- ❑ Similar notions hold, for example, for a steam generation plant that uses some fossil fuel for combustion:

FOSSIL – FUEL FIRED STEAM GENERATION PLANT



EFFICIENCY NOTIONS

- ☐ The engineering notion of efficiency is focused on the measure of two quantities
  - ○ input
  - ○ output
- ☐ For a given system with an input and an output

input → *system* → *output*

EFFICIENCY NOTIONS

we define efficiency, denoted by η, as

$$\eta \;\; ! \;\; \frac{output}{input}$$

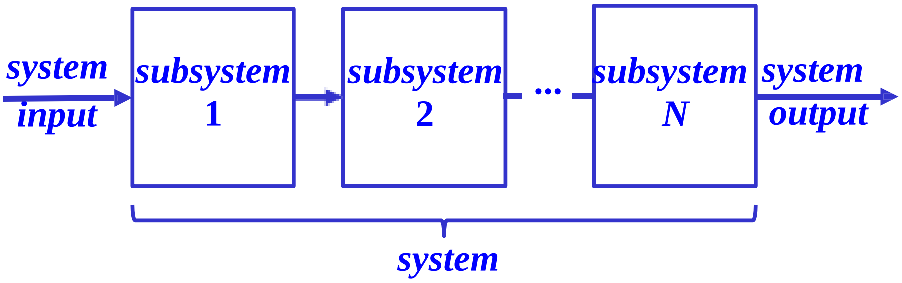
where both input and output are measured in the same units and so η is *unitless*

- ❑ **Many engineers prefer to express efficiency in *%* and so**

$$\eta_{\%} = \eta \times 100 \; \%$$

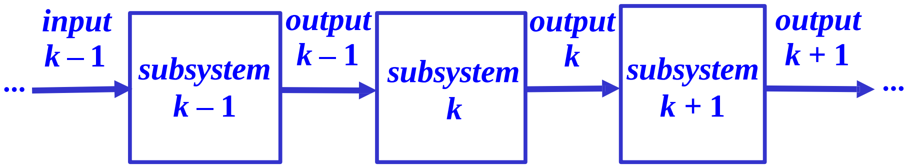
EFFICIENCY NOTIONS

- ❑ **Systems may be complex and comprise of multiple subsystems: for example, consider a string of *N* subsystems connected in series**



EFFICIENCY NOTIONS

- The relationships between two sequential subsystems $k - 1$ and k in this system we are



readily apparent; in particular,

$$\textit{output } k = \textit{input } k + 1 \qquad k = 0, 1, \ldots, N - 1$$

EFFICIENCY NOTIONS

or, equivalently

$$\textit{input } k = \textit{output } k - 1 \qquad k = 1, 2, \ldots, N$$

with

$$\textit{input } 1 = \textit{system input}$$

$$\textit{output } N = \textit{system output}$$

- The efficiency η_k of each subsystem k is given by

$$\eta_k = \frac{\textit{output } k}{\textit{input } k} \qquad k = 1, 2, \ldots, N$$

EFFICIENCY NOTIONS

- ❑ The overall system efficiency is obtained by the application of the relations above

$$
\begin{aligned}
\eta_{system} &= \frac{system\ output}{system\ input} \\
&= \frac{output\ N}{input\ 1} \\
&= \frac{output\ N}{input\ N} \times \frac{output\ N-1}{input\ N-1} \times \cdots \times \frac{output\ 1}{input\ 1} \\
&= \eta_1 \times \eta_2 \times \cdots \times \eta_N
\end{aligned}
$$

APPLICATION OF EFFICIENCY NOTIONS

- ❑ Consider an energy focused process that consists of multiple individual stages *connected in series*: we consider each stage to be a *subsystem* in a system of connected subsystems in series to represent the multi-stage process and evaluate its overall efficiency

APPLICATION OF EFFICIENCY NOTIONS

- For each stage, we use the *Energy Conservation Principle* to examine the energy consumption and losses to evaluate the inputs and outputs
- As an example, we consider the process to deploy a fossil fuel – be it gasoline or diesel – in an *ICEV* to drive a specific distance; we refer to the entire sequence of stages that make up this process as the *well–to–wheels* process

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THE *WELLS-TO-WHEELS* PROCESS

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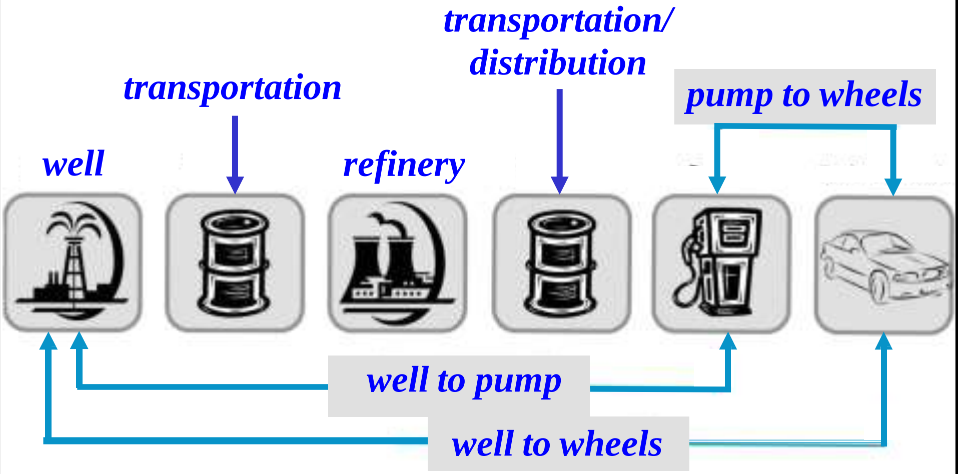

THE *WELLS-TO-WHEELS* PROCESS

- **The well output – called the feedstock – is the initial stage of the process, which includes the remaining stages to reach the *ICEV* wheels**
- **Efficiency is a key engineering/economic metric to assess the effectiveness of the overall process**
- **The energy consumption and attendant losses incurred at each stage are key measures that we use in the determination of the both the *well-to-wheels* efficiency and the associated emissions**