ECE 398GG – ELECTRIC VEHICLES

10a. AC Analysis

Prof. Andrew Stillwell

Department of Electrical and Computer Engineering

University of Illinois at Urbana-Champaign

EV Charging Part 1

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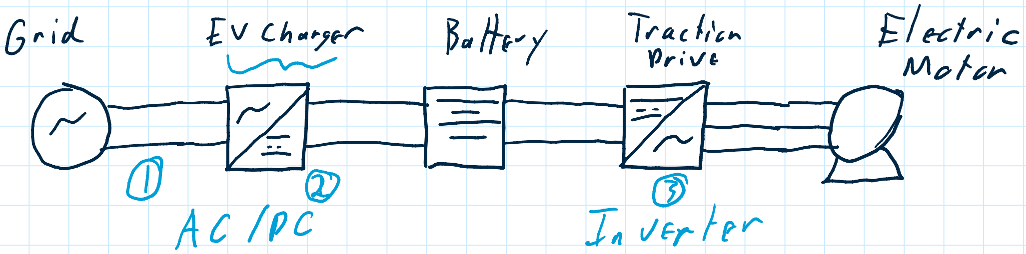
Today:

- EV Charging System Overview
- AC Circuits

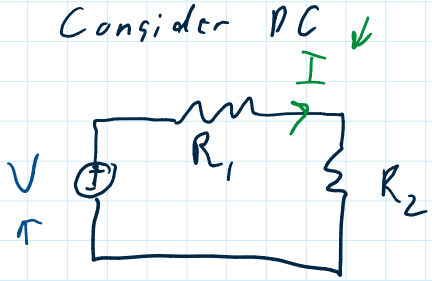
High Level system:

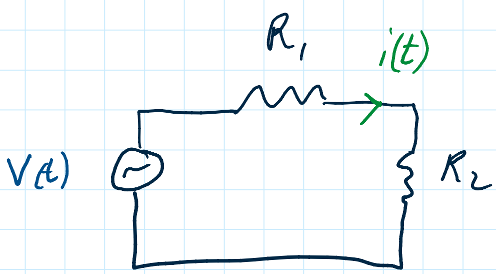


1. AC Power flow
2. AC/DC Power Conversion
   ↳ DC/DC Power conversion
3. we learn about this for free
   ↳ similar to 2

AC circuit analysis

Consider DC





$V = I(R_1 + R_2)$

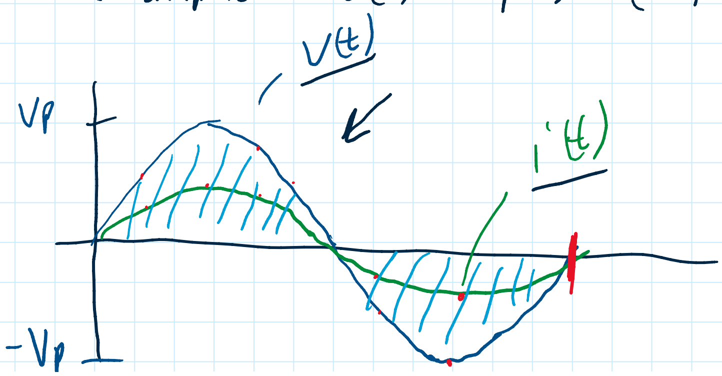
$P = V \cdot I$

↳ Algebraic

$V(t) = i(t)(R_1 + R_2)$

$P(t) = V(t) \cdot i(t)$

example: $V(t) = V_P \cdot \sin(\omega t)$ $\omega = 2\pi \cdot f$

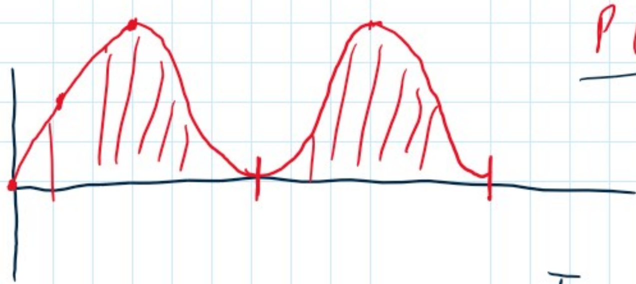


$i(t) = V(t)/(R_1 + R_2)$

$i(t) = I_P \cdot \sin(\omega t)$

$I_P = V_P/(R_1 + R_2)$

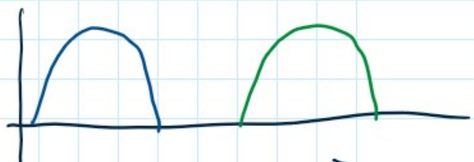
$P(t) = V(t)\cdot i(t)$



Typically want Average Power

$$P = \langle P(t)\rangle = \frac{1}{T}\int_0^T P(t)\,dt \qquad T = \frac{1}{f}$$

