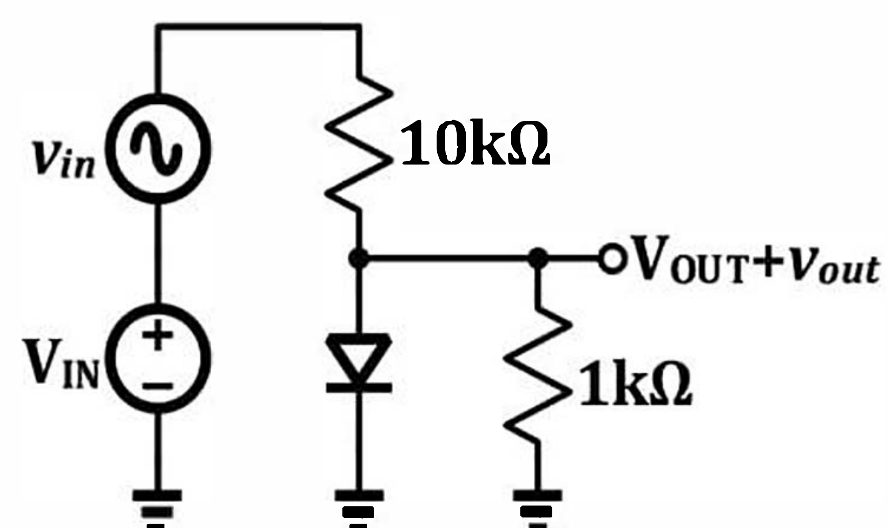
1. Consider the circuit shown in the figure below. Assume V_T=25mV, V_{IN}=10V, v_{in}=0.1V.

a. Using constant voltage model for the diode with V_{D0}=0.7V, compute diode current, I_{D0}, and its incremental resistance, r_d.

b. Sketch incremental model and calculate amplitude of the incremental output voltage, v_{out1}, and incremental diode current, i_{d1}.

c. Recompute incremental diode current, i_{d2}, by including the second order term in the Taylor series expansion of diode current (you may use v_{out1} as incremental output voltage). Recompute new incremental output voltage, v_{out2}, using the newly calculated incremental diode current, i_{d2}. Calculate error voltage, $v_e = v_{out1} - v_{out2}$.

d. Repeat parts (b) and (c) for $v_{in} = 5$V.



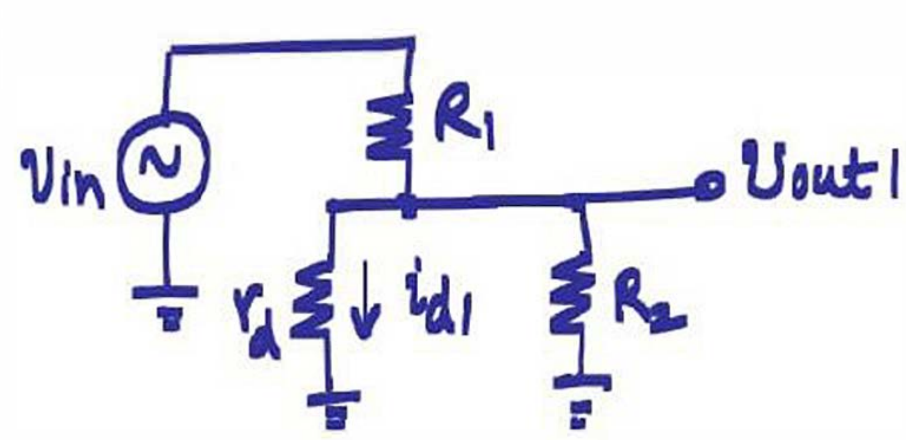
(a) $I_{R1} = \dfrac{V_{IN} - V_{D0}}{R_1} = \dfrac{10 - 0.7}{10k} = 0.93\,mA$

$I_{R2} = \dfrac{V_{D0}}{R_2} = \dfrac{0.7}{1k} = 0.7\,mA$

$I_{D0} = I_{R1} - I_{R2} \Rightarrow \boxed{I_{D0} = 0.23\,mA}$

$r_d = \dfrac{V_T}{I_{D0}} = \dfrac{25mV}{0.23mA} \Rightarrow \boxed{r_d = 108.7\,\Omega}$

(b)



$v_{out1} = v_{in} \cdot \dfrac{\overbrace{r_d \parallel R_2}^{98\Omega}}{R_1 + r_d \parallel R_2}$

