ECE 398GG – ELECTRIC VEHICLES

10b. Complex Power Flow Concepts

Prof. Andrew Stillwell

Department of Electrical and Computer Engineering

University of Illinois at Urbana-Champaign

EV Charging Pt 2

Thursday, March 10, 2022 9:11 AM

Today:

- Complex Power
- 3 Phase Power
- Simple Power Flow

Last time: (Review)

→ Phasors

For AC systems ($v(t) = \sqrt{2} \cdot V \cdot \cos(\omega t + \theta_v)$) $V = V_{RMS}$

Can solve circuits algabraically with phasors

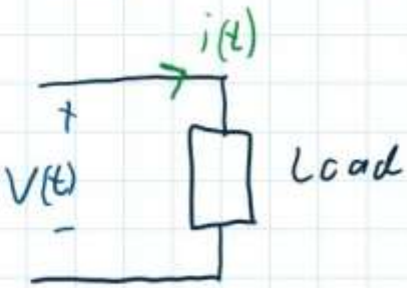
→ $V(t) = \sqrt{2} \cdot V \cdot \cos(\omega t + \theta_v) \iff \bar{V} = V \angle \theta_v$ ←

$\bar{J} = I \angle \theta_i$

P, average Power for a resistor

$$P = \frac{V^2}{R} = I^2 \cdot R$$

Once we introduce reactive elements (L, C)



$$P = V \cdot I \cdot \cos(\theta_v - \theta_i)$$

$VI \triangleq S$ = Apparent Power [VA]

$\cos(\theta_v - \theta_i)$ = Power factor

Complex Power : $\bar{S}$

$$\bar{S} \triangleq P + jQ$$

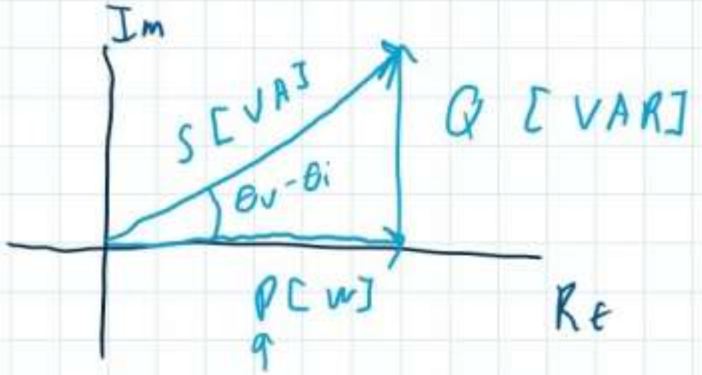
Q = Reactive Power ⇒ "Power" supplied / Absorbed by reactive elements [VAR]

↳ $Q_C = \frac{V_C^2}{|\bar{Z}_C|}$ $Q_L = I_L^2 \, |\bar{Z}_L|$

↳ $Q_C = \frac{V_C^2}{|\bar{Z}_C|}$ $\quad Q_L = I_L^2 |Z_L|$

↳ $\underline{Q}$ is still very important (ECE 476)

Can represent Complex power with Power triangle



$\bar{S} = P + jQ$ - complex

→ $S = \sqrt{P^2 + Q^2}$ - Apparent

$\cos(\theta_v - \theta_i) = \frac{P}{S}$

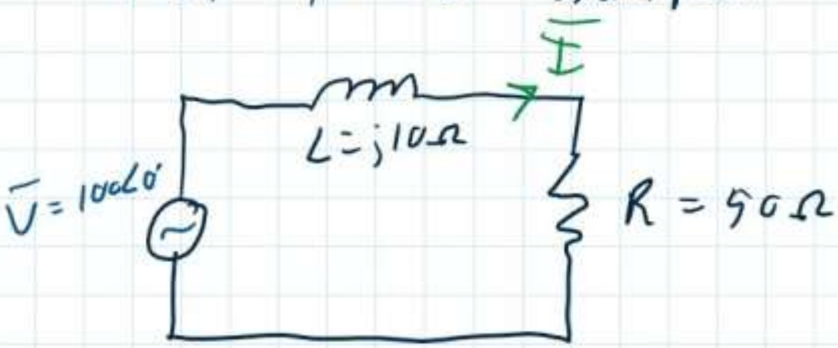
$\underline{P = S \cdot \cos(\theta_v - \theta_i)}$ ✓

