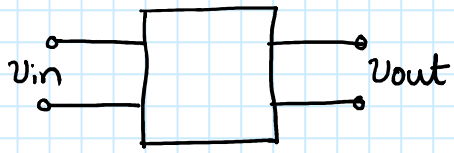
Lecture 4 [Linear & non-linear systems, incremental analysis]

Linear systems:



- satisfy superposition

$$v_{in1} \longrightarrow v_{out1}$$
$$v_{in2} \longrightarrow v_{out2}$$

$$\Rightarrow \quad a v_{in1} + b v_{in2} \longrightarrow a v_{out1} + b v_{out2} \quad \forall a, b, v_{in1}, v_{in2}$$

- Impulse response defines a Linear Time Invariant (LTI) system

$$v_{out}(t) = v_{in}(t) \otimes h(t)$$

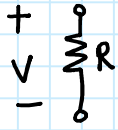
($\otimes$: convolution; $h(t)$: impulse response)

or $$V_{out}(s) = V_{in}(s) \cdot H(s)$$

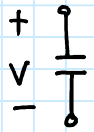
($H(s)$: Transfer function)

Linear elements: Resistors, Capacitors, Inductors

- 2 terminal element defined by V-I relationship

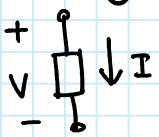


$$I = V/R$$



$$I = C\frac{dV}{dt}$$ still linear

- For any 2-terminal element



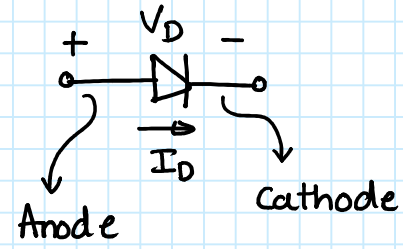
$$I = f(V)$$

or

conversly $$V = g(I)$$

Nonlinear elements:

Diode:



$$I_D = I_S\left[\exp\left(\frac{V_D}{V_T}\right) - 1\right]$$

where I_S : saturation current

V_T : Thermal voltage

$= \frac{kT}{q} \approx 26$ mV @ room temperature

Regions of operation:

(i) $V_D > 0$ forward biased

for $V_D \gg V_T$

$$I_D \simeq I_S \cdot \exp\left(\frac{V_D}{V_T}\right)$$

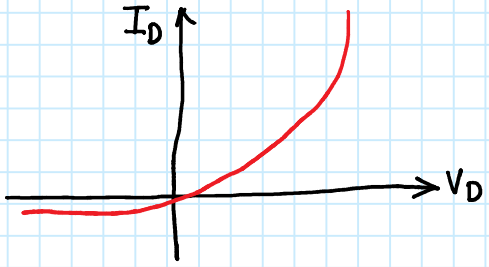
or $V_D = V_T \ln\left(\frac{I_D}{I_S}\right)$

(ii) $V_D < 0$ reverse biased

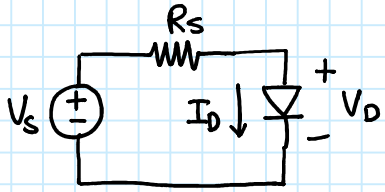
for $V_D \ll 0$

$\exp\left(\frac{V_D}{V_T}\right) \ll 1$

$\Rightarrow I_D \simeq -I_S$



Circuit using a diode: (Non-linear network)



Given V_S, R_S, find I_D & V_D

$$V_S = I_D R_S + V_D \quad -(1)$$

$$I_D = I_S\left\{\exp\left(\frac{V_D}{V_T}\right) - \right\} \quad -(2)$$

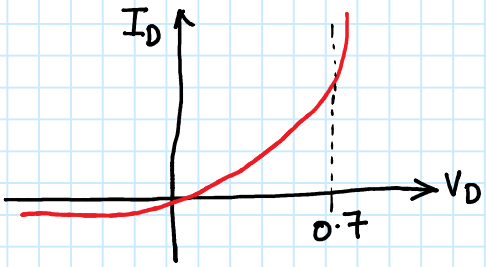
Solve (1) & (2) for I_D, V_D

Possible methods:

(1) Graphical method

(2) Iterative method

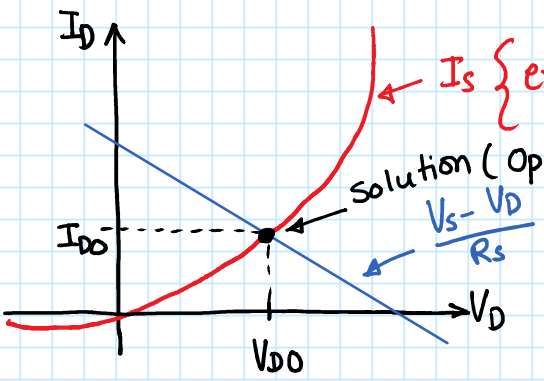
(3) Coarse approximation



$V_D \simeq 0.7$

$I_D \simeq \frac{V_S - 0.7}{R_S}$

Graphical method :



$$I_D = \frac{V_S - V_D}{R_S} \quad (1)$$

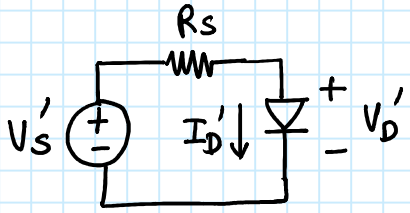
$$I_D = I_S\left\{\exp\left(\frac{V_D}{V_T}\right) - 1\right\} \quad (2)$$

Solution is when (1) = (2)

$\Rightarrow$ intersection of 2 curves

Summary: Given V_S & R_S we found $I_D = I_{DO}$ & $V_D = V_{DO}$

Question: How should we determine V_D & I_D if V_S changes to V_S'?



$$I_D' = \frac{V_S' - V_D'}{R_S}$$

$$I_D' = I_S\left\{\exp\left(\frac{V_D'}{V_T}\right) - 1\right\}$$

Can repeat graphical method with new V_S'. BUT too tedious!

Idea: Use previously calculated V_{DO}, I_{DO} to estimate V_D' & I_D'

We represent :

$$V_S' = V_{SO} + \Delta V_S$$

$$V_D' = V_{DO} + \Delta V_D$$

$$I_D' = I_{DO} + \Delta I_D$$

operating pt | increments

$$\frac{(V_{SO} + \Delta V_S) - (V_{DO} + \Delta V_D)}{R_S} = I_{DO} + \Delta I_D$$

$$= I_S \exp\left\{\left(\frac{V_{DO} + \Delta V_D}{V_T}\right) - 1\right\}$$

$$\frac{V_{SO} - V_{DO}}{R_S} + \frac{\Delta V_S - \Delta V_D}{R_S} = I_{DO} + \Delta I_D$$

$\Rightarrow$ $$\boxed{\Delta V_S = \Delta I_D \cdot R_S + \Delta V_D}$$

$$I_D' = I_S\left\{\exp\left(\frac{V_{DO}+\Delta V_D}{V_T}\right)-1\right\}$$

Taylor's series expansion of $y = f(x)$ at $x = X_0$:

$$y = \sum_{n=0}^{\infty} \frac{f^n(X_0)}{n!}(x-X_0)^n$$

$$y = \boxed{f(X_0)} + f'(X_0)(x-X_0) + \frac{f''(X_0)\cdot(x-X_0)^2}{2} + \cdots$$

operating point

Taylor series expansion of I_D' around V_{DO} gives:

$$I_D' = I_S\left\{\exp\left(\frac{V_{DO}}{V_T}\right)-1\right\} + \frac{I_S}{V_T}\exp\left(\frac{V_{DO}}{V_T}\right)\cdot\left(V_{DO}+\Delta V_{DO}-V_{DO}\right)+\cdots$$

$$\approx I_{DO} + \frac{I_S}{V_T}\cdot\exp\left(\frac{V_{DO}}{V_T}\right)\cdot\Delta V_{DO} \quad \text{(ignoring higher order terms)}$$

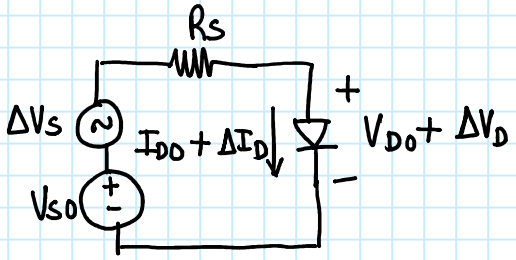
$$\Delta V_S = \Delta I_D \cdot R_S + \Delta V_D$$

$$\Delta I_D = \frac{1}{r_d}\cdot\Delta V_D \quad \text{where } r_d = \frac{V_T}{I_S\exp\left(\frac{V_{DO}}{V_T}\right)} \approx V_T/I_{DO}$$

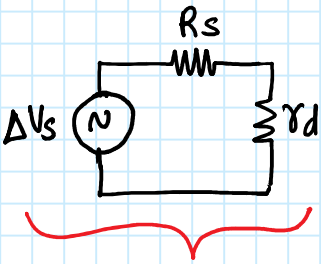
(r_d: incremental resistance; $\frac{1}{r_d}$: constant)

Key observation : Above equations are linear in incremental quantities : ΔV_S, ΔV_D & ΔI_D

Remember absolute voltages & currents, V_S', V_D', & I_D' are still related by non-linear equations.



$$\Delta I_D = \frac{\Delta V_S}{R_S + r_d}$$



Incremental model of diode network