$\Rightarrow \boxed{v_{out1} = 970\,\mu V}$

$i_{d1} = \dfrac{v_{out1}}{r_d} \Rightarrow \boxed{i_{d1} = 8.92\,\mu A}$

(c) Second term in the Taylor series is equal to:

$$\Delta i_{d2} = \frac{1}{2}\frac{d^2 I_D}{dV_D^2}\cdot(V_{D0}+\Delta v_d - V_{D0})^2 = \frac{1}{2}\cdot\frac{I_S}{V_T^2}\cdot\exp\left(\frac{V_{D0}}{V_T}\right)\cdot\Delta v_d^2$$

$$\Rightarrow \Delta i_{d2} \approx \frac{1}{2}\cdot\frac{I_{D0}}{V_T^2}\cdot\Delta v_d^2$$

Using $\Delta v_d = v_{out1}$: $\Delta i_{d2} = \frac{1}{2}\cdot\frac{0.23\,mA}{(25\,mV)^2}\cdot(970\,\mu V)^2 = 0.173\,\mu A$

$$i_{d2} = i_{d1} + \Delta i_{d2} \Rightarrow \boxed{i_{d2} = 9.093\,\mu A}$$

$$v_{out2} = i_{d2}\times r_d \Rightarrow \boxed{v_{out2} = 988.41\,\mu V} \Rightarrow$$ Assuming r_d is constant

$$v_e = v_{out2} - v_{out1} \Rightarrow \boxed{v_e = 18.4\,\mu V = 1.9\%}$$

(d) Since V_{IN} (& operating point) did not change, r_d remains unchanged

$$v_{out1} = \frac{5\times 98}{10k + 98} \Rightarrow \boxed{v_{out1} = 48.5\,mV}$$

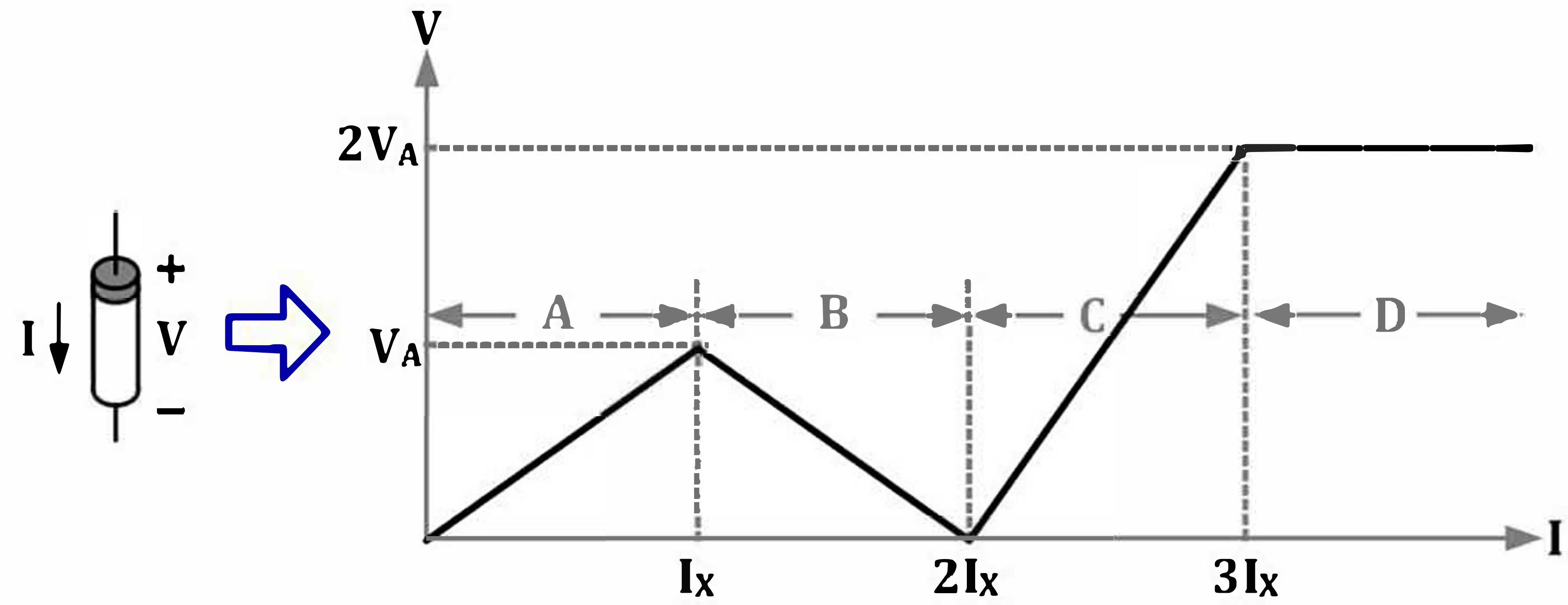
$$i_{d1} = \frac{v_{out1}}{r_d} \Rightarrow \boxed{i_{d1} = 446\,\mu A}$$

