ECE 342

2. PN Junctions and Diodes

Jose E. Schutt-Aine
Electrical & Computer Engineering
University of Illinois
jschutt@emlab.uiuc.edu

Definitions

B: material dependent parameter = 5.4 × 10^{31} for Si
E_G: Bandgap energy = 1.12 eV
k: Boltzmann constant=8.62×10^{-5} ev/K
n_i: intrinsic carrier concentration
At T = 300 K, n_i = 1.5 × 10^{10} carriers/cm^3
J_p: current density A/m^2
q: electron charge
D_p: Diffusion constant (diffusivity) of holes

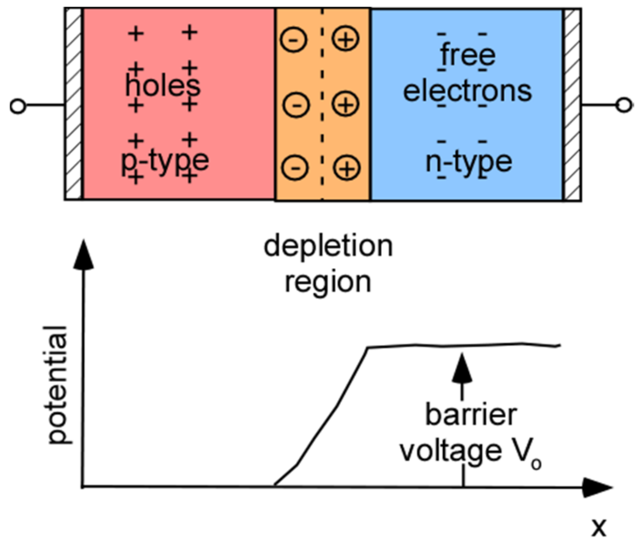
μ_p: mobility for holes = 480 cm^2 /V sec
μ_n: mobility for electrons = 1350 cm^2 /V sec

N_D: concentration of donor atoms
n_{no}: concentration of free electrons at thermal equilibrium

N_A: concentration of acceptor atoms
p_{po}: concentration of holes at thermal equilibrium

$$\text{Einstein Relation}: \frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{q} = V_T : \textit{thermal voltage}$$

PN Junction



- **When a p material is connected to an n-type material, a junction is formed**
  - Holes from p-type diffuse to n-type region
  - Electrons from n-type diffuse to p-type region
  - Through these diffusion processes, recombination takes place
  - Some holes disappear from p-type
  - Some electrons disappear from n-type

A depletion region consisting of bound charges is thus formed
Charges on both sides cause electric field ➔ potential = V_o

PN Junction

- **Potential acts as barrier that must be overcome for holes to diffuse into the n-region and electrons to diffuse into the p-region**
- **Open circuit: No external current**

Junction built-in voltage

From principle of detailed balance and equilibrium we get:

$$V_o = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

For Si, V_o is typically 0.6V to 0.8V

Charge equality in depletion region gives:

$$q x_p A N_A = q x_n A N_D$$

A: cross-section of junction
x_p: width in p side
x_n : width in n side

ε_s : silicon permittivity
$\varepsilon_s = 11.7\varepsilon_o = 1.04 \times 10^{-8}$ F/m

$$\frac{x_n}{x_p} = \frac{N_A}{N_D}$$

$$W_{dep} = x_n + x_p = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)V_o}$$

Example

Find the barrier voltage across the depletion region of a silicon diode at T = 300 K with $N_D=10^{15}/cm^3$ and $N_A=10^{18}/cm^3$.

Use $$V_o = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

@ 300K,

$$n_i = 1.5\times 10^{10}/\text{cm}^3$$

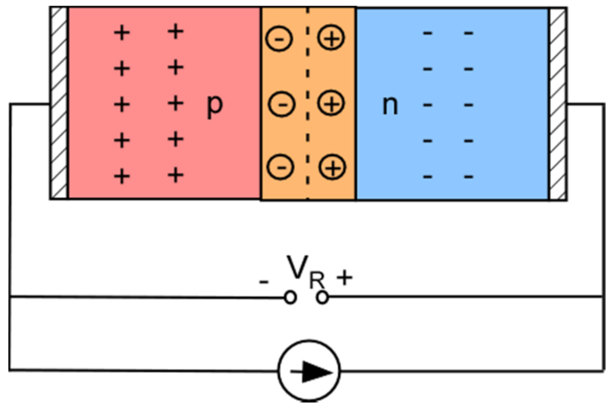
$$V_T = 0.026 \text{ V}$$

$$V_o = \psi_o = 0.026\ln\left(\frac{10^{18}\cdot 10^{15}}{(1.5)^2\times 10^{20}}\right) = 0.026\ln\left[\frac{10^{13}}{2.25}\right]$$

$$V_o = \psi_o = 0.026\times 29.12 = 0.7571 \text{ volts}$$

$$V_o = \psi_o = 0.7571 \text{ volts}$$

PN Junction under Reverse Bias



- **When a reverse bias is applied**
  - Transient occurs during which depletion capacitance is charged to new bias voltage
  - Increase of space charge region
  - Diffusion current decreases
  - Drift current remains constant
  - Barrier potential is increased
  - A steady state is reached
  - After transient: steady-state reverse current = I_S-I_D (I_D is very small) ➔ reverse current ~ I_S ~10^{-15} A

Under reverse bias the current in the diode is negligible

Depletion Layer Stored Charge

$$q_j = q_N = qN_D x_n A$$

A: cross section area
q_j: stored charge

Let W_{dep}= depletion-layer width

$$q_j = q\frac{N_A N_D}{N_A + N_D} AW_{dep}$$

The total voltage across the depletion layer is $V_o + V_R$

$$W_{dep} = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_o + V_R)}$$

Depletion Capacitance

$$C_j = \left.\frac{dq_j}{dV_R}\right|_{V_R=V_Q}$$

Q is bias point
V_R is reverse voltage

$$C_j = \frac{\varepsilon_s A}{W_{dep}} = \frac{C_{jo}}{\sqrt{1+\frac{V_R}{V_o}}}$$

$$C_{jo} = A\sqrt{\frac{\varepsilon_s q}{2}\left(\frac{N_A N_D}{N_A + N_D}\right)\left(\frac{1}{V_o}\right)}$$

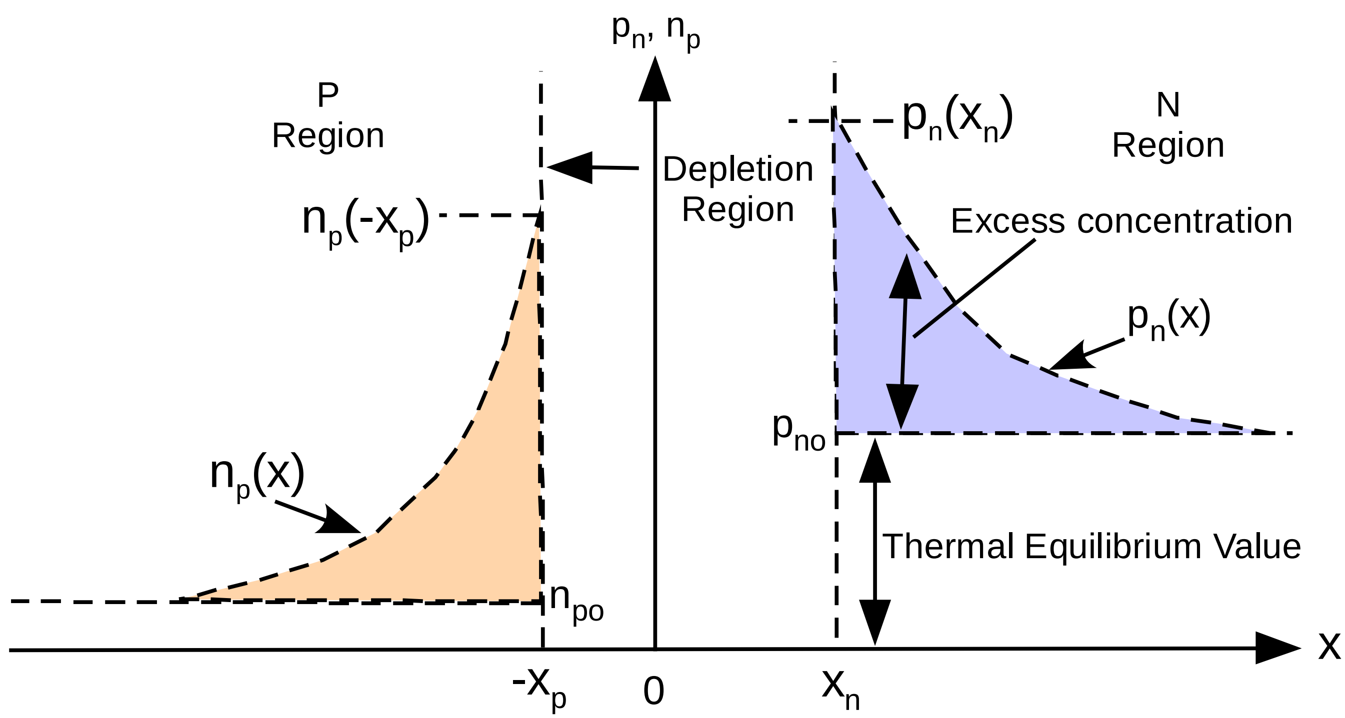
$$C_j = \frac{C_{jo}}{\left(1+\frac{V_R}{V_o}\right)^m}$$

m is the grading coefficient and depends on how the concentration varies from the p side to the n side

$1/3 < m < 1/2$

For an abrupt junction, m=0.5

Forward-Biased Junction Carrier Distribution



$N_A >> N_D$

Barrier voltage is now lower than V_o

In steady state, concentration profile of <u>excess minority carriers</u> remains constant

Forward-Biased PN Junction

Diode equation: $I_D = I_S\left(e^{V/nV_T} - 1\right)$

$$I_S = Aqn_i^2\left(\frac{D_p}{L_pN_D} + \frac{D_n}{L_nN_A}\right)$$

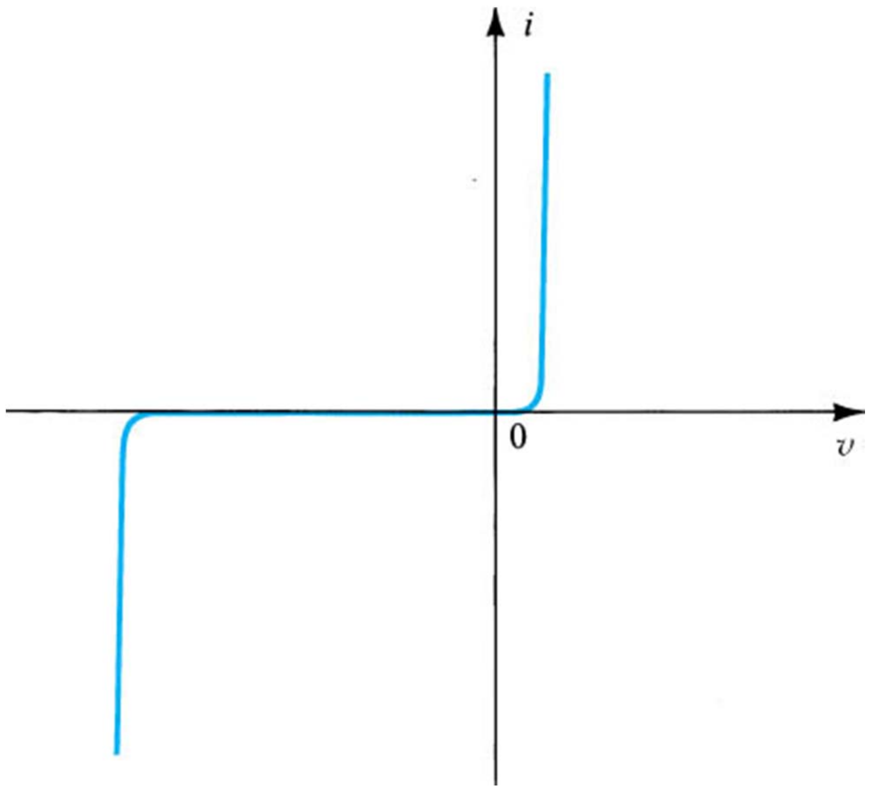
since n_i^2 is a strong function of temperature; thus I_s is a strong function of temperature

n has a value between 1 and 2. Diodes made using standard IC process have $n=1$; discrete diodes have $n=1$