How do we find P, given v(t), i(t)



$$\langle P(t)\rangle = \frac{1}{T}\int_0^T P(t)\,dt = \frac{1}{T}\int_0^T v(t)\cdot i(t)\,dt$$

$$V(t) = i(t)\cdot R$$

$$P = \frac{1}{T}\int_0^T i(t)^2\cdot R\,dt = \underbrace{\frac{1}{T}\int_0^T i(t)^2\,dt}\cdot R$$

$$P = I_{RMS}^2\cdot R$$

$$\boxed{I_{RMS} = \sqrt{\frac{1}{T}\int_0^T i(t)^2\,dt}}$$

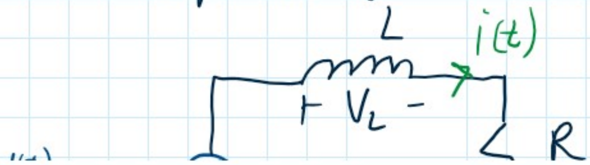
Root Mean Squared

$$P = I_{RMS}^2\cdot R = \frac{V_{RMS}^2}{R}$$

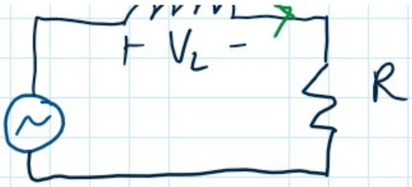
for $v(t) = V_p\cdot\cos(\omega t + \theta_v)$ or $i(t) = I_p\cdot\cos(\omega t + \theta_i)$

$$V_{RMS} = \frac{V_P}{\sqrt{2}} = V \qquad I_{RMS} = \frac{I_P}{\sqrt{2}} = I$$

$$V_P = \sqrt{2}\cdot V \qquad I_P = \sqrt{2}\cdot I$$



$$V(t) = L\frac{di(t)}{dt} + R\cdot i(t)$$



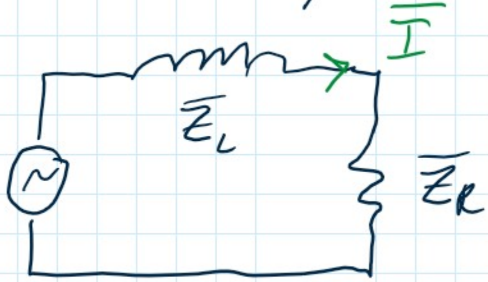
$V(t) = \sqrt{2} \cdot V \cos(\omega t)$

$$\underbrace{V(t) = L\frac{di(t)}{dt} + R \cdot i(t)}$$

↳ First order Diff. Eqn.

We can use Phasors to solve this circuit algebraically

$\bar{V}$



$$\bar{V} = \bar{I}(\bar{Z}_L + \bar{Z}_R)$$

$P = ?$

Euler's Formula

$$v(t) = V \cdot \sqrt{2} \cdot \cos(\omega t + \theta_v) \iff \bar{V} = V \cdot e^{j\theta_v} = V \angle \theta_v$$

$$i(t) = I \cdot \sqrt{2} \cdot \cos(\omega t + \theta_i) \iff \bar{I} = I\, e^{j\theta_i} = I \angle \theta_i$$

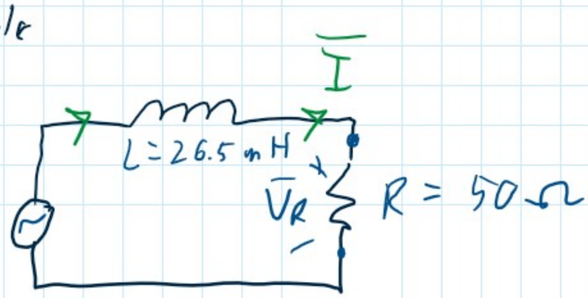
$$L \iff \bar{Z}_L = j\omega L = \omega L \angle 90^\circ$$

$$C \iff \bar{Z}_C = \frac{1}{j\omega C} = \frac{1}{\omega C} \angle -90^\circ$$

$$R \iff \bar{Z}_R = R = R \angle 0^\circ$$

Example

$V(t)$



$V(t) = 100\sqrt{2}\cos(\omega t)\ V$

$\omega = 2\pi \cdot f, \quad f = 60\ Hz$

$\omega = 377\ rad/s$

$\bar{V} = 100 \angle 0^\circ\ V$

$$\bar{Z}_L = j\omega L = j377 \cdot 26.5 \times 10^{-3} = j10 = 10 \angle 90^\circ$$

$$\bar{Z}_R = 50$$

$\bar{I}$?