$$\Delta i_{d2} = 432.8\,\mu A \Rightarrow \boxed{i_{d2} = 878.8\,\mu A}$$

$$v_{out2} = i_{d2}\times r_d \Rightarrow \boxed{v_{out2} = 95.5\,mV}$$

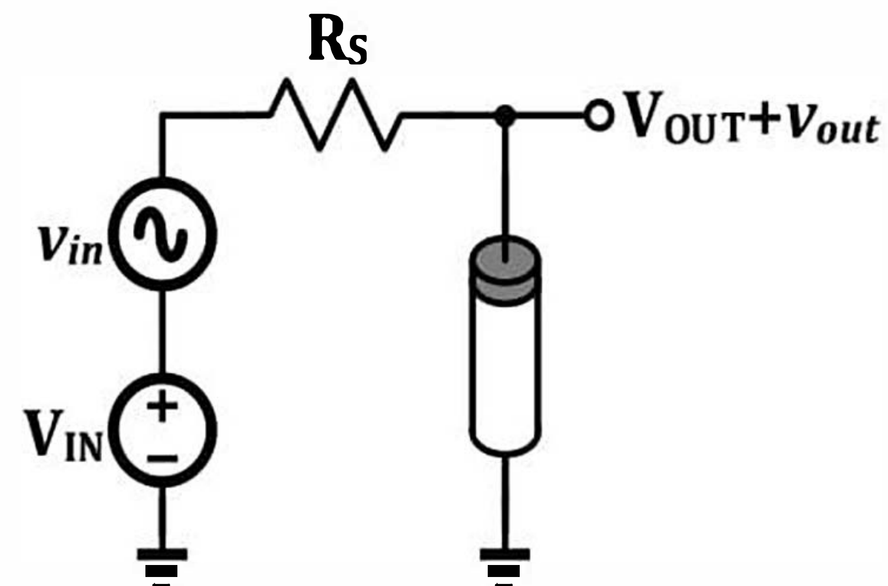
$$v_e = v_{out2} - v_{out1} \Rightarrow \boxed{v_e = 47\,mV = 97\%}$$

2. Consider a device whose I-V characteristic is shown below:



a. Find "incremental resistance" of the device in all regions of operation.

b. Sketch the incremental model of the circuit shown below.



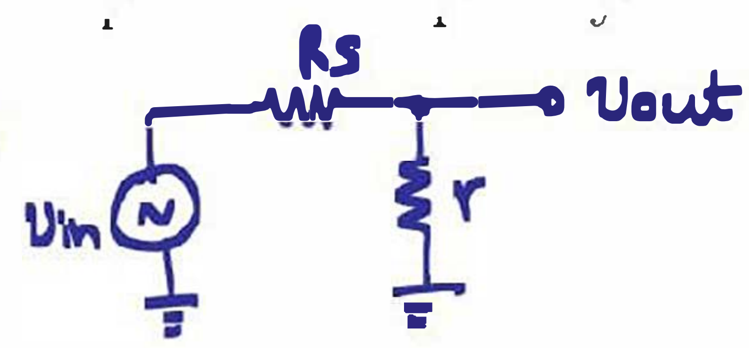
c. Determine the region (A, B, C, or D) in which the device should be operated to maximize incremental gain, $A_v = \left|\dfrac{v_{out}}{v_{in}}\right|$.

d. Compute the error in incremental voltage, v_{out}, (in regions A, B and C) if the second order term is included in the Taylor series expansion. Explain your answer.

(a)

$$r = \frac{dV}{dI} = \begin{cases} \dfrac{V_A}{I_x} & \text{in regim A} \\ -\dfrac{V_A}{I_x} & \text{in regim B} \\ \dfrac{2V_A}{I_x} & \text{in regim C} \\ 0 & \text{in regim D} \end{cases}$$

(b)