In general, assume *n=1*

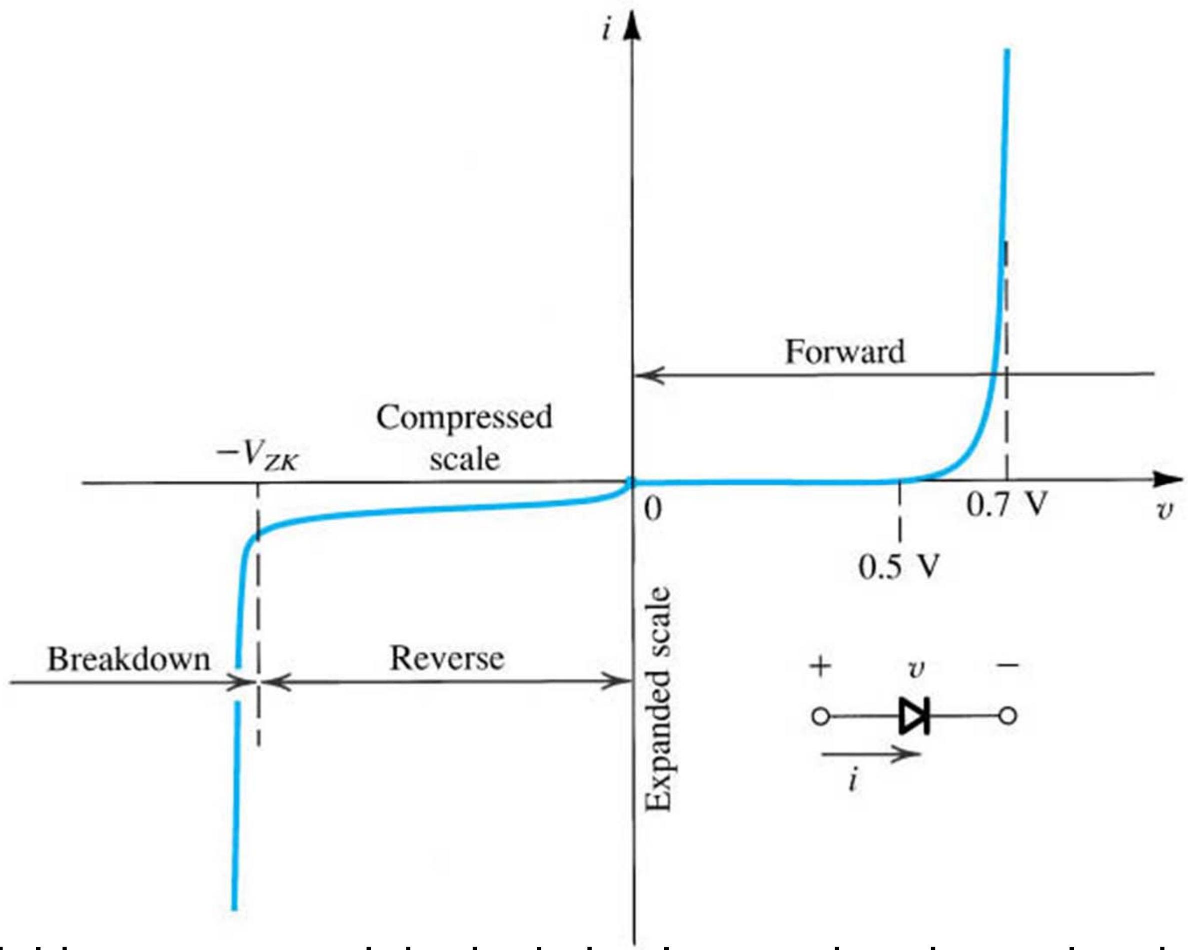
If $V \gg V_T$, we can use $I_D \approx I_S e^{V/V_T}$

Diode Characteristics



- **Three distinct regions**
  - The forward-bias region, determined by $v > 0$
  - The reverse-bias region, determined by $v < 0$
  - The breakdown region, determined by $v < -V_{ZK}$

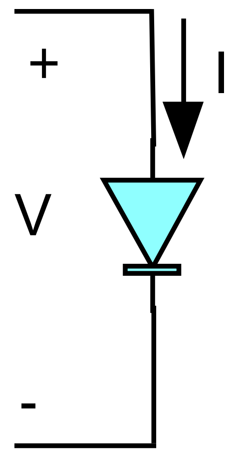
Diode I-V Relationship



Breakdown

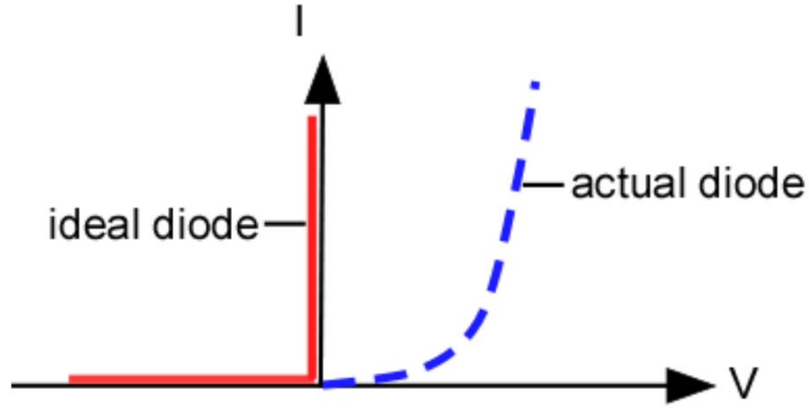
– Electric field strong enough in depletion layer to break covalent bonds and generate electron-hole pairs. Electrons are then swept by E-field into the n-side. Large number of carriers for a small increase in junction voltage

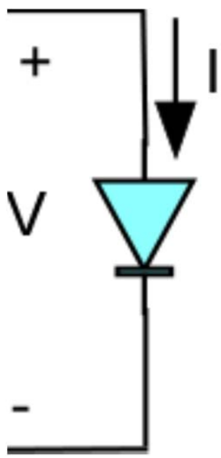
The Diode

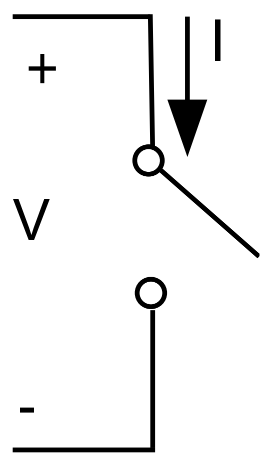


- **Diode Properties**
  - Two-terminal device that conducts current freely in one direction but blocks current flow in the opposite direction.
  - The two electrodes are the anode which must be connected to a positive voltage with respect to the other terminal, the cathode in order for current to flow.