$$\bar{I} = \frac{\bar{V}}{\bar{Z}_L + \bar{Z}_R} = \frac{100 \angle 0^\circ}{\underbrace{j10 + 50}} = \frac{100 \angle 0^\circ}{51 \angle 11.3^\circ}$$

$\rightarrow$ —

$Z_L + Z_R$ $\qquad j\underbrace{10 + 50}$ $\qquad 51 \angle 11.3°$

$\rightarrow \vec{\bar{I}} = 1.96 \angle -11.3° \text{ A}$

$\bar{V}_R = \bar{I} R = 98 \angle -11.3° \text{ V}$

P_R? $\quad P_R = \dfrac{V_{RMS}^2}{R}$ or $I_{RMS}^2 \cdot R$

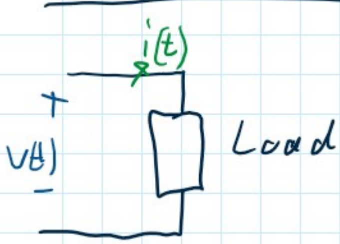
$\rightarrow \dfrac{98^2}{50} = 192 \text{ W} \qquad 1.96^2 \cdot 50 = 192 \text{ W}$ ✓

Note: Purely reactive elements Do not dissipate power - just store energy

To supply power (P) to RL load, we must supply 1.96 A_{RMS} of current

$100 \cdot 1.96 = 196 \text{ VA} > 192 \text{ W}$ - sent to resistive load

Instantaneous Power



$p(t) = V(t) \cdot i(t) \leftarrow$

$V(t) = V \cdot \sqrt{2} \cdot \cos(\omega t + \theta_v) \leftarrow$

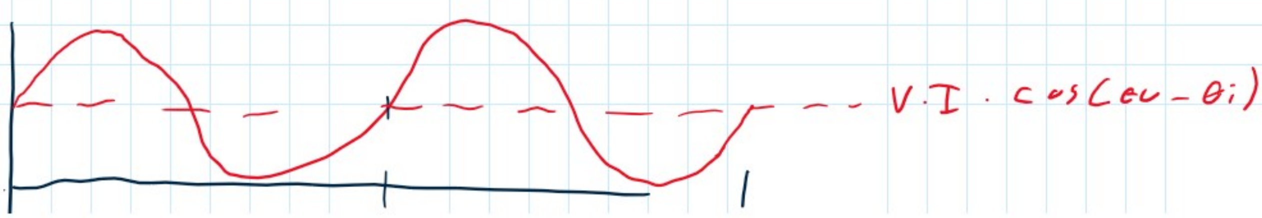
$i(t) = I \cdot \sqrt{2} \cdot \cos(\omega t + \theta_i) \leftarrow$

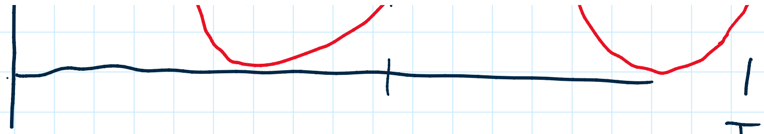
$p(t) = 2 \cdot V \cdot I \cos(\omega t + \theta_v) \cdot \cos(\omega t + \theta_i)$

Trig identity: $\cos\theta_1 \cdot \cos\theta_2 = \frac{1}{2}\left[\cos(\theta_1 - \theta_2) + \cos(\theta_1 + \theta_2)\right]$

$p(t) = V \cdot I \left[\underbrace{\cos(\theta_v - \theta_i)}_{\text{constant}} + \cos(2\omega t + \theta_v + \theta_i)\right]$

$p(t)$





$$T = 2\pi/\omega = 1/60$$

$$P = \frac{1}{T}\int_0^T p(t)\,dt$$

$$= \frac{1}{T}\int_0^T V\cdot I\left[\underbrace{\cos(\theta v - \theta i)}_{const} + \cancelto{0}{\cos(2\omega t + \theta v + \theta i)}\right]dt$$

$$\underbrace{P = V\cdot I\cdot\cos(\theta v - \theta i)}$$

$$S \triangleq V\cdot I = \text{Apparent Power}$$

$$\cos(\theta v - \theta i) = \text{Power factor} = PF$$