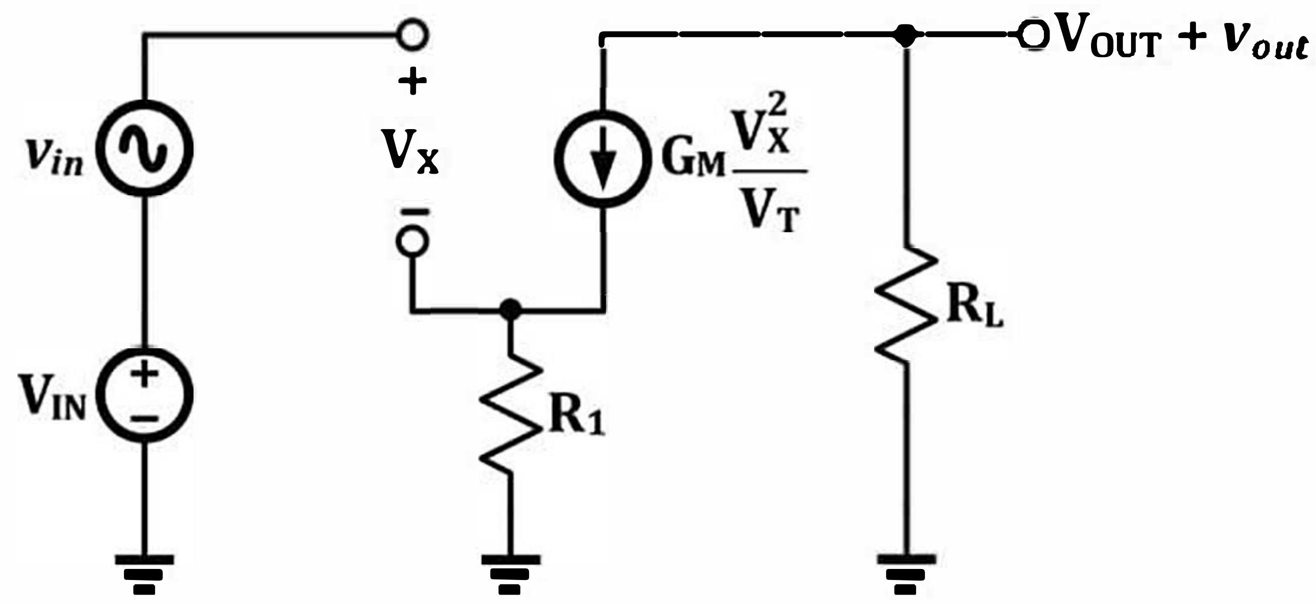
(c) $v_{out} = \dfrac{r}{R+r} \cdot v_{in}$

$\Rightarrow A_v = \dfrac{r}{R+r}$ is maximum when r is negative

$\Rightarrow$ The device must be biased in regim B

(d) Error is zero because of piecewise linear V-I characteristics

3. Consider the circuit that uses a non-linear voltage controlled current source shown below.



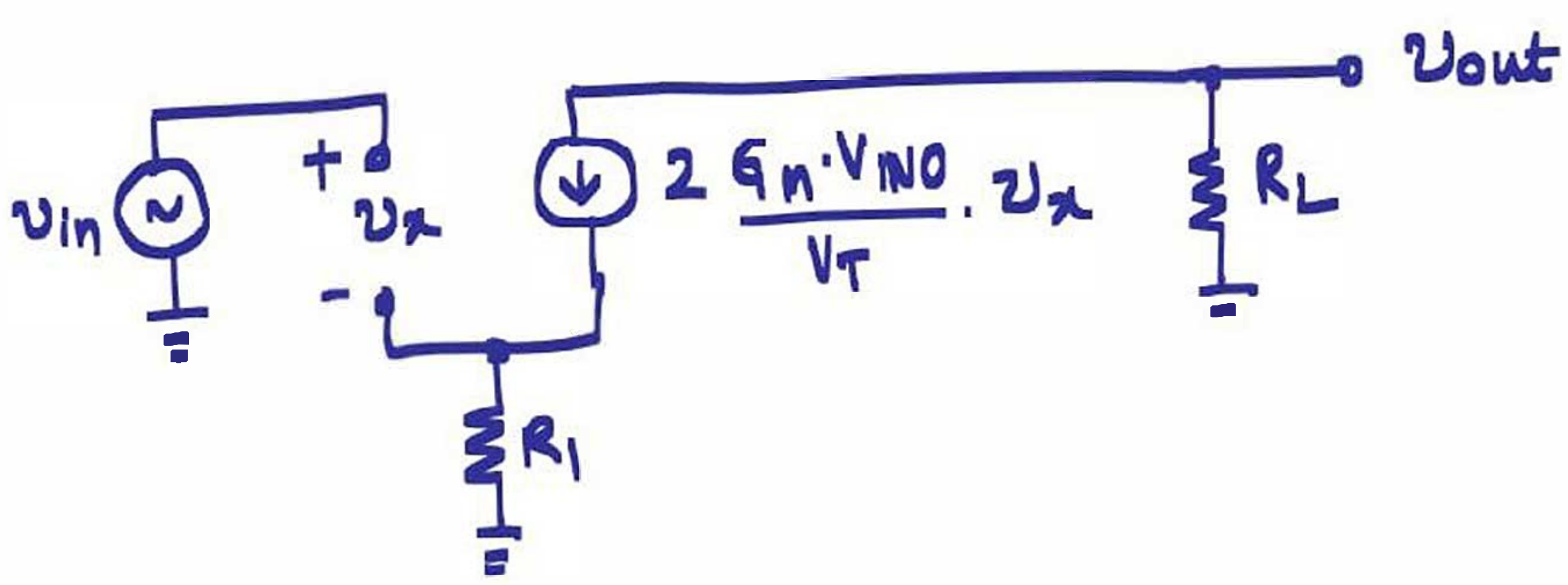
a. Draw incremental model. Note V_T is a constant with units of Volts.

b. Determine incremental output voltage, v_{out}, in terms of incremental input voltage, v_{in}, and circuit parameters, G_M, R_1 and R_L.

c. What value of R_1 maximizes incremental gain, $A_v = \left|\frac{v_{out}}{v_{in}}\right|$.