Ideal Diode Characteristics

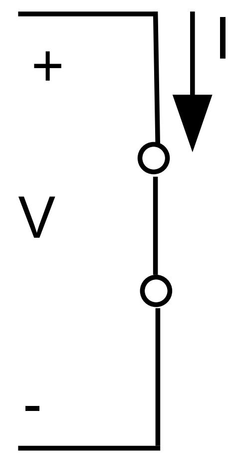






V < 0

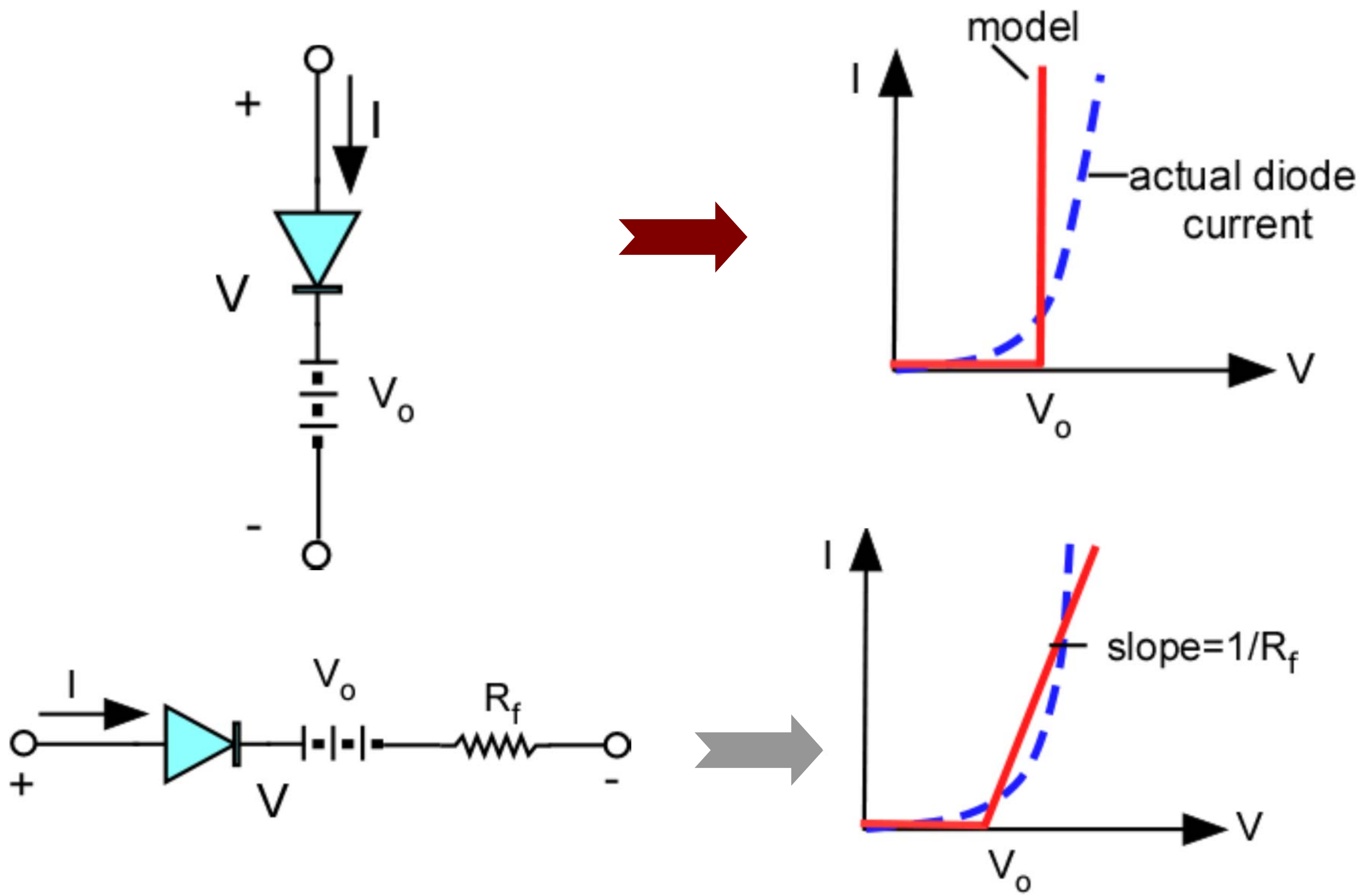
OFF



I > 0

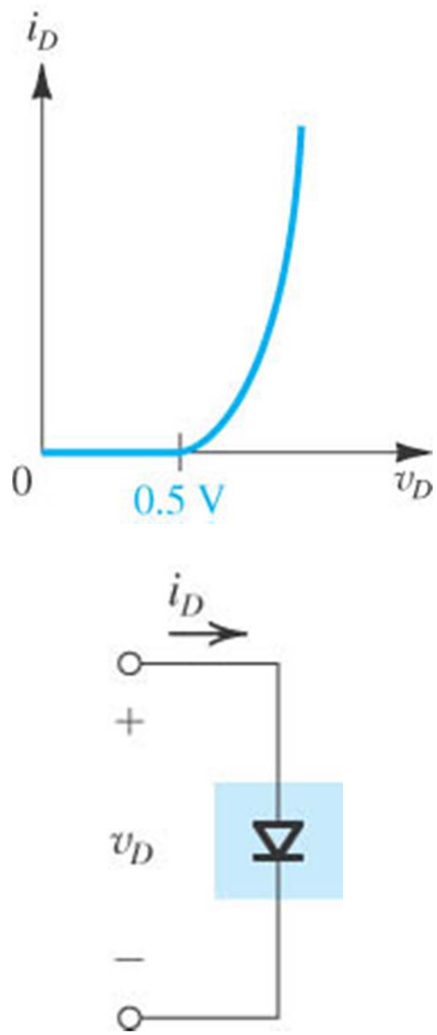
ON

Ideal Diode Characteristics

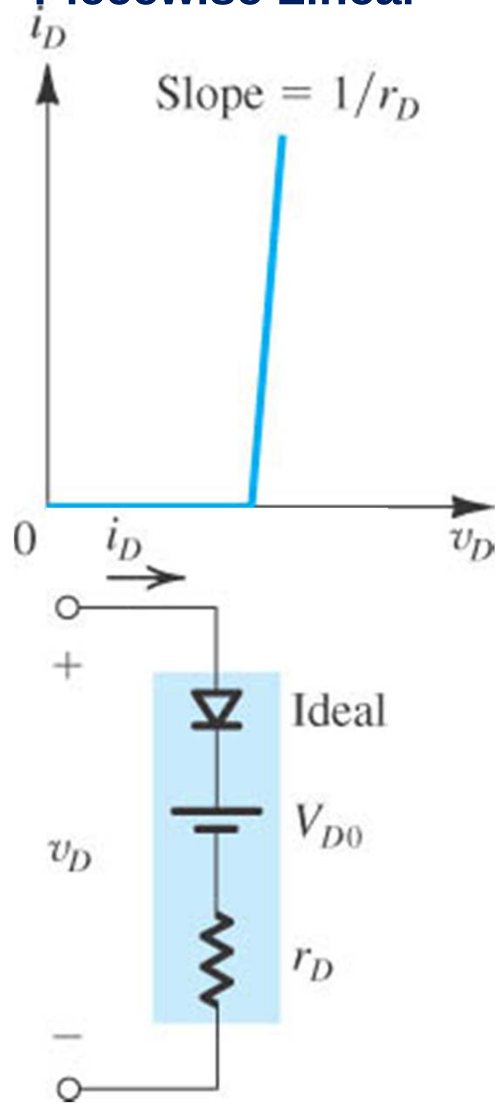


Diode Models

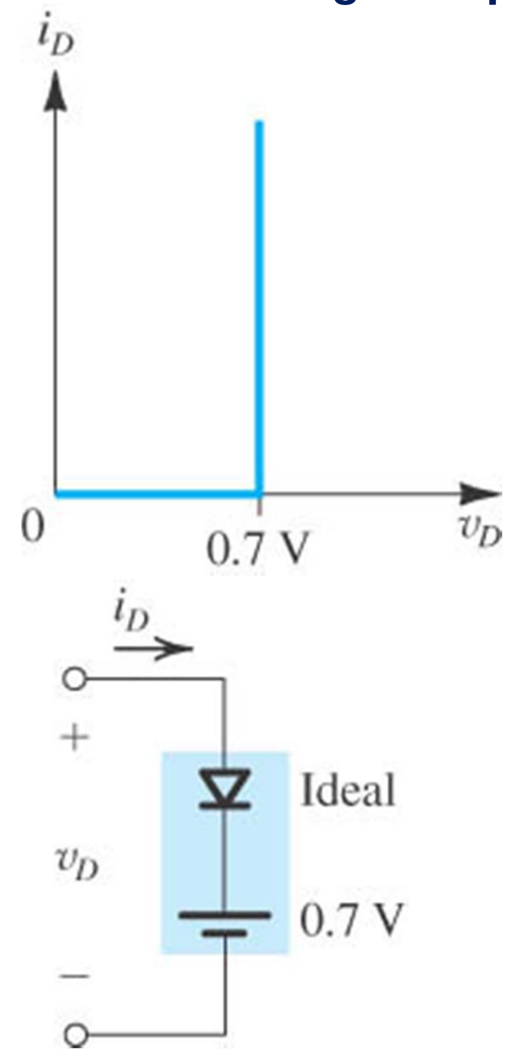
Exponential



Piecewise Linear

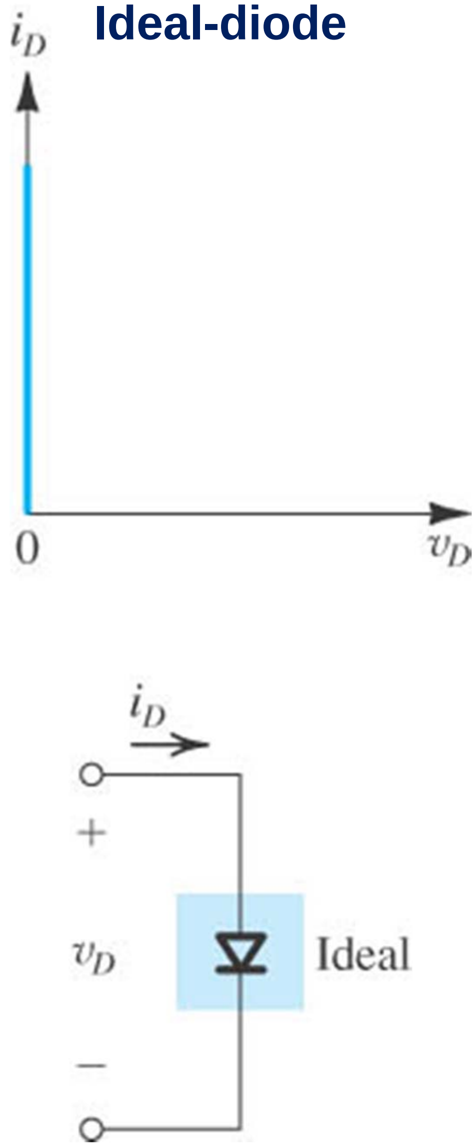


Constant-Voltage-Drop

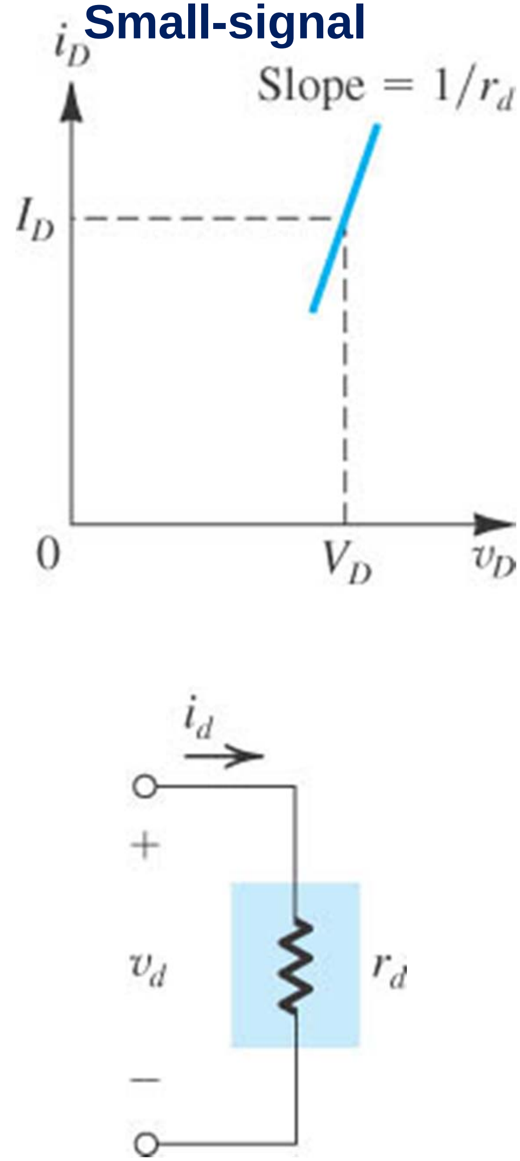


Diode Models

Ideal-diode



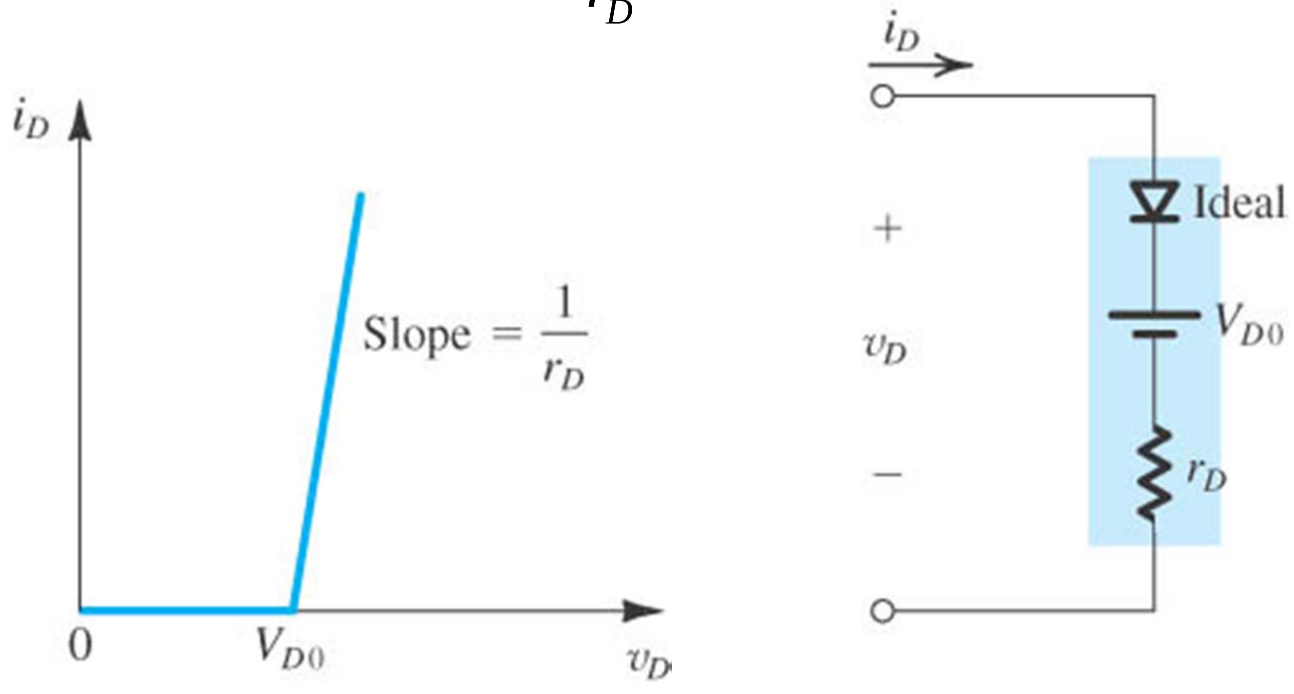
Small-signal



Piecewise-Linear Model

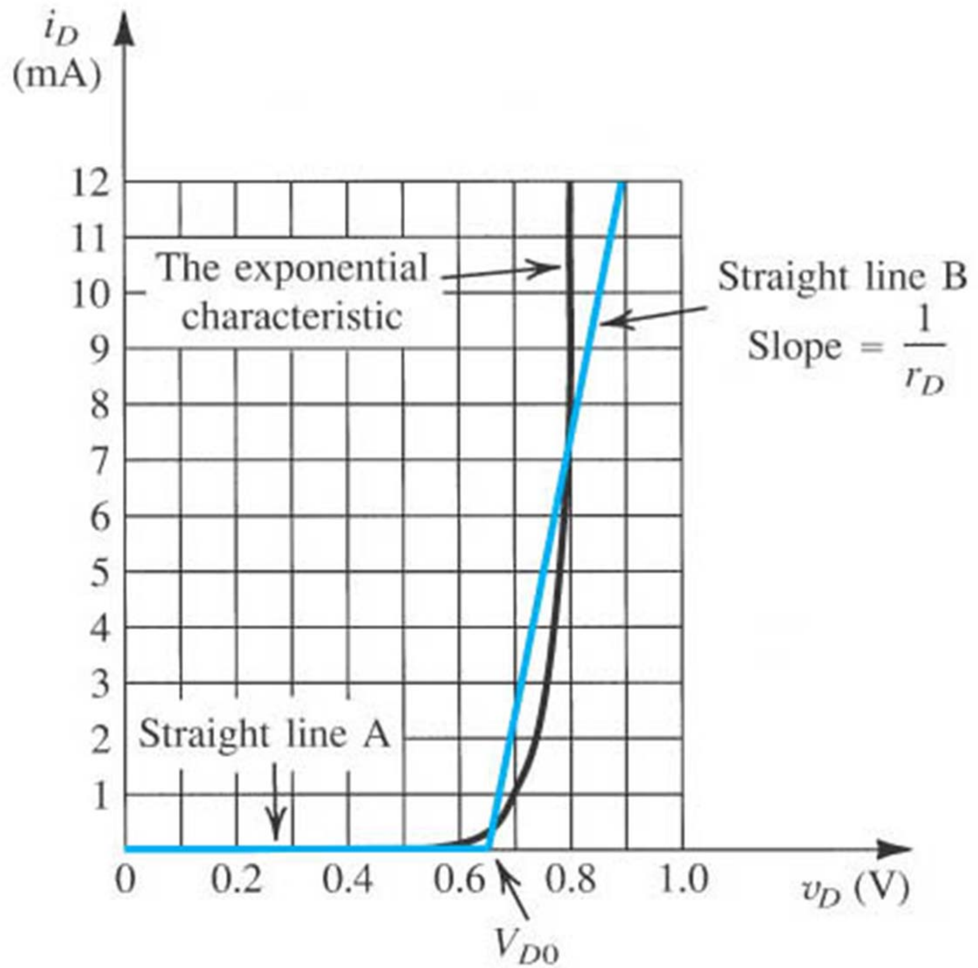
$$for\ v_D \le V_{D0}:\ i_D = 0$$

$$for\ v_D \ge V_{D0}:\ i_D = \frac{1}{r_D}\left(v_D - V_{D0}\right)$$



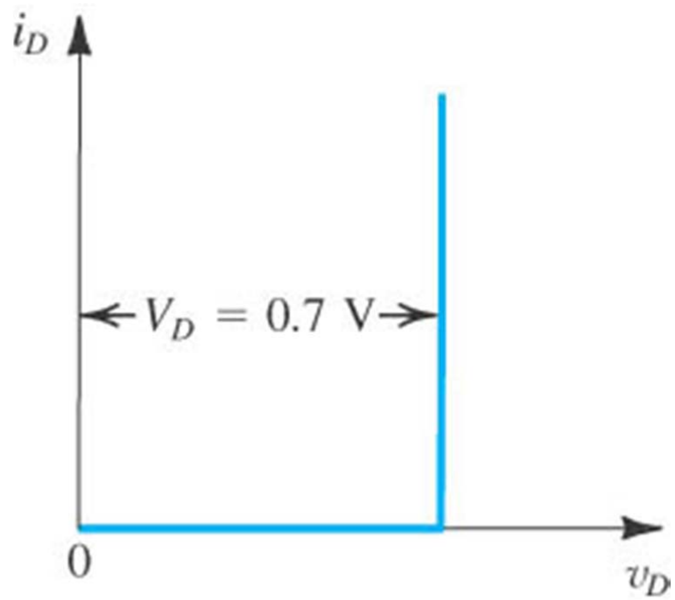
(a) (b)

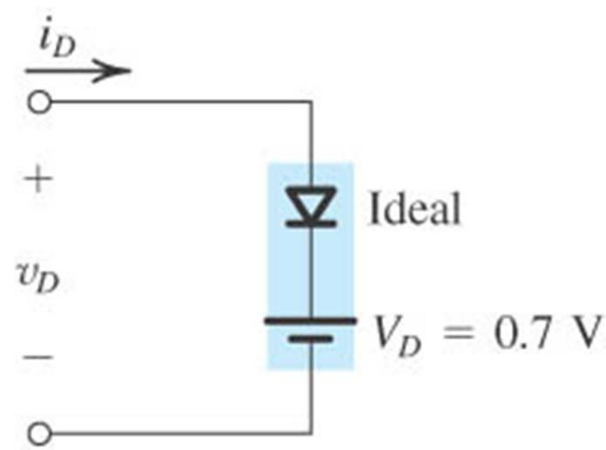
Piecewise-Linear Model



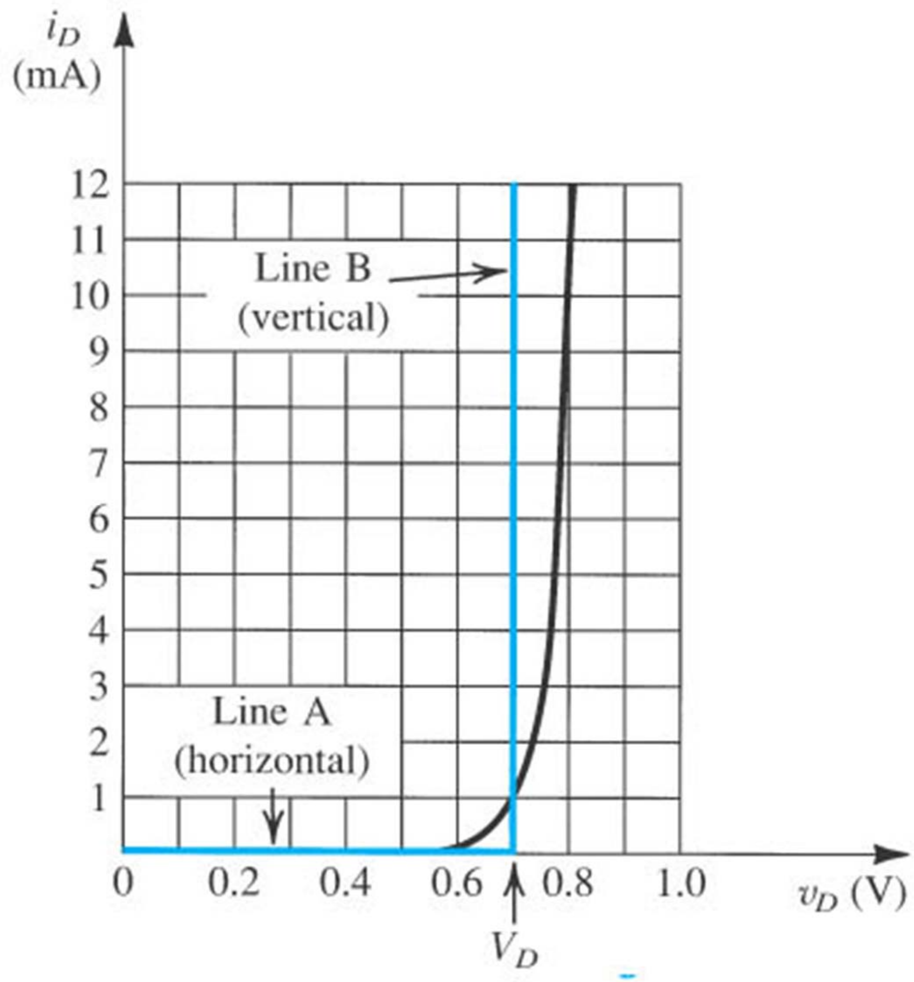
Constant-Voltage-Drop Model

$$for\ i_D > 0:\ v_D = 0.7\,V$$

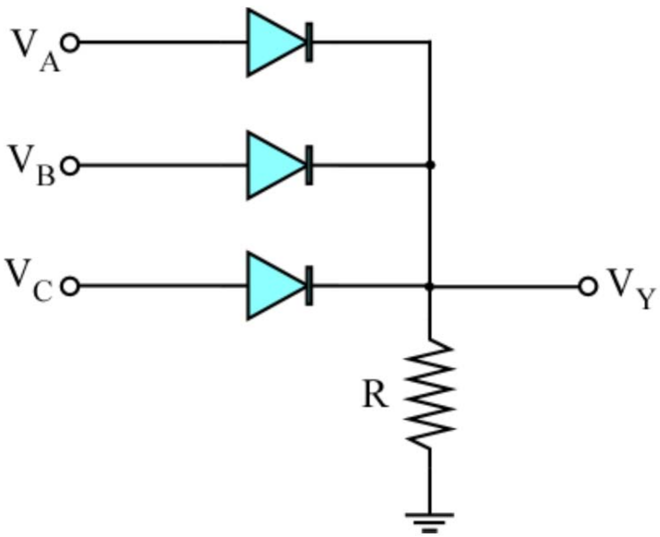


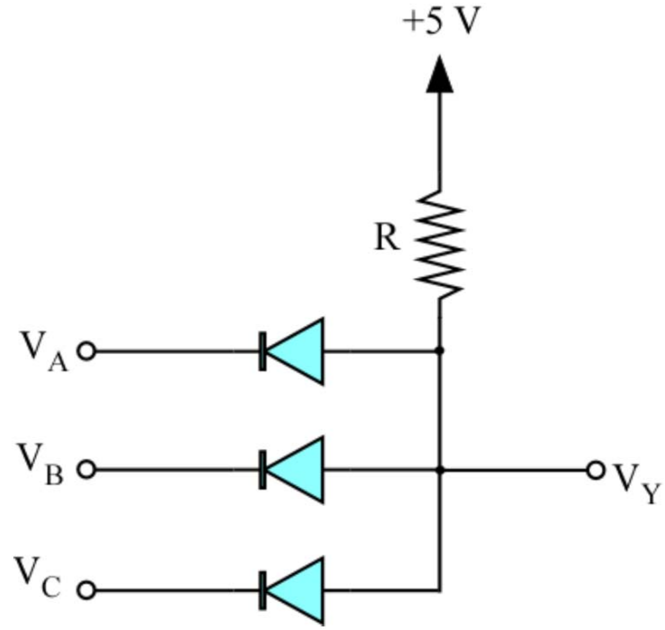


