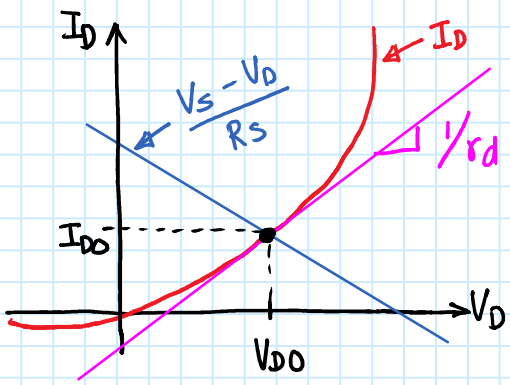
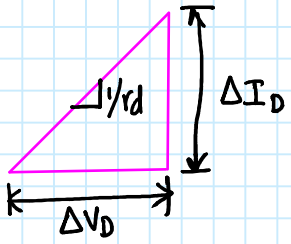
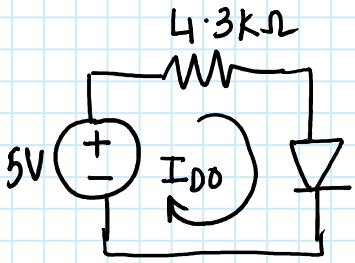
Lecture 5 (Incremental models)

Graphical View:



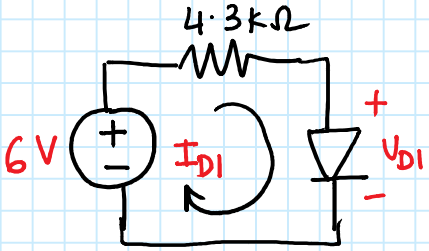


Assumptions: Valid only for small changes i.e ΔV_D is *small*

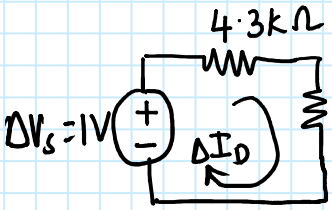


$V_{DO} = 0.7V$

$I_{DO} = 1mA$



Find I_{D1}, V_{D1}



$r_d = \frac{V_T}{I_{DO}} = 25\Omega$

$\Delta I_D = \frac{1V}{4{\cdot}3k + 25} = 231\mu A$

$\Delta V_D = r_d \times \Delta I_D = 5mV$

$I_{D1} = I_{DO} + \Delta I_D = 1{\cdot}231\,mA$ $\qquad V_{D1} = V_{DO} + \Delta V_D = 0{\cdot}705V$

BUT how accurate are these calculations?

Recall: We ignored higher order terms in the Taylor's series expansion of I_D around V_{DO}

Let us inculde second term: $\frac{f''(V_{DO})}{2}.(\Delta V_D)^2$

$f''(V_{DO}) = \frac{I_S}{V_T^2}.\exp\left(\frac{V_{DO}}{V_T}\right). \simeq \frac{I_{DO}}{V_T^2} \Rightarrow$ second term $\simeq \frac{I_{DO}}{2V_T}.(\Delta V_D)^2$

More accurate $I_D = I_{DO} + \frac{I_{DO}}{V_T}.\Delta V_D + \frac{I_{DO}}{2V_T^2}.(\Delta V_D)^2 = I_{DO} + \frac{I_{DO}}{V_T}.\Delta V_D(1+\alpha)$

$\simeq I_{DO} + \frac{I_{DO}}{V_T}.\Delta V_D$ if $\Delta V_D \ll 2V_T$

In the above example: Error $\alpha = \Delta V_D/2V_T = \frac{5mV}{2\times 25m} = 0{\cdot}1 \equiv 10\%$

Generalization to any 2-terminal element:

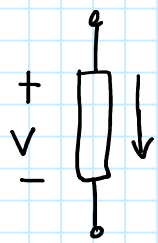
$I = f(V)$ can be linearized around an operating point V_0, I_0

$$I = I_0 + \Delta I$$

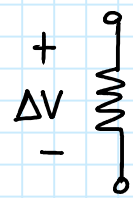
$$V = V_0 + \Delta V$$

$$I \approx \cancel{f(V_0)} + f'(V_0)\cdot \Delta V = \cancel{I_0} + \Delta I$$

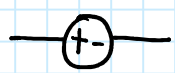
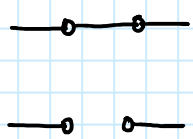
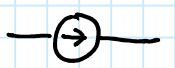
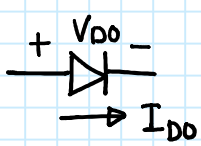
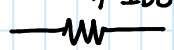
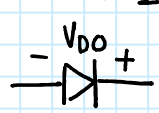
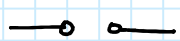
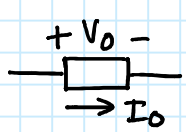
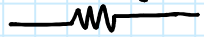
$$\Delta I = f'(V_0)\cdot \Delta V$$