(a) $I_X = f(v_X) = \frac{G_M \cdot V_X^2}{V_T}$; $f'(v_X)\Big|_{V_X = V_{IN0}} = \frac{2 G_M \cdot V_{IN0}}{V_T}$

$$\Rightarrow I_X \approx \frac{G_M \cdot V_{IN0}^2}{V_T} + 2\frac{G_M \cdot V_{IN0}}{V_T} \cdot v_x$$



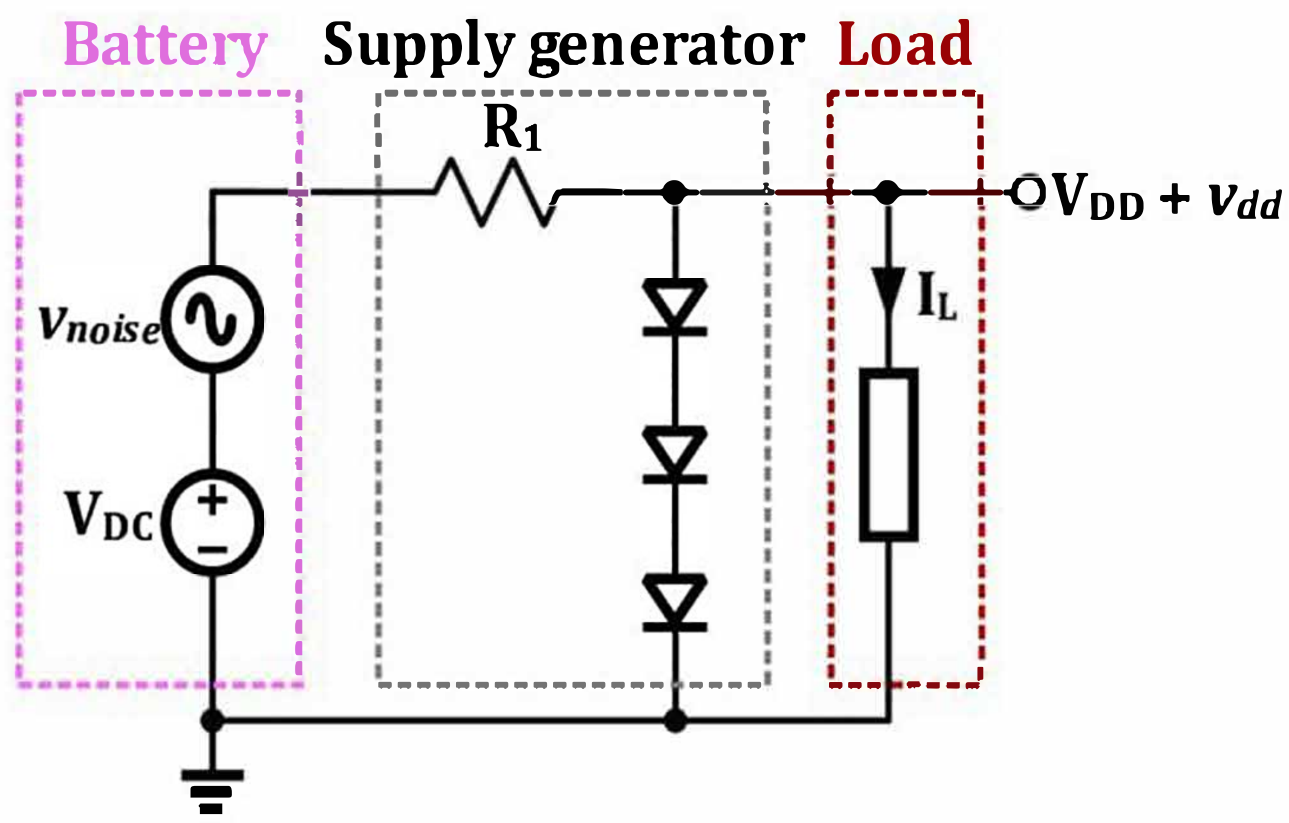
(b) $v_{in} = v_x + 2 \cdot \frac{G_M \cdot V_{IN0}}{V_T} v_x \cdot R_1 \Rightarrow v_x = \frac{v_{in}}{1 + 2\frac{G_M V_{IN0}}{V_T} \cdot R_1}$

$$v_{out} = -\frac{2 G_M \cdot V_{IN0}}{V_T} \cdot v_x \cdot R_L$$

$$\Rightarrow \boxed{A_v = \frac{v_{out}}{v_{in}} = -\frac{2 G_M \cdot V_{IN0}/V_T \cdot R_L}{1 + 2 G_M \frac{V_{IN0}}{V_T} \cdot R_1}}$$