Constant-Voltage-Drop Model



Diodes Logic Gates





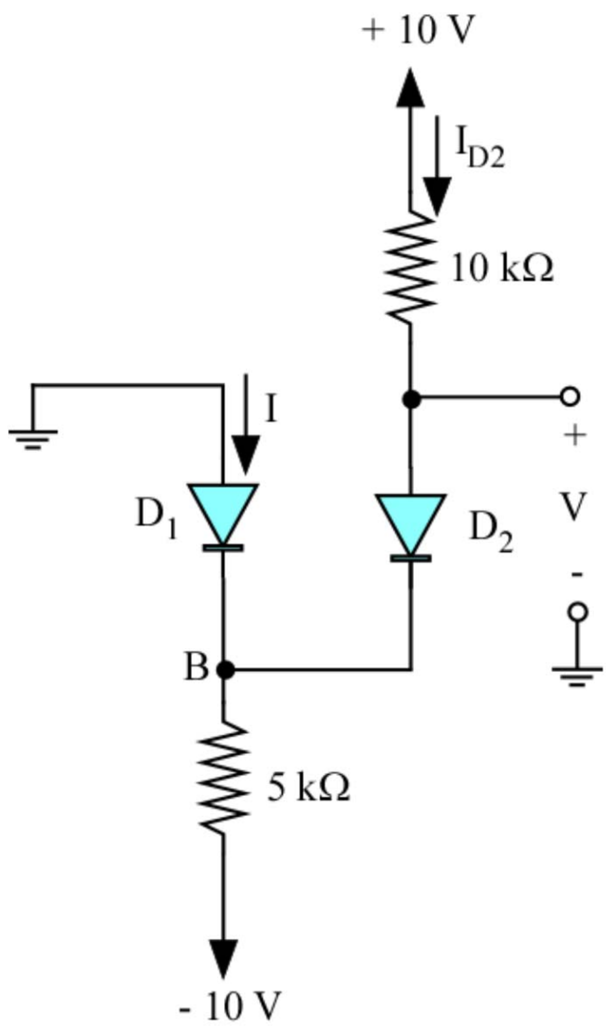
OR Function

$$Y = A + B + C$$

AND Function

$$Y = A \cdot B \cdot C$$

Diode Circuit Example 1



IDEAL Diodes

<u>Assume</u> both diodes are on; then

$$V_B = 0 \quad and \quad V = 0$$

$$I_{D2} = \frac{10-0}{10} = 1\,mA$$

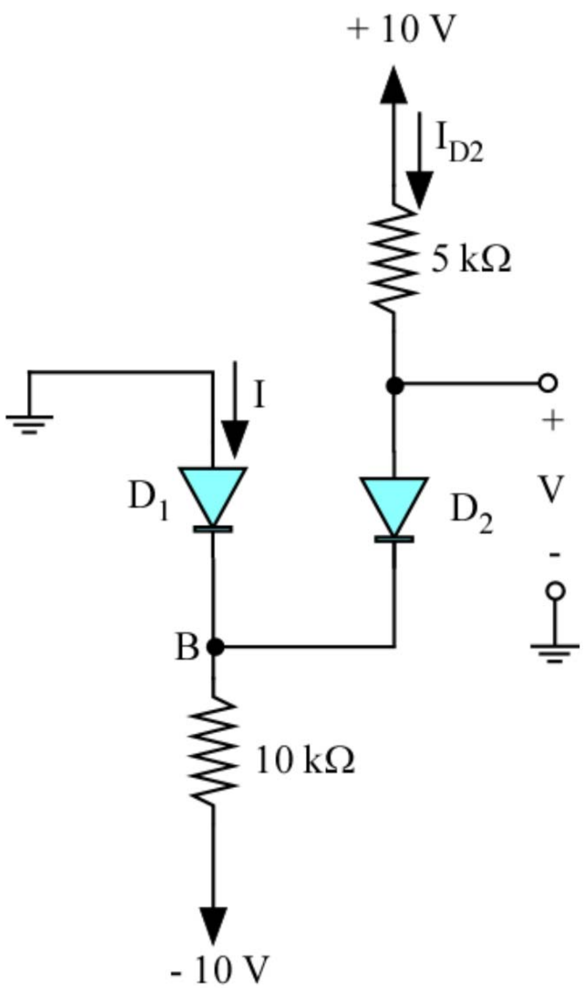
At node B

$$I + 1 = \frac{0-(-10)}{5} \Rightarrow I = 1\,mA, V = 0\,V$$

D_1 is conducting as originally assumed

Diode Circuit Example 2

IDEAL Diodes



Assume both diodes are on; then

$$V_B = 0 \quad and \quad V = 0$$

$$I_{D2} = \frac{10-0}{5} = 2\ mA$$

At node *B*

$$I + 2 = \frac{0-(-10)}{10} \Rightarrow I = -1\ mA \Rightarrow wrong$$

original assumption is not correct …
assume D_1 is off and D_2 is on

$$I_{D2} = \frac{10-(-10)}{15} = 1.33\ mA$$

$$V_B = -10 + 10 \times 1.33 = +3.3\ V$$

D_1 is reverse biased as assumed

Example

The diode has a value of $I_S = 10^{-12}$ mA at room temperature (300° K)