$\underset{V \cdot I}{\uparrow}$

$\sin(\theta_v - \theta_i) = \frac{Q}{S}$

$\underline{Q = S \cdot \sin(\theta_v - \theta_i)}$

Back to our example:



$\bar{I} = 1.96 \angle -11.3°$ A $\quad \underline{P} = 192$ W $\quad V \cdot I = 196$ VA $= \underline{S}$

Q_L? $\quad Q_L = I^2 \cdot \omega L = 1.96^2 \cdot 10 = 38.4$ VAR

or $S^2 = P^2 + Q^2 \Rightarrow Q = \sqrt{S^2 - P^2} = 38.4$ VAR

or $Q = S \cdot \sin(\theta_v - \theta_i) = 196 \sin(0 + 11.3°) = 38.4$ VAR

→ $\bar{S} = P + jQ = 192 + j38.4 = 196 \angle 11.3°$

$\rightarrow \bar{S} = P + jQ = 192 + j38.4 = 196\angle 11.3^\circ$

$\bar{S} = \bar{V}\cdot\bar{I}^* = 100\angle 0^\circ \times 1.96\angle +11.3^\circ = 196\angle +11.3^\circ$

Measurements

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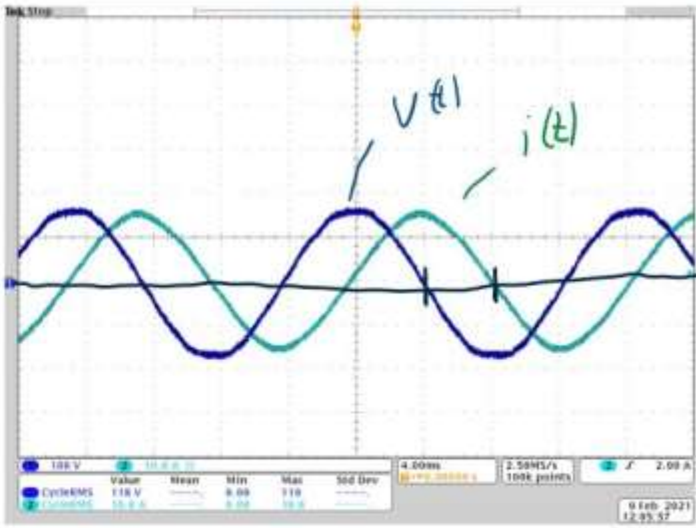
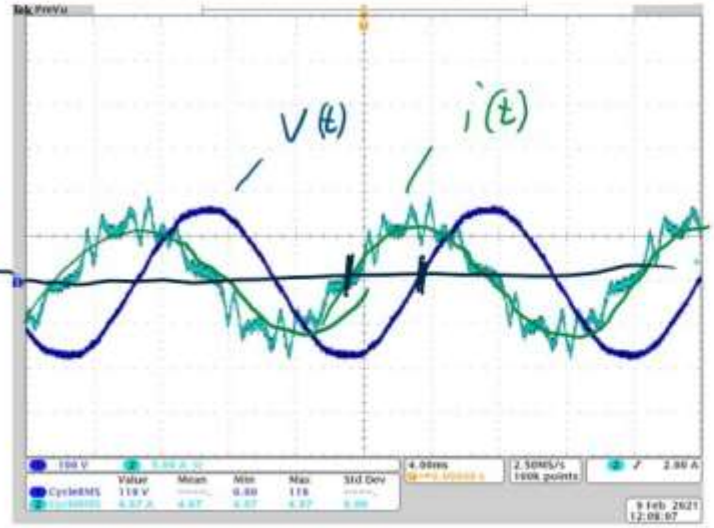
Inductor Load



Capacitor Load



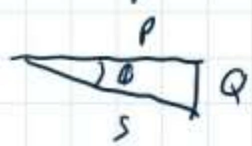
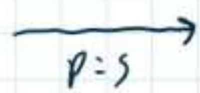
Load Based Notation

$\bar{Z}_L = = j\omega L = \omega L \angle 90°$

$\bar{Z}_C = 1/j\omega C = 1/\omega C \angle -90°$

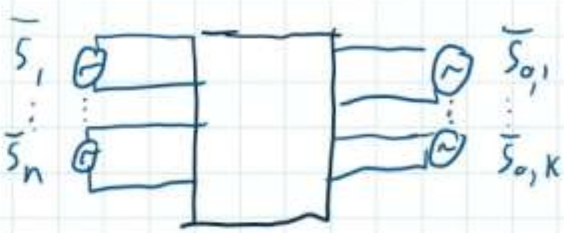
$Q > 0 \Rightarrow$ Load consumes VARS ←