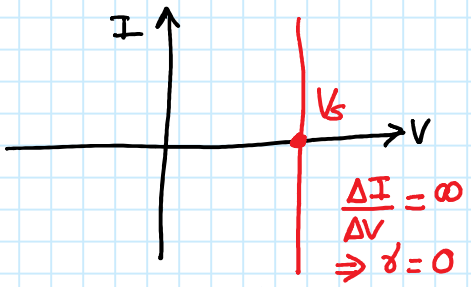
$I = f(V)$ $\equiv$



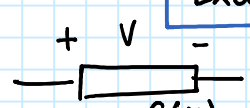
$r = \dfrac{1}{f'(V_0)}$ & $\Delta I = \dfrac{\Delta V}{r}$.

Element	Incremental model
R	R
	
	
Forward biased 	V_T / I_{DO} 
Reverse biased 	
 $I = f(V)$	$1/f'(V_0)$ 

Voltage source I-V char.



Example



$I = f(V)$

$$I = \alpha (V - V_T)^2 \qquad V \geq V_T$$

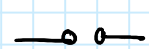
$$= 0 \qquad V < V_T$$



$\equiv$



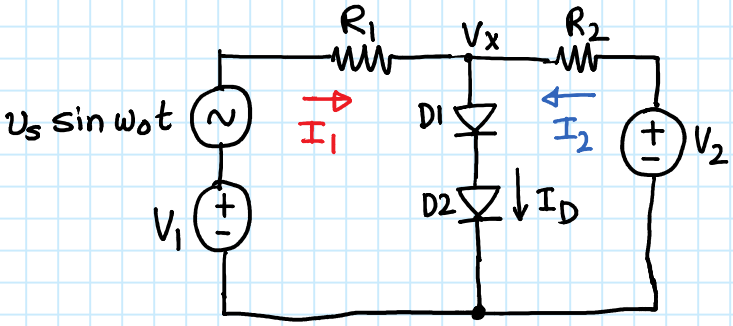
$V \geq V_T$



$V < V_T$

Analysis of circuits with non-linear elements

Step # 1 Find operating point by solving non-linear equations. You may ignore incremental perturbations



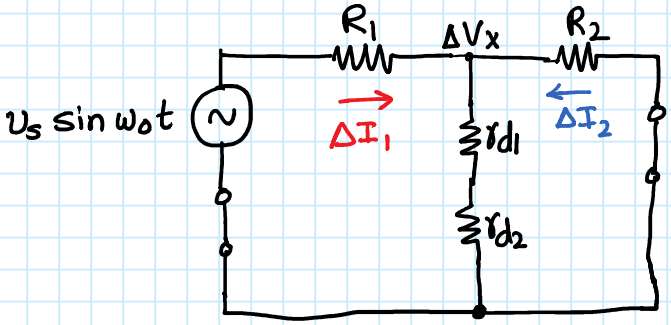
Quiescent point

$V_{X0} = 0.7 + 0.7 = 1.4$ assuming constant voltage drop model.

$$I_{10} = \frac{V_1 - V_X}{R_1} \quad ; \quad I_{20} = \frac{V_2 - V_X}{R_2}$$

$$I_{D0} = I_{10} + I_{20}$$

Step # 2 : Draw the incremental equivalent circuit by replacing circuit elements with their incremental models



$$r_{d1} = \frac{V_T}{I_{D0}} \quad \& \quad r_{d2} = \frac{V_T}{I_{D0}}$$

$$\Delta V_X = \frac{R_2 \,||\, (r_{d1} + r_{d2})}{R_1 + \left[R_2 \,||\, (r_{d1} + r_{d2})\right]}$$

$$\Delta I_1 = \frac{v_s \sin \omega_0 t - V_X}{R_1}$$

$$\Delta I_2 = \frac{-V_X}{R_2}$$

Step # 3 : Calculate total node voltage & branch currents by summing quiescent & incremental voltage/current

$$I_1 = I_{10} + \Delta I_1 \qquad I_2 = I_{20} + \Delta I_2 \qquad V_X = V_{X0} + \Delta V_X$$

Step # 4 : (Optional) Estimate error due to incremental analysis