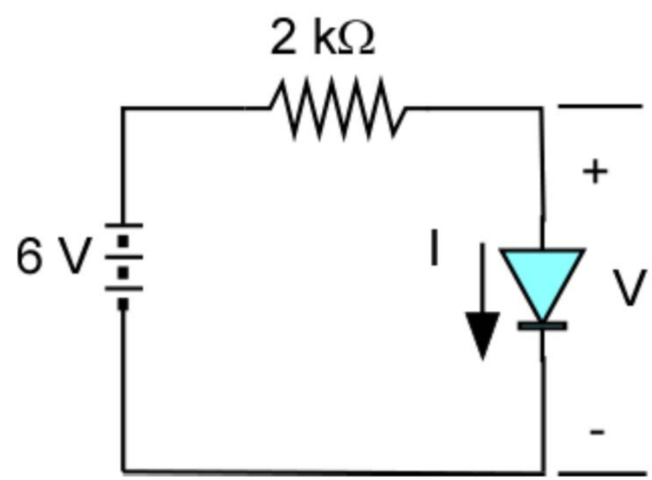
(a) Approximate the current I assuming the voltage drop across the diode is 0.7V

(b) Calculate the accurate value of I

(c) If I_S doubles for every 6° C increase in temperature, repeat part (b) if the temperature increases by 40° C

(a) The resistor will have an approximate voltage of 6-0.7 = 5.3 V. Ohm's law then gives a current of

$$I = \frac{5.3}{2} = 2.65\, mA$$



(b) The current through the resistor must equal the diode current; so we have

$$I = \frac{6-V}{2} (resistor\, current) \qquad I = I_S e^{V/V_T} (diode\, current)$$

Example (cont'd)

$$\frac{6-V}{2}=10^{-12}e^{V/0.026}$$

Nonlinear equation ➔ must be solved iteratively
Solution: V = 0.744 V

Using this value of the voltage, we can calculate the current

$$I=\frac{6-V}{2}=\frac{6-0.744}{2}=2.63\,mA$$

When the temperature changes, both I_s and V_T will change. Since $V_T=kT/q$ varies directly with T, the new value is:

$$V_T(340)=V_T(300)\times\frac{340}{300}=0.0295$$

Example (cont'd)

The value of I_s doubles for each 6° C increase, thus the new value of I_s is

$$I_S(340) = I_S(300) \times 2^{40/6} = 1.016 \times 10^{-10}\ mA$$

The equation for I is then

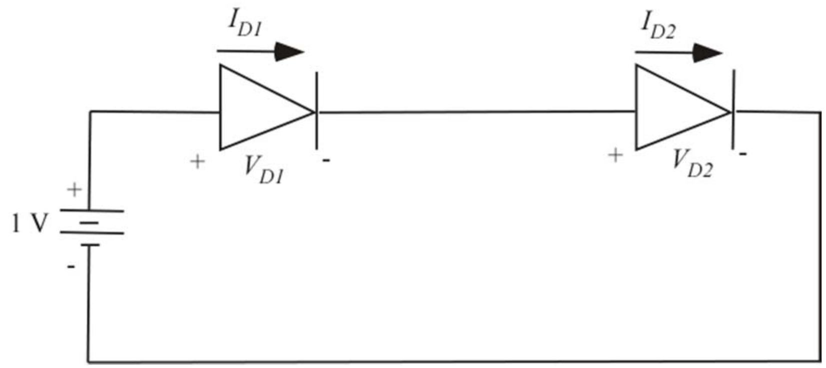
$$I = \frac{6 - V}{2} = 1.016 \times 10^{-10} \times e^{V/0.0295}$$

Solving iteratively, we get

$$V = 0.640\, V \quad and \quad I = 2.68\, mA$$

Example

Two diodes are connected in series as shown in the figure with I_{s1} $=10^{-16}$ A and $I_{s2}=10^{-14}$ A. If the applied voltage is 1 V, calculate the currents I_{D1} and I_{D2} and the voltage across each diode V_{D1} and V_{D2}.



The diode equations can be written as:

$$I_{D1} = I_{S1} e^{V_{D1}/V_T} \qquad I_{D2} = I_{S2} e^{V_{D2}/V_T} \qquad \frac{I_{S1}}{I_{S2}} e^{\frac{V_{D1}-V_{D2}}{V_T}} = \frac{I_{D1}}{I_{D2}} = 1$$

from which $V_{D1} - V_{D2} = -V_T \ln\left(\frac{I_{S1}}{I_{S2}}\right) = -0.12$

Using KVL, we get $V_{D1} + V_{D2} = 1$ from which $V_{D2} = 0.44$ V and $V_{D1} = 0.56$ V

$$I_{D1} = 10^{-16} e^{0.56/0.026} = 0.22\ \mu\text{A} = I_{D2}$$

Small Signal Model

Approximation - valid for small fluctuations about bias point

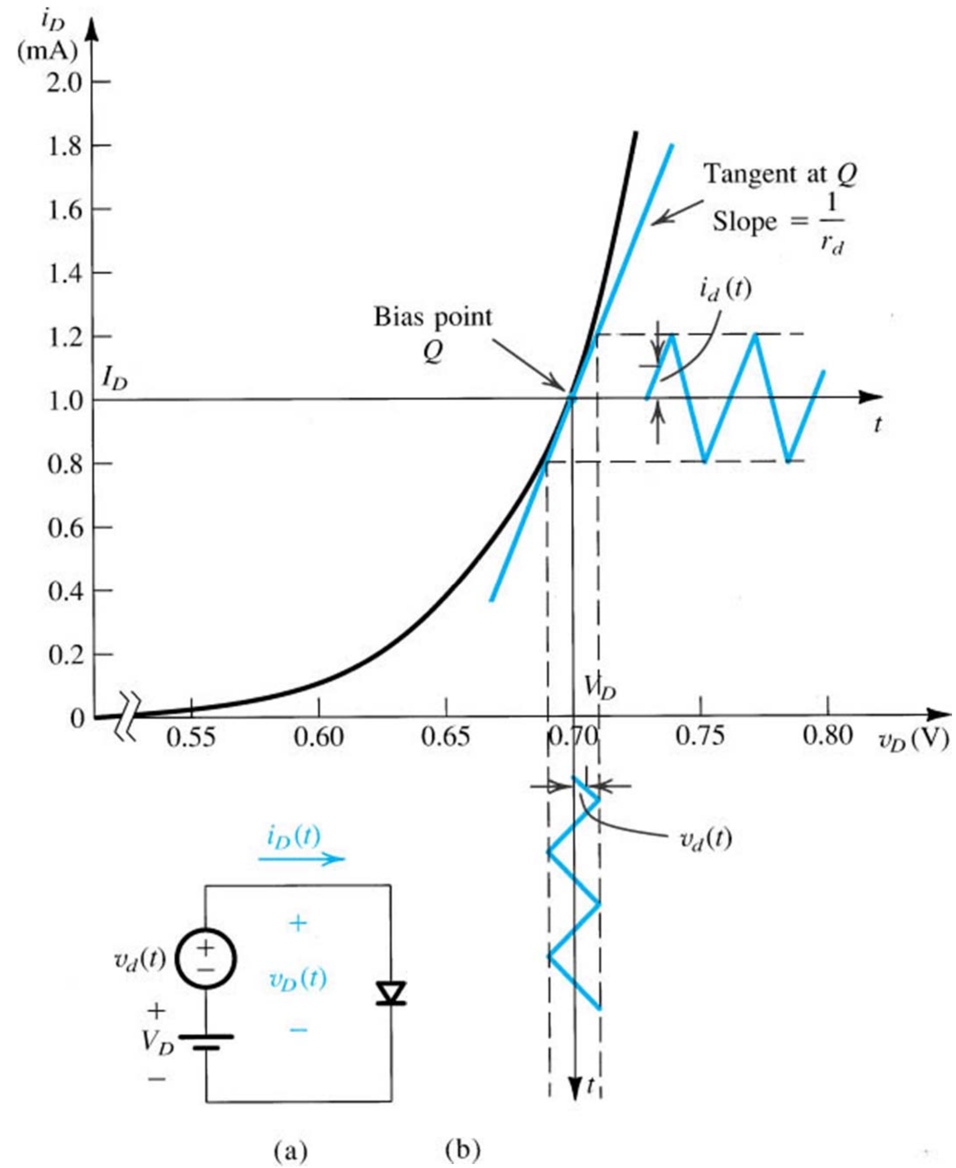
$$i_D = I_D e^{v_d/nV_T}$$

$$r_d = \frac{1}{\left[\frac{\partial i_D}{\partial v_D}\right]_{i_D = I_D}} = \frac{nV_T}{I_D}$$

$$i_D = I_D + i_d$$

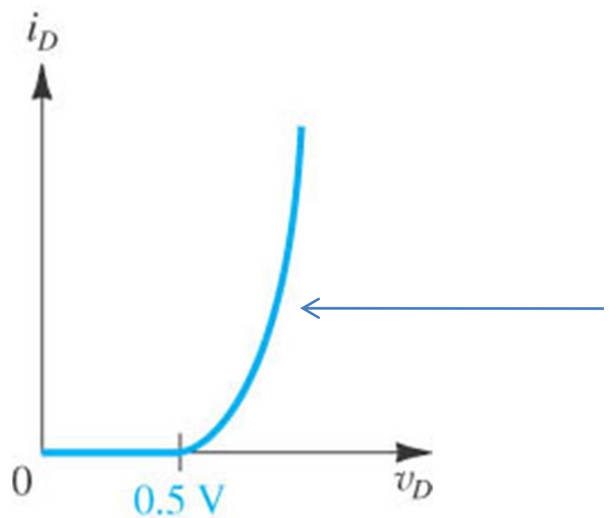
Total — DC — applied (small)

$$v_D = V_D + v_d$$



Diodes as Voltage Regulators

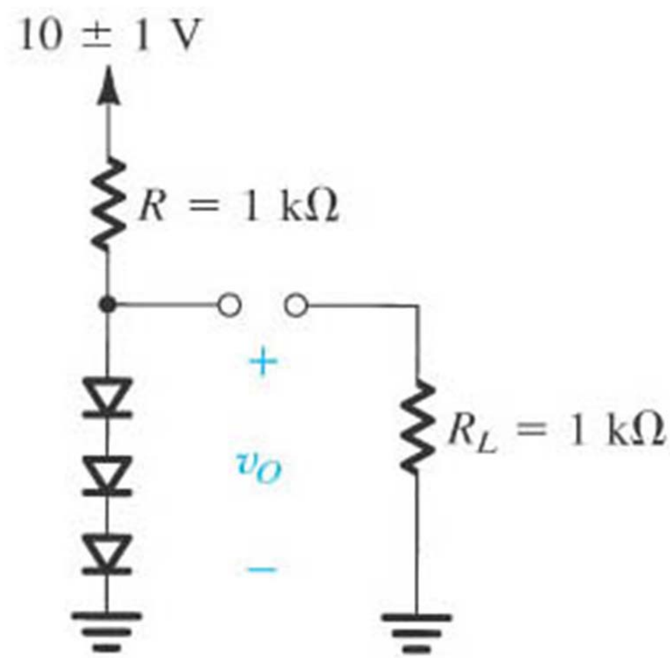
- **Objective**
  - Provide <u>constant</u> dc voltage between output terminals
  - Load current changes
  - Dc power supply changes
  - Take advantage of diode I-V exponential behavior



Big change in current correlates to small change in voltage

Voltage Regulator - Example

Assume $n=2$ and calculate % change caused by a ±10% change in power-supply voltage (a) with no load (b) with 1-kΩ load