$Q < 0 \Rightarrow$ Load supplies VARS

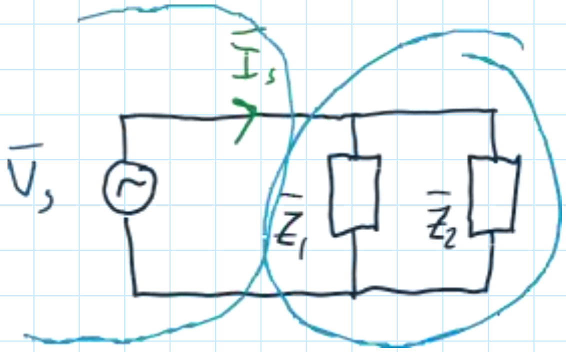
Loads	$\phi = \theta_v - \theta_i$	PF	Power triangle	Load Q
Inductive	> 0	< 1 "Lagging"		> 0
capacitive	< 0	< 1 "Leading"		< 0
Resistive	$= 0$	1		0

Power triangle labels: S, Q, P, ϕ; resistive: $P = S$

Conservation of complex Power



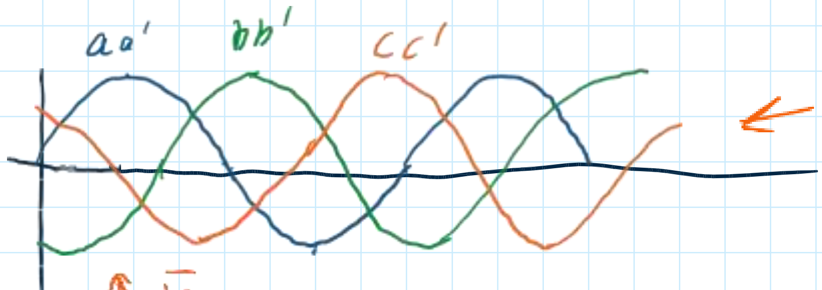
Complex power into network = Complex power out of network

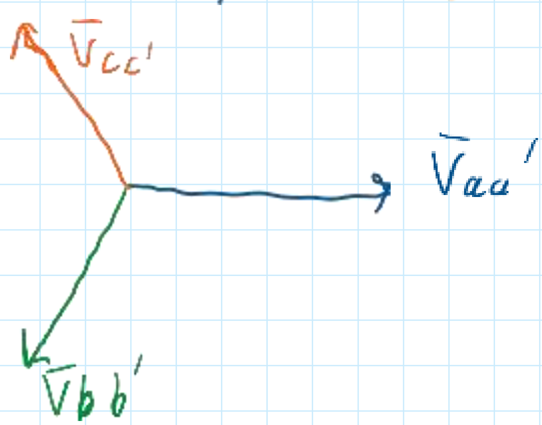


$$\bar{S}_s = \bar{S}_1 + \bar{S}_2 = \bar{V}_s \cdot \bar{I}_s^*$$

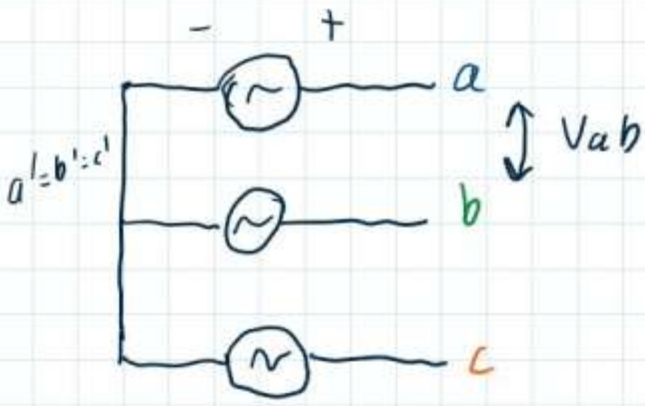
3 Phase Power

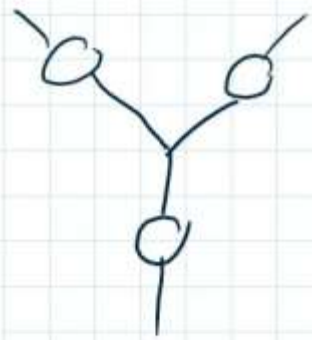
$$\left.\begin{aligned} \bar{V}_{aa'} &= V \angle \theta_v \\ \bar{V}_{bb'} &= V \angle \theta_v - 120^\circ \\ \bar{V}_{cc'} &= V \angle \theta_v + 120^\circ \end{aligned}\right\} \text{Positive Sequence}$$