(c) For maximum gain : $\boxed{R_1 = 0}$

4. Diodes can be used to generate desired supply voltage, V_{DD}, from a battery voltage as shown below.



The battery is modeled as a DC voltage source, V_{DC} in series with another voltage source, v_{noise}, that models small noise in the battery voltage. I_L is the current drawn by the load, which could be your iPad.

a. Assuming nominal DC voltage of the battery, $V_{DC} = 5V$, $R_1 = 320\Omega$, and constant voltage model of the diode with $V_{D0} = 0.6V$, calculate the DC supply voltage, V_{DD} under no load condition (i,e. $I_L = 0$).

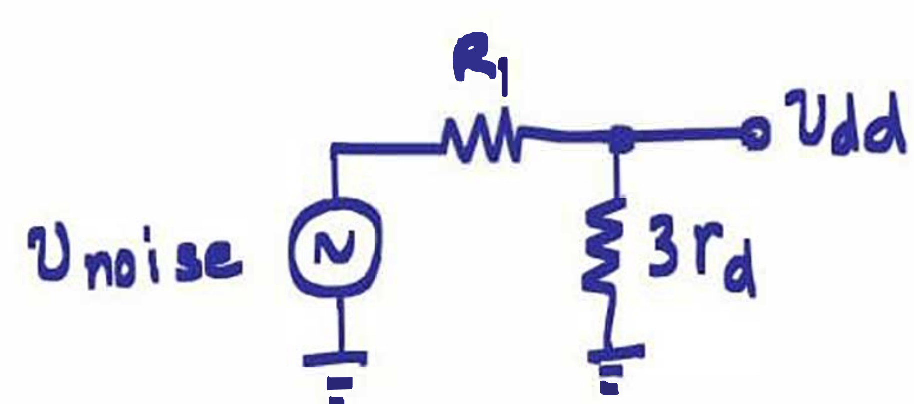
b. Calculate perturbations in the supply voltage, v_{dd}, caused by noise in the battery voltage under no load condition.

c. Repeat parts (a) and (b) when a load is connected to the supply, which draws a current of $I_L = 2mA$.

d. Repeat parts (a) and (b) when battery discharges to $V_{DC} = 4.5V$.

(a) $V_{DD} = 3V_{D0} \Rightarrow \boxed{V_{DD} = 1.8\text{ V}}$

(b) $I_{R1} = \dfrac{5-1.8}{320} = 10mA = I_{D0}$



$r_d = \dfrac{V_T}{I_{D0}} = \dfrac{25m}{10m} = 2.5\,\Omega$

$v_{dd} = \dfrac{3 \cdot r_d}{R_1 + 3 \cdot r_d} = \dfrac{3 \times 2.5}{320 + 3 \times 2.5} \cdot v_{noise}$

$\Rightarrow \boxed{v_{dd} = 0.023\, v_{noise} \approx \dfrac{v_{noise}}{43}}$ $\Rightarrow$ v_{noise} is attenuated when it appears at the output

(c) $I_{DO} = I_{R1} - I_L = 8\,mA$ $\boxed{V_{DD} = 1.8V}$ $\Rightarrow$ V_{DD} is independent of I_L

$$r_d = \frac{25m}{8m} = 3.125\Omega$$

$$v_{dd} = \frac{9.375}{329.375} \cdot v_{noise} \Rightarrow \boxed{v_{dd} = 0.028\, v_{noise} \approx \frac{v_{noise}}{35}}$$

(d) $\boxed{V_{DD} = 3V_{DO} = 1.8V}$ $\Rightarrow$ V_{DD} is independent of battery voltage

$$I_{R1} = I_{DO} = \frac{4.5 - 1.8}{320} = 8.44\,mA$$

$$r_d = \frac{25}{8.44} = 2.96\,m\Omega$$

$$v_{dd} = \frac{3 \times 2.96}{320 + 3 \times 2.96} \cdot v_{noise}$$

$$\Rightarrow \boxed{v_{dd} = 0.027 \cdot v_{noise} \approx \frac{v_{noise}}{37}}$$