Nominal value of current is:

$$I = \frac{10-2.1}{1} = 7.9\,mA$$

Incremental resistance for each diode:

$$r_d = \frac{nV_T}{I} = \frac{2\times 25}{7.9} = 6.3\Omega$$

Resistance for all 3 diodes:

$$r = 3r_d = 18.9\,\Omega$$

Voltage change

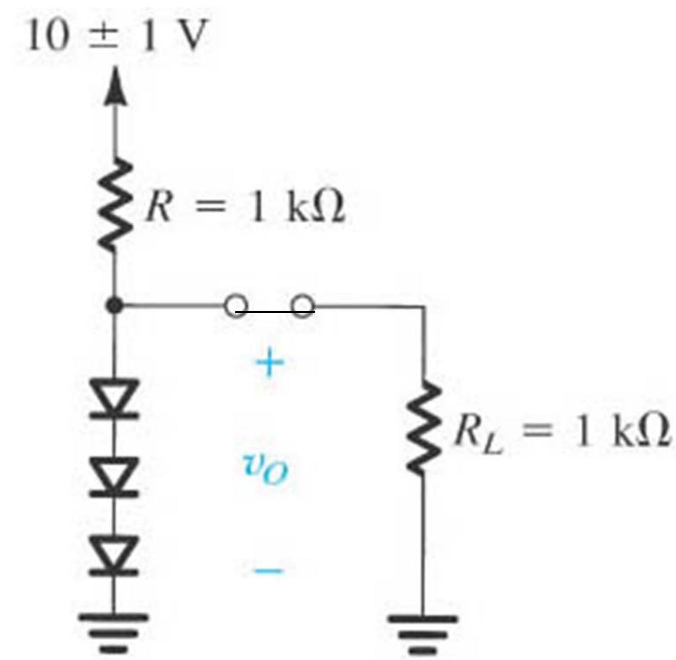
$$\Delta v_o = 2\frac{r}{r+R} = 2\frac{0.0189}{0.0189+1} = 37.1\,mV \rightarrow \pm 18.5\,mV \rightarrow \pm 0.9\%$$

Voltage Regulator – Example (con't)

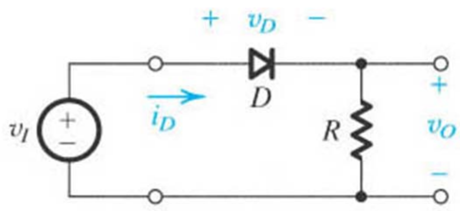
When 1kΩ load is connected, it draws a current of 2.1 mA resulting in a decrease in voltage across the 3 diodes given by

$$\Delta v_o = -2.1 \times r$$

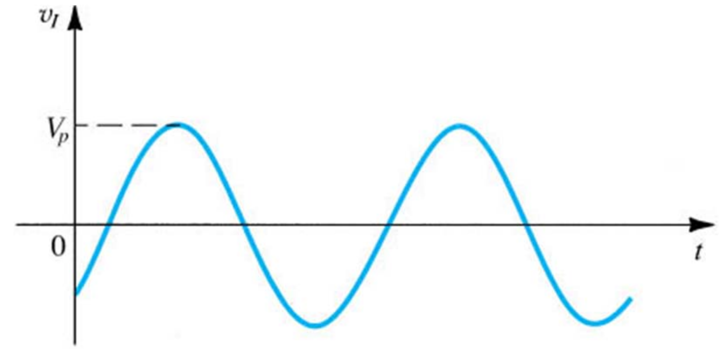
$$\Delta v_o = -2.1 \times 18.9 = -39.7\ mV$$

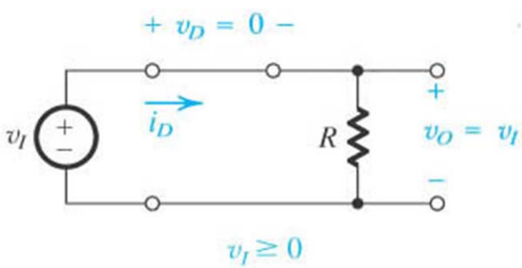


Diode as Rectifier

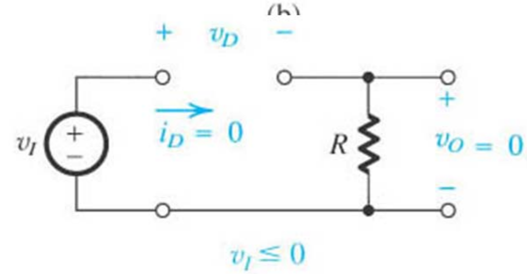


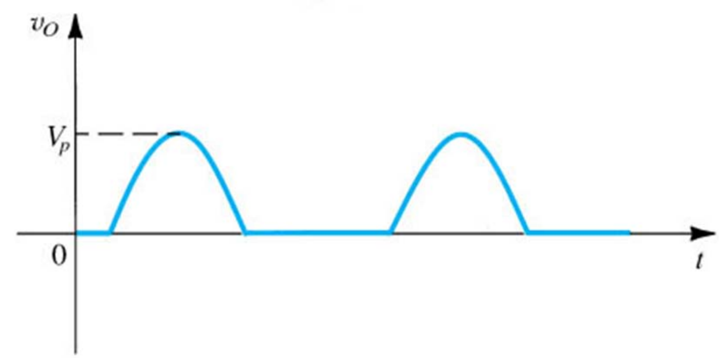
(a)





(c)

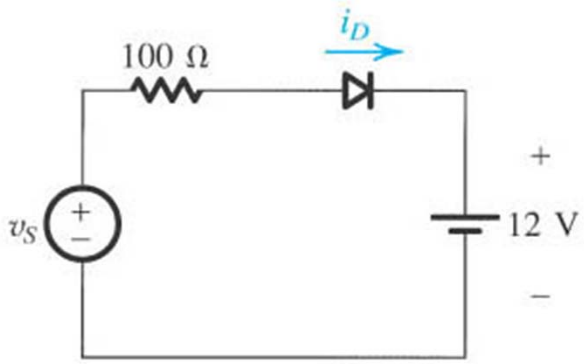


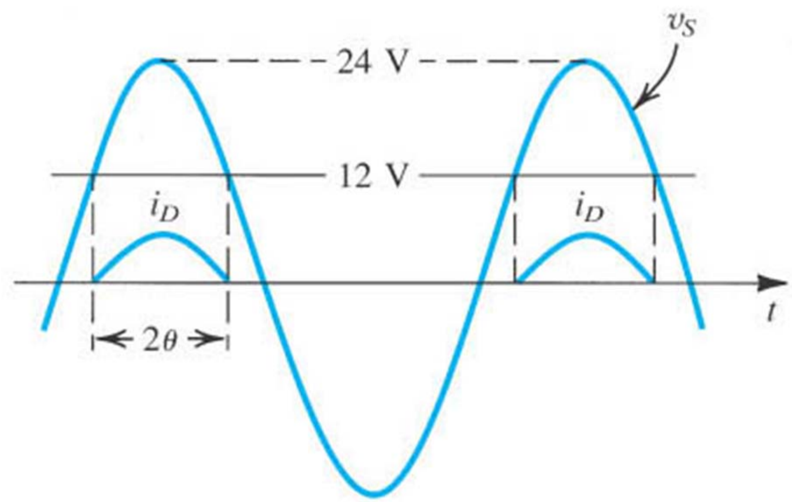


(e)

While applied source alternates in polarity and has zero average value, output voltage is unidirectional and has a finite average value or a *dc component*

Diode as Rectifier





v_s is a sinusoid with 24-V peak amplitude. The diode conducts when v_s exceeds 12 V. The conduction angle is 2θ where θ is given by

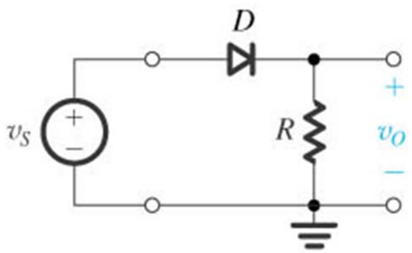
$$24\cos\theta = 12 \Rightarrow \theta = 60^\circ$$

The conduction angle is 120°, or one-third of a cycle. The peak value of the diode current is given by

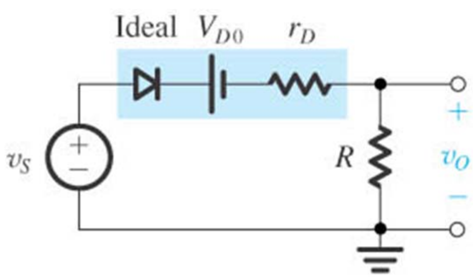
$$I_d = \frac{24-12}{100} = 0.12\ A$$

The maximum reverse voltage across the diode occurs when v_s is at its negative peak: 24+12=36 V

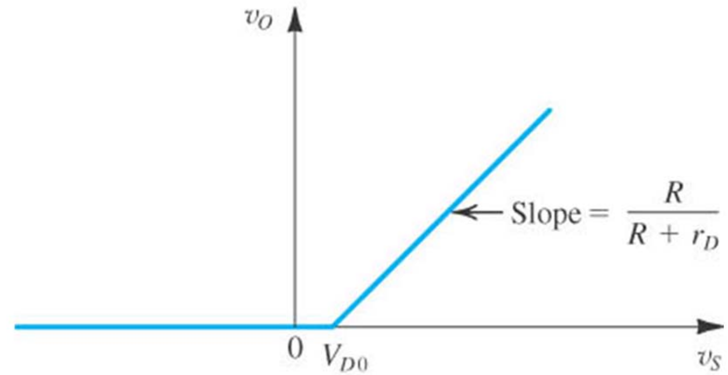
Half-Wave Rectifier

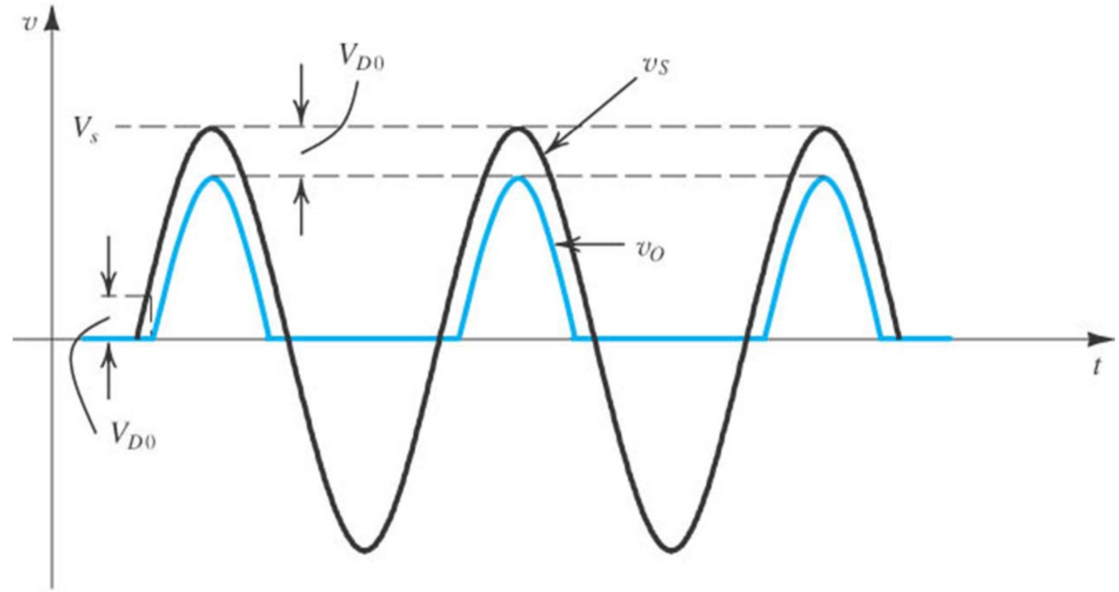


(a)