Three Phase Connections

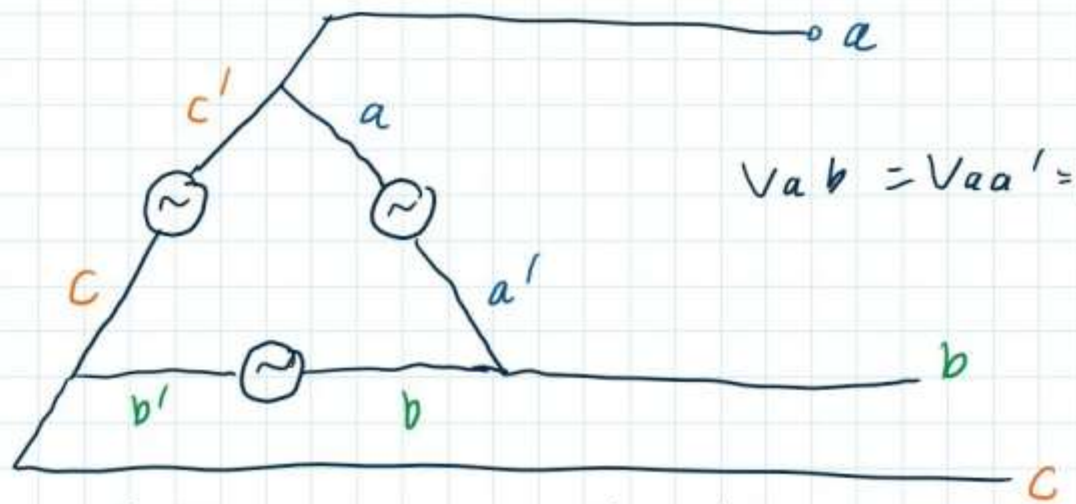




WYE
Connection
(Y)

$V_{ab} = \sqrt{3} \cdot V_{aa'}$

$V_{Line} = \sqrt{3} \cdot V_{phase}$

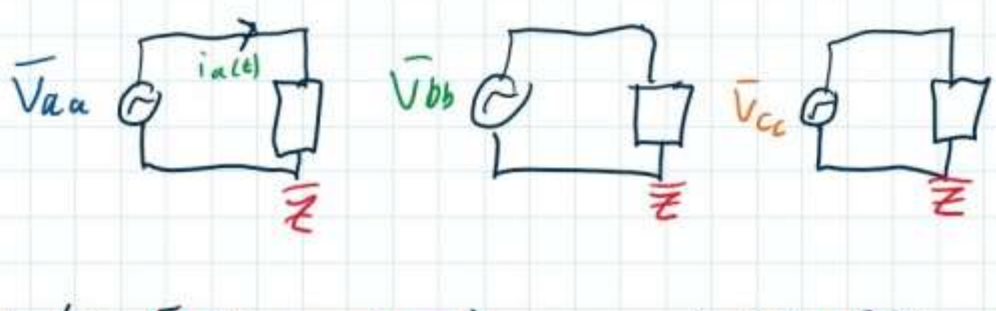


$V_{ab} = V_{aa'} \Rightarrow V_L = V_{phase}$

Delta connection (Δ)

why three phase ?
↳ if balanced, no neutral line current

Just as important for Power converters! $P(t)$



$V_{aa'}(t) = \sqrt{2} \cdot V \cdot \cos(\omega t)$ $\quad$ $i_a(t) = \sqrt{2} \cdot I \cdot \cos(\omega t - \phi)$

$V_{bb'}(t) = \sqrt{2} \cdot V \cdot \cos(\omega t - 120°)$ $\quad$ $i_b(t) = \sqrt{2} \cdot I \cdot \cos(\omega t - \phi - 120°)$

$V_{cc'}(t) = \sqrt{2} \cdot V \cdot \cos(\omega t + 120°)$ $\quad$ $i_c(t) = \sqrt{2} \, I \cdot \cos(\omega t - \phi + 120°)$

