Problem 1: Relationship between DTFT and DFT

Consider the signal $\mathbf{x[n] = cos(\omega_0 n)}$

1. Compute the the DTFT, $\mathbf{X_d(\omega)}$, of $\mathbf{x[n]}$
2. Obtain the $\mathbf{N}$-length DFT, $\mathbf{X[k], k = 0, 1, \ldots, N-1}$, of $\mathbf{x[n]}$ by sampling $\mathbf{X_d(\omega)}$ when $\omega_0 = \dfrac{2\pi k_0}{N}$ and when $\omega_0 \neq \dfrac{2\pi k_0}{N}$, where $\mathbf{k_0}$ is an integer.
3. How many non zero samples of $\mathbf{X[k]}$ do you obtain when $\omega_0 = \dfrac{2\pi k_0}{N}$

Problem 2: Spectral Analysis

Consider the signals shown below,

$$\begin{aligned} \mathbf{x_1[n]} &= \mathbf{cos}\left(\frac{\pi n}{4}\right) + \mathbf{cos}\left(\frac{17\pi n}{64}\right) \\ \mathbf{x_2[n]} &= \mathbf{cos}\left(\frac{\pi n}{4}\right) + \mathbf{0.8cos}\left(\frac{21\pi n}{64}\right) \\ \mathbf{x_2[n]} &= \mathbf{cos}\left(\frac{\pi n}{4}\right) + \mathbf{0.001cos}\left(\frac{21\pi n}{64}\right) \end{aligned}$$

A length-64 DFT is computed of each of the above signals. Which of the signals will have two distinct peaks.

Problem 3: Spectral analysis

The signal $\mathbf{x_a(\Omega_0 t)}$ is sampled with sampling period $\mathbf{T}$ to obtain the DT signal $\mathbf{x[n] = cos(\omega_0 n)}$. Let $\mathbf{X[k], k = 0, 1, \ldots, N-1}$ denote the $\mathbf{N}$-length DFT of $\mathbf{x[n]}$. For what values of $\mathbf{T}$ will $\mathbf{|X[k]|}$ have exactly two non-zero values.

Problem 4: Parseval's Theorem

Prove Parseval's Theorem,

$$\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2$$

where $\mathbf{x[n]}$ denotes a DT signal of length $\mathbf{N}$ and $\mathbf{X[k], k = 0, 1, \ldots, N-1}$ denotes the $\mathbf{N}$ length DFT of $\mathbf{x[n]}$.