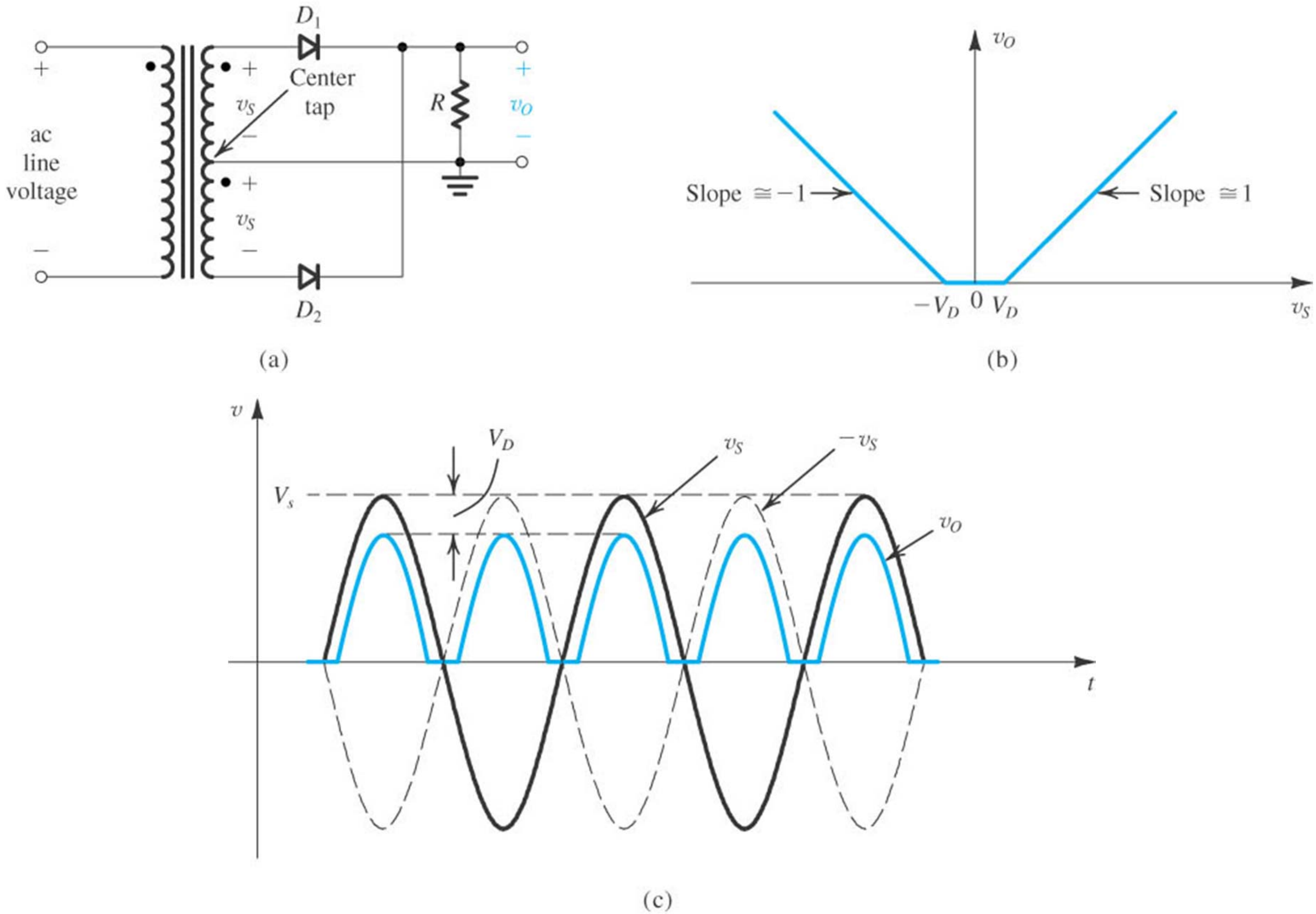
(b)



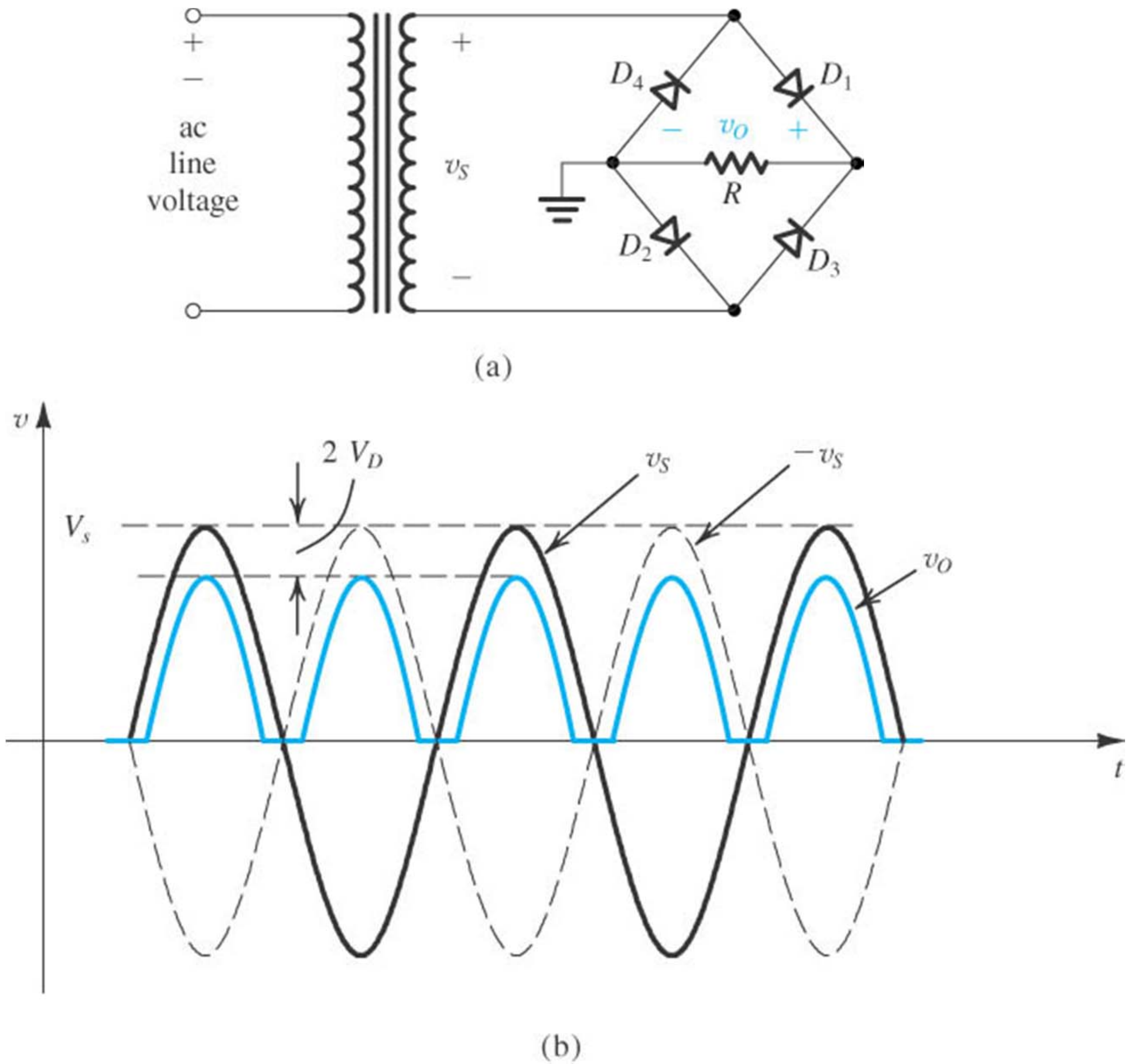


(d)

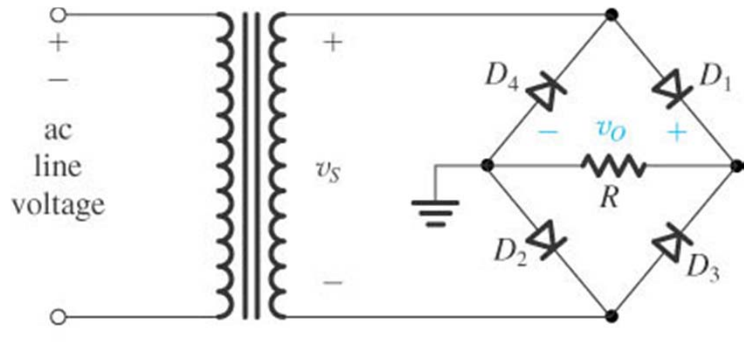
Full-Wave Rectifier



Bridge Rectifier



Bridge Rectifier



(a)

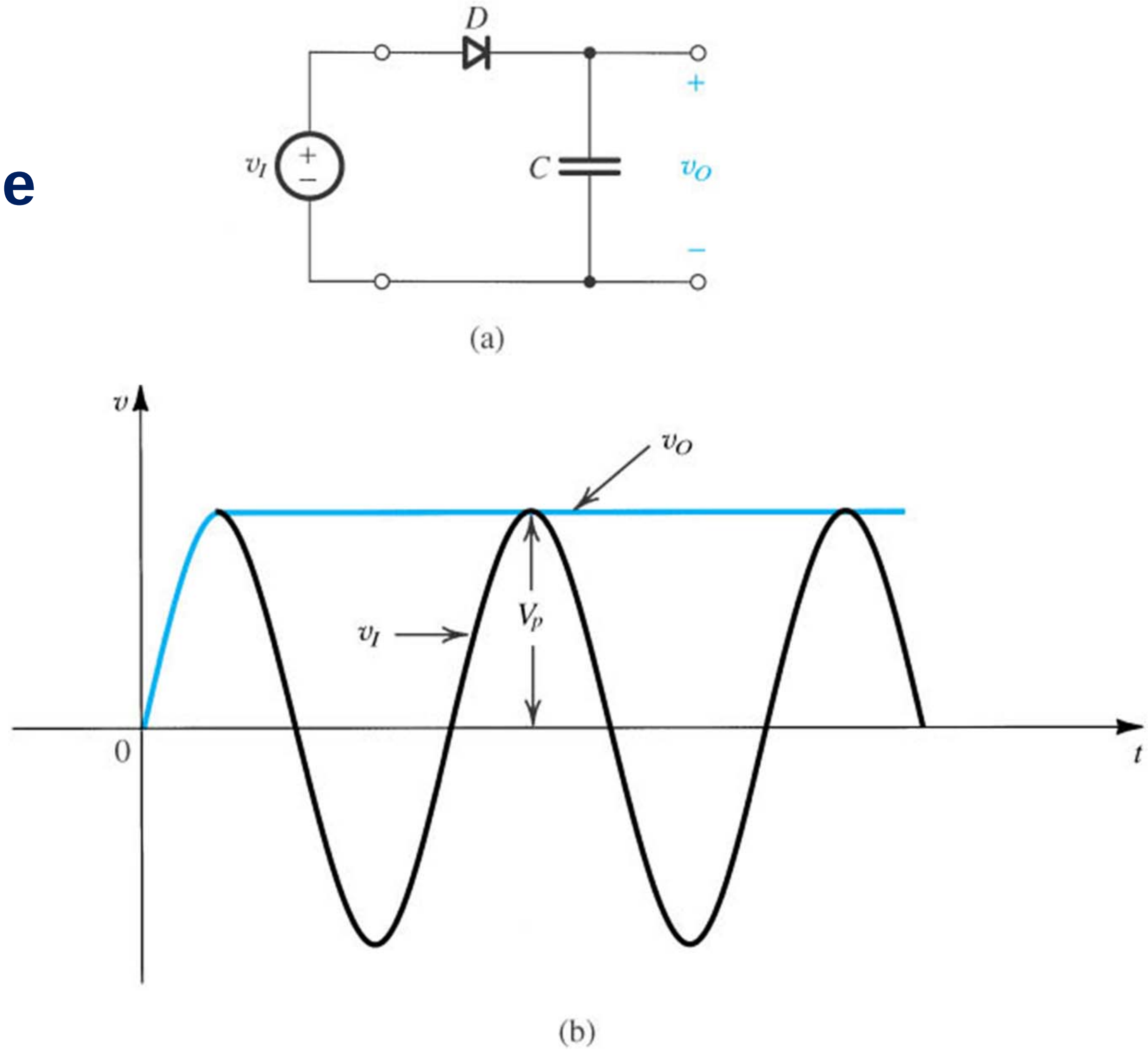
- **Properties**
  - Uses four diodes.
  - v_o is lower than v_s by two diode drops.
  - Current flows through R in the same direction during both half cycles.