$V_{bb'}(t) = \sqrt{2}\cdot V\cdot \cos(\omega t - 120°)$ $\qquad$ $i_b(t) = \sqrt{2}\cdot I \cdot \cos(\omega t - \phi - 120°)$

$V_{cc'}(t) = \sqrt{2}\cdot V\cdot \cos(\omega t + 120°)$ $\qquad$ $i_c(t) = \sqrt{2}\, I \cdot \cos(\omega t - \phi + 120°)$

$$P_a(t) = V\cdot I \cdot [\cos(\phi) + \cos(2\omega t - \phi)]$$

$$P_b(t) = V\cdot I\,[\cos(\phi) + \cos(2\omega t - \phi - 240°)]$$

$$P_c(t) = V\cdot I\,[\cos(\phi) + \cos(2\omega t - \phi + 240°)]$$

$$P(t) = P_a(t) + P_b(t) + P_c(t)$$

$$= 3V\cdot I\cos\phi + \underbrace{V\cdot I\,[\cos(2\omega t - \phi) + \cos(2\omega t - \phi - 240°) + \cos(2\omega t - \phi + 240°)]}_{=0}$$

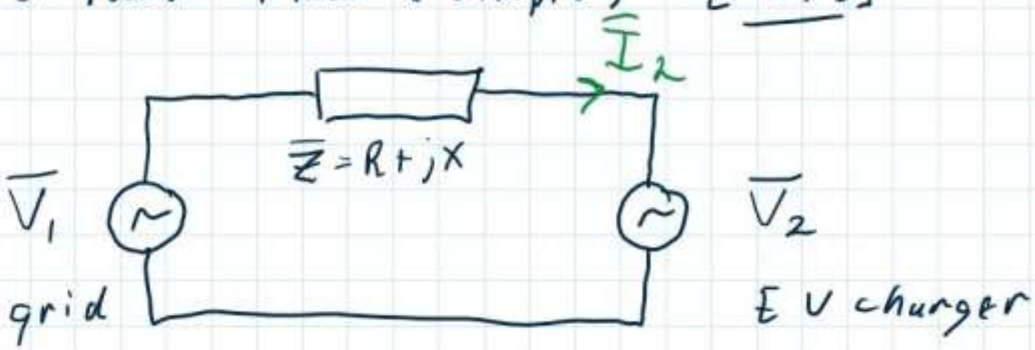
*

$$\boxed{P(t) = 3\cdot V\cdot I\cdot\cos\phi} = P$$

Instantaneous power is actually constant!!!

can also show: $\bar{S} = 3\cdot V\cdot I\cos(\phi) + 3\cdot V\cdot I\sin(\phi)$

AC Power Flow (Simple) [476]



→ assume line is lossless

$$\bar{Z} = jX = Z e^{j90°} = Z\angle 90° \quad (Z = X = \omega L)$$

$\bar{V}_1 = V_1\angle\theta_1$, $\bar{V}_2 = V_2\angle\theta_2$

$\bar{S}_2 = \bar{V}_2\cdot\bar{I}_2^*$ $\qquad$ $\bar{I}_2 = \dfrac{\bar{V}_2 - \bar{V}_1}{\bar{Z}}$

$S_2 = V_2 \cdot I_2^* \qquad I_2 = \dfrac{V_2 - V_1}{Z}$

$$= \bar{V}_2 \cdot \frac{\bar{V}_2^* - \bar{V}_1^*}{\bar{Z}^*} = V_2 \angle \theta_2 \cdot \frac{V_2 \angle -\theta_2 - V_1 \angle -\theta_1}{Z \angle -90^\circ}$$

$$\bar{S}_2 = \frac{V_2^2}{Z} \angle 90^\circ - \frac{V_1 \cdot V_2}{Z} \angle \theta_2 - \theta_1 + 90^\circ$$

From Euler's $\quad e^{j\theta} = \underbrace{\cos\theta}_{P} + j \underbrace{\sin\theta}_{Q}$

$$P_2 = \frac{V_2^2}{Z} \cdot \underbrace{\cos(90^\circ)}_{0} - \frac{V_1 V_2}{Z} \cos(\theta_2 - \theta_1 + 90^\circ)$$

$$\boxed{P_2 = \frac{V_1 \cdot V_2}{Z} \cdot \sin(\theta_2 - \theta_1)}$$