5. A non-linear device has its output current, i_{O}, related to its input voltage, v_{I}, by $i_{\mathrm{O}} = \mathrm{I_T} e^{(v_{\mathrm{I}}/\mathrm{V_T})}$, where $\mathrm{I_T}$ and $\mathrm{V_T}$ are constant current and voltage, respectively.

a. Write the Taylor series expansion of i_{O} at an input voltage of $v_{\mathrm{I}} = v_i + \mathrm{V_I}$.

b. Assuming $\mathrm{I_T} =$, $\mathrm{V_T} =$ and $\mathrm{V_I} =$, plot i_{O} versus v_i.

c. Plot the error between the exact i_{O} and Taylor series approximated i_{O} versus v_i.

Solution

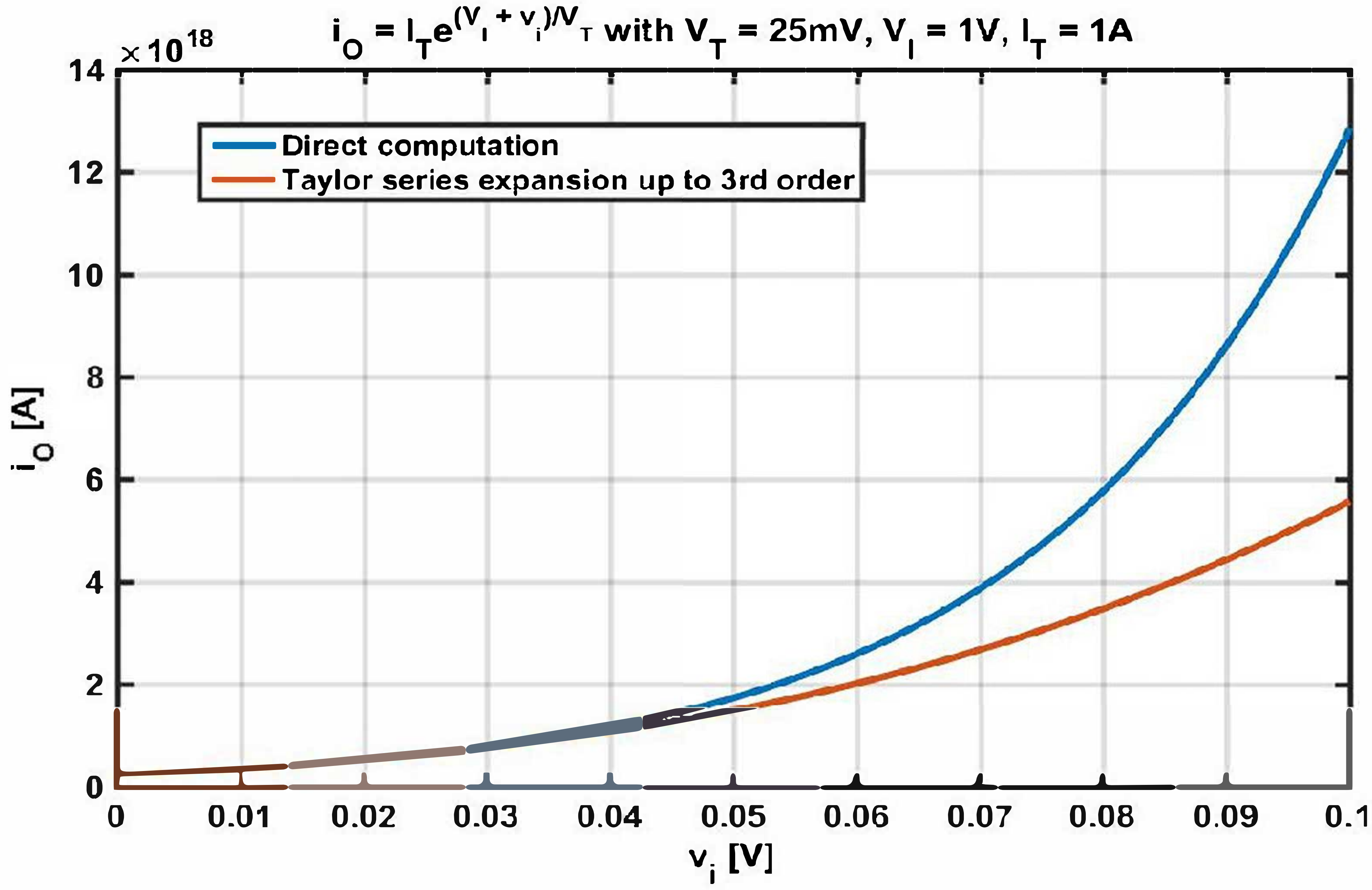
a. The Taylor series expansion of exponential is given as