The peak inverse voltage (PIV) of each diode:

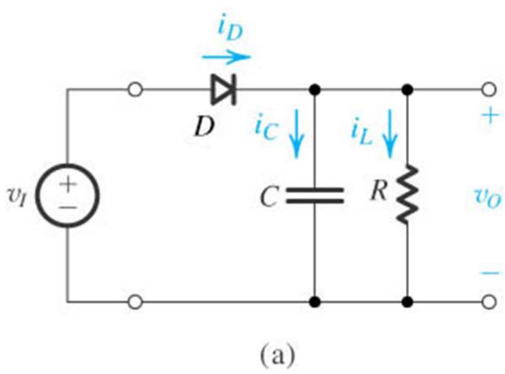
$$PIV = v_s - 2v_D + v_D = v_s - v_D$$

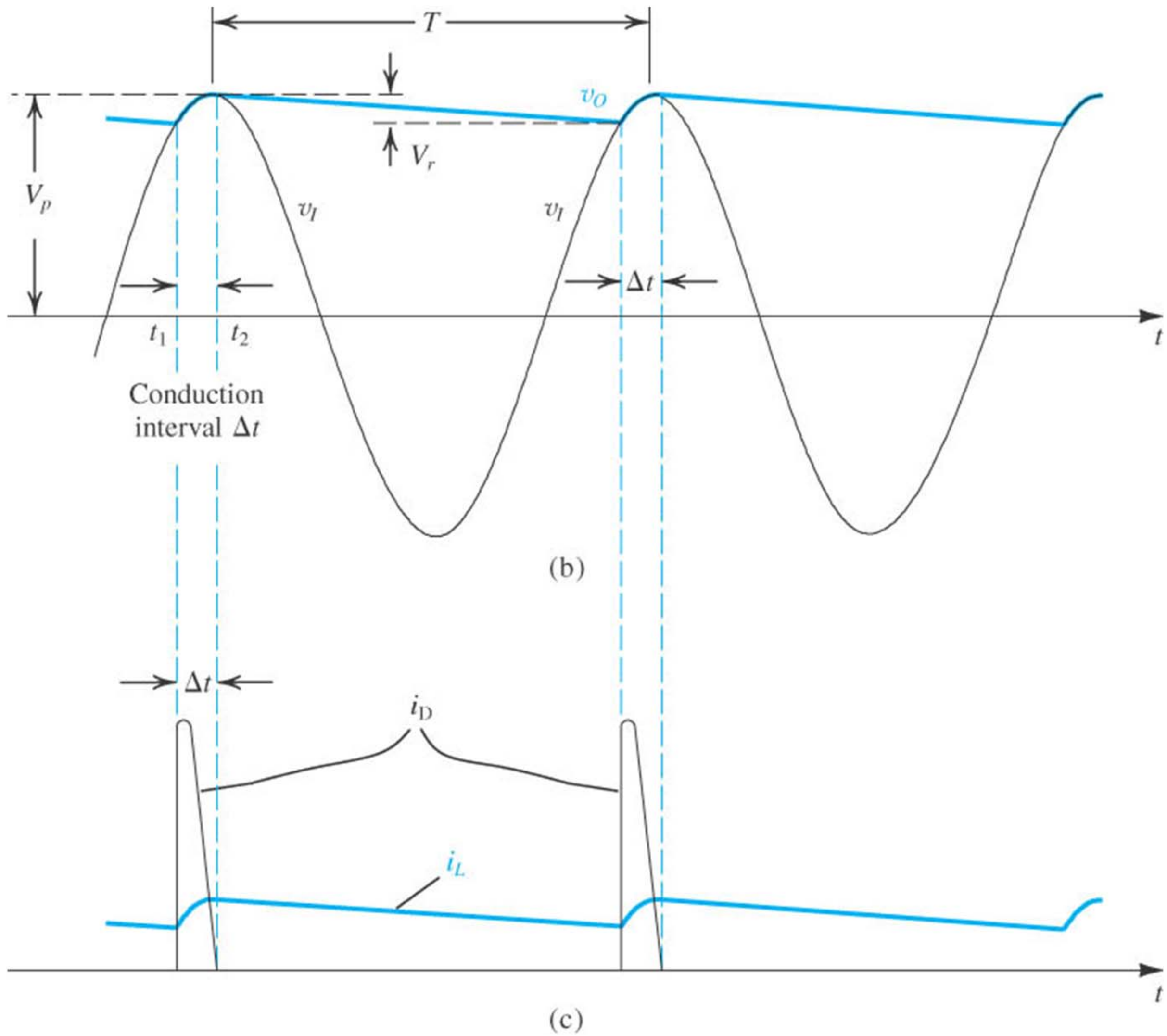
Peak Rectifier

Filter capacitor is used to reduce the variations in the rectifier output



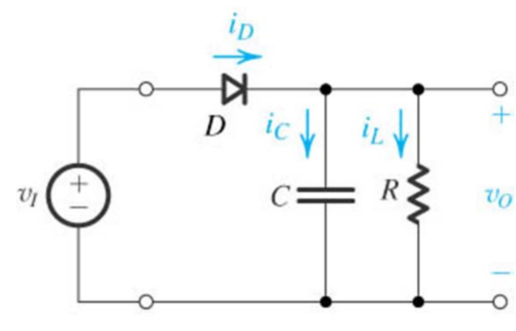
Rectifier with Filter Capacitor





Rectifier with Filter Capacitor

- **Operation**
  - Diode conducts for brief interval Δt
  - Conduction stops shortly after peak
  - Capacitor discharges through R
  - $CR >> T$
  - V_r is peak-to-peak ripple



(a)

$$i_L = v_o / R \qquad I_L = V_p / R$$

$$i_D = i_C + i_L = C\frac{dv_I}{dt} + i_L$$

$$i_{Dav} = I_L\left(1 + \pi\sqrt{2V_p / V_r}\right)$$

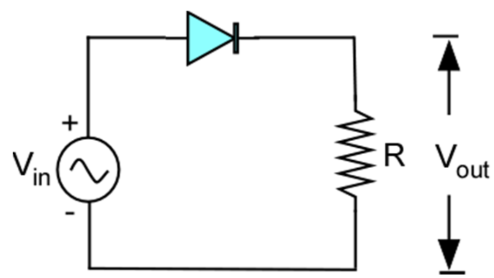
$$v_o = V_p e^{-t/CR}$$

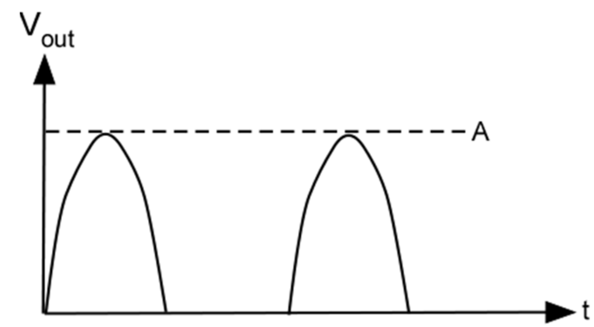
$$i_{D\max} = I_L\left(1 + 2\pi\sqrt{2V_p / V_r}\right)$$

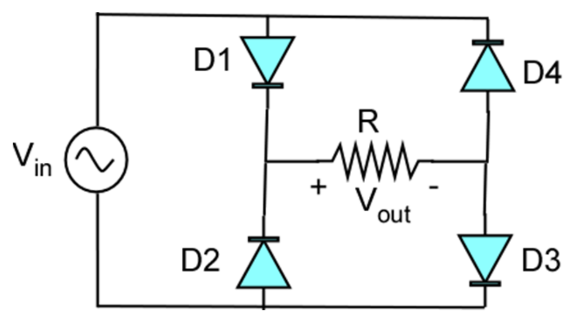
$$V_r \simeq V_p \frac{T}{CR} = \frac{V_p}{fCR} = \frac{I_L}{fC}$$

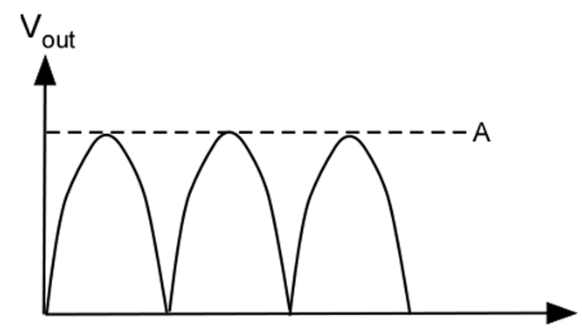
Diode Circuits - Rectification

$$V_{in} = A\sin\omega t$$



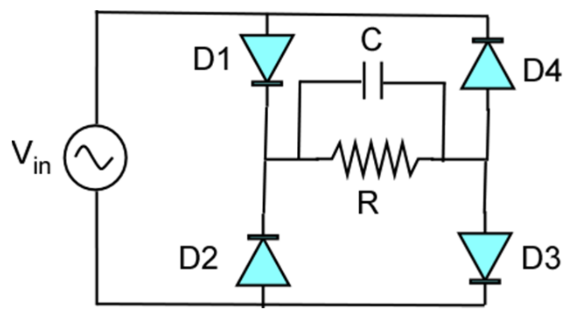


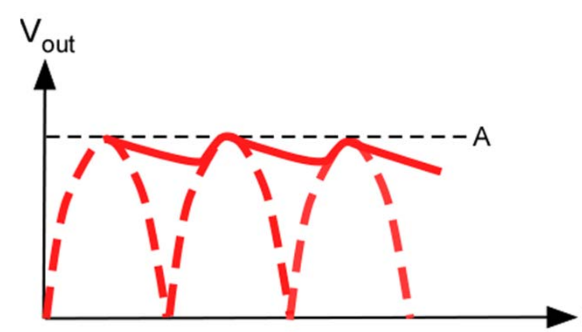




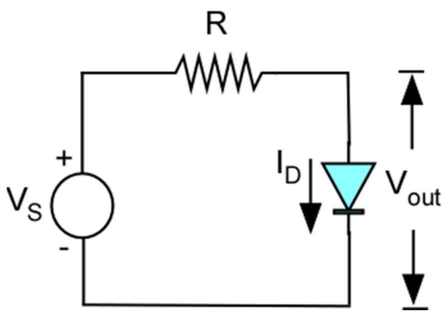
Rectification with ripple reduction.

C must be large enough so that RC time constant is much larger than period





Diode Circuits

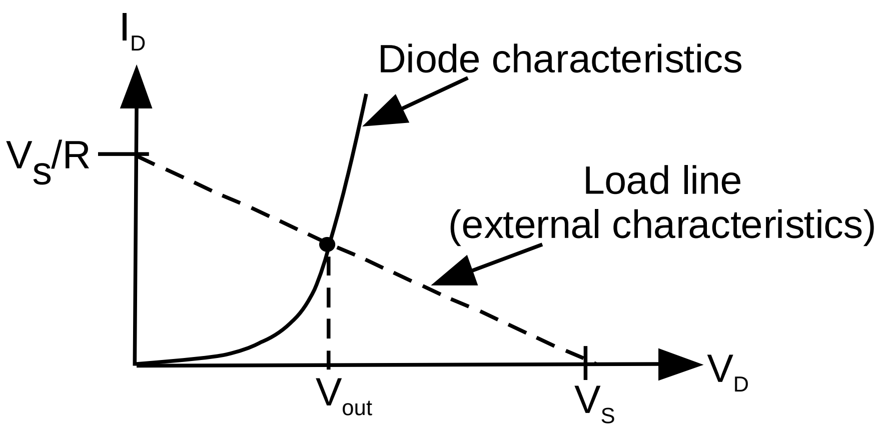


$$V_{out} = V_D$$

$$I_D = I_S\left(e^{V_D/V_T} - 1\right)$$

$$V_S = RI_D + V_D = RI_D(V_D) + V_D$$

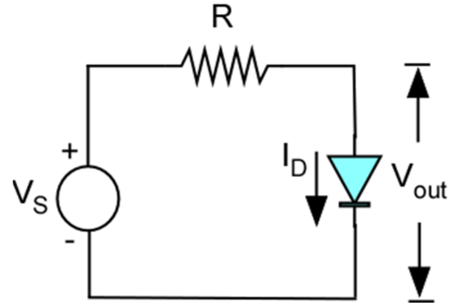
Nonlinear transcendental system ➔ Use graphical method



Solution is found at itersection of load line characteristics and diode characteristics

Diode Circuits – Iterative Methods

Newton-Raphson Method



Wish to solve *f(x)=0* for *x*

$$V_{out} = V_D$$

Use: $x_{k+1} = x_k - \left[f'(x_k) \right]^{-1} f(x_k)$

$$x^{(k+1)} = x^{(k)} - \left[f'(x^{(k)}) \right]^{-1} f(x^{(k)})$$

$$f(V_D) = \frac{V_D - V_S}{R} + I_S \left(e^{V_D/V_T} - 1 \right) = 0$$

$$f'(V_D) = \frac{1}{R} + \frac{I_S}{V_T} e^{V_D/V_T}$$

$$V_D^{(k+1)} = V_D^{(k)} - \frac{\dfrac{V_D^{(k)} - V_S}{R} + I_S \left(e^{V_D^{(k)}/V_T} - 1 \right)}{\dfrac{1}{R} + \dfrac{I_S}{V_T} e^{V_D^{(k)}/V_T}}$$

Where $V_D^{(k)}$ is the value of V_D at the *k*th iteration

Procedure is repeated until convergence to final (true) value of V_D which is the solution. Rate of convergence is quadratic.