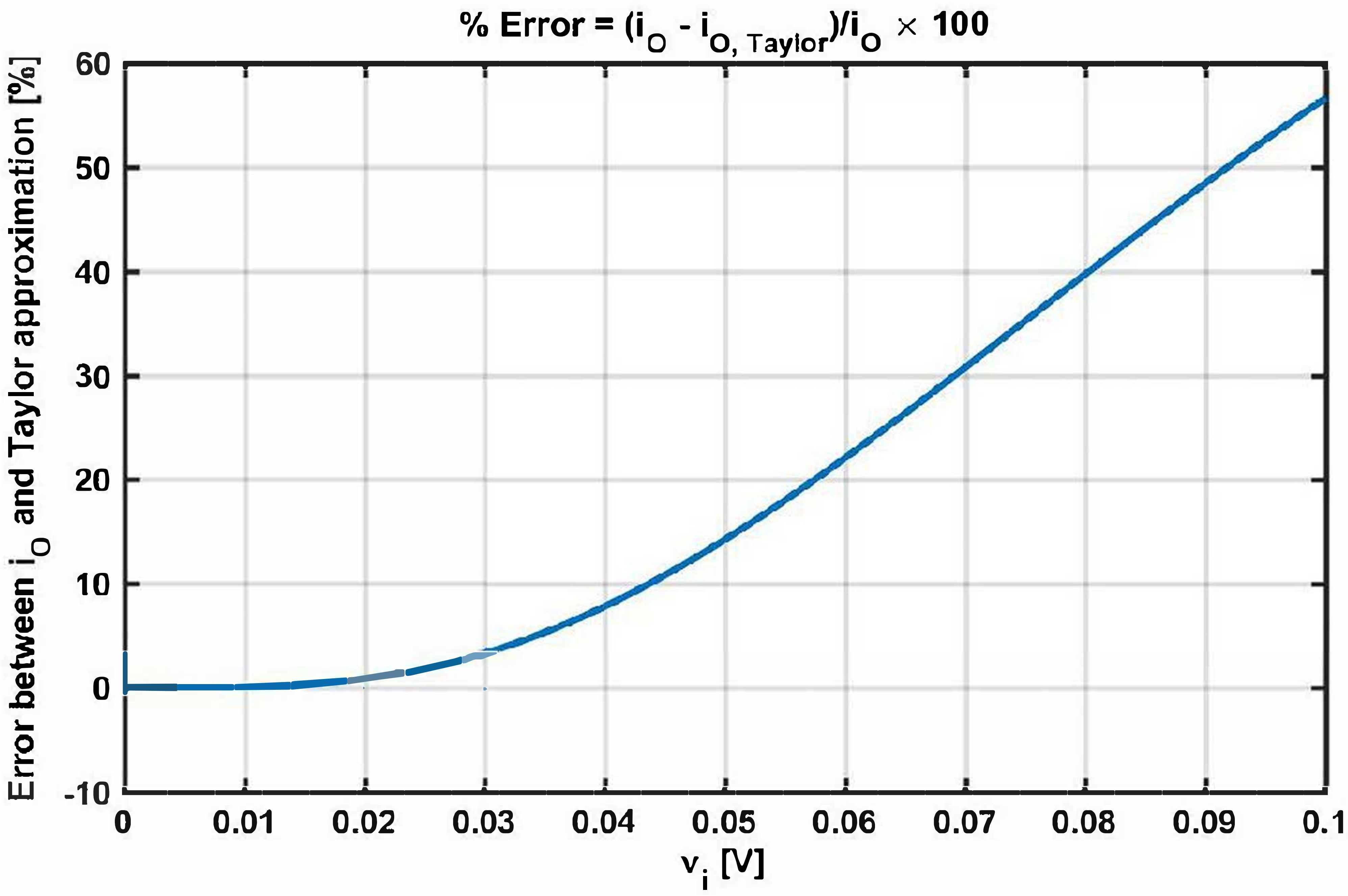
$$e^x = 1 + x + x^2 + \cdots$$

Assuming that V_I is a constant and we write the Taylor series expansion with respect to v_i (i.e we try to capture the change of i_O vs. v_i around a fixed V_I biasing), we have the following

$$i_O = I_T e^{\frac{V_I}{V_T}} \left[1 + \frac{v_i}{V_T} + \frac{1}{2!}\left(\frac{v_i}{V_T}\right)^2 + \cdots \right]$$

Note that Taylor series expansion contains an *infinite* number of terms.




% Error = $(i_O - i_{O,\,Taylor})/i_O \times 100$
Error between i_O and Taylor approximation [%]
60
50
40
30
20
10
0
-10
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
v_i [V]