Useful Formulae

A photon with a wavelength of 1 μm has an energy of 1.239 eV.
GaAs electron effective mass: $0.067m_o$
GaAs light hole effective mass: $0.12m_o$
GaAs heavy hole effective mass: $0.5m_o$
Free electron mass m_o: 9.11×10^{-31} kg
Atomic weight of Ga: 69.723 g/mole; Atomic weight of As: 74.922 g/mole
Atomic weight of Al: 26.982 g/mole; Atomic weight of In: 114.818 g/mole
Atomic weight of P: 30.974 g/mole
Avogadro's Number: 6.022×10^{23} molecules/mole
Boltzmann's Constant: 1.38×10^{-23} J/K = 8.62×10^{-5} eV/K
Plank's Constant: 6.626×10^{-34} J-s = 4.136×10^{-15} eV-s
Free Space Permittivity: $\varepsilon_0 = 8.85 \times 10^{-14}$ F/cm = 8.85×10^{-12} F/m (use 9×10^{-12} F/m for hand calculation)
1 eV = 1.602×10^{-19} Joules; q = 1.6×10^{-19} C; Speed of Light c = 2.998×10^{10} cm/s
Bohr Radius: 0.5292 Å; Rydberg: 13.6 eV; 1/e ~ 0.37
kT = 0.0259 eV @ Room Temperature; kT = 0.025 eV @ 290K; ln(x) = 2.3 log(x); ln(10) = 2.3

IIB	IIIA	IVA	VA	VIA
	5 B	6 C	7 N	8 O
	13 Al	14 Si	15 P	16 S
30 Zn	31 Ga	32 Ge	33 As	34 Se
48 Cd	49 In	50 Sn	51 Sb	52 Te

$$p = m^* v = \hbar k = \frac{h}{\lambda} \qquad E = h\nu = \hbar\omega \qquad E = \frac{1}{2}m^* v^2 = \frac{p^2}{2m^*} = \frac{\hbar^2 k^2}{2m^*}$$

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}} \cong e^{-(E-E_F)/kT} \text{ for } E >> E_F \qquad N(E) = DOS(E) = \frac{\sqrt{2}}{\pi^2}\left(\frac{m^*}{\hbar^2}\right)^{3/2} E^{1/2}$$

$$n_0 = \int_{E_c}^{\infty} f(E)N(E)dE$$ Triangle Area: A=0.5x(base)x(height) $$m^* = \frac{\hbar^2}{d^2E/dk^2}$$

Note: N(E) is referenced to the bottom of the conduction band for electrons and the top of the valence band for holes.

$$p_0 = N_V\left[1 - f(E_V)\right] = N_V e^{-(E_F-E_V)/kT} \qquad \frac{D}{\mu} = \frac{kT}{q} \qquad \mu = \frac{q\bar{t}}{m^*} \qquad n_i = \sqrt{N_C N_V}\, e^{-E_g/2kT}$$

$$N_C = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}; \; N_V = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2} \qquad n_i = N_C e^{-(E_C-E_i)/kT} \qquad p_i = N_V e^{-(E_i-E_V)/kT}$$

For $ax^2 + bx + c = 0, \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $$R = \frac{\rho L}{wt} = \frac{L}{\sigma wt}$$

$$n_0 = n_i e^{(E_F-E_i)/kT} \qquad p_0 = n_i e^{(E_i-E_F)/kT} \qquad n_0 p_0 = n_i^2 \qquad \sigma = q(n\mu_n + p\mu_p)$$

$$n = N_C e^{-(E_C-F_n)/kT} = n_i e^{(F_n-E_i)/kT} \qquad p = N_V e^{-(F_p-E_V)/kT} = n_i e^{(E_i-F_p)/kT} \qquad np = n_i^2 e^{(F_n-F_p)/kT}$$

$$\mathcal{E}(x) = -\frac{d\mathcal{V}(x)}{dx} = \frac{1}{q}\frac{dE_i}{dx} \qquad \frac{d\mathcal{E}(x)}{dx} = -\frac{d^2\mathcal{V}(x)}{dx^2} = \frac{\rho(x)}{\epsilon} = \frac{q}{\epsilon}(p - n + N_d^+ - N_a^-)$$

$\delta n = g_0\tau_n$; $\delta p = g_0\tau_p$ $$\delta p(t) = \delta p(0)\, e^{-\alpha_r(p_0+n_0)t} = \delta p(0) e^{-t/\tau_p} \qquad J_{drift} = \sigma\mathcal{E}$$

$p - n + N_d^+ - N_a^- = 0$ ←(neutral or quasi-neutral regions)

$$J_n(x) = q\mu_n n(x)\mathcal{E}(x) + qD_n \frac{dn(x)}{dx} \qquad J_p(x) = q\mu_p p(x)\mathcal{E}(x) - qD_p \frac{dp(x)}{dx}$$

$$\frac{\partial p(x,t)}{\partial t} = \frac{\partial \delta p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - \frac{\delta p}{\tau_p} \qquad \frac{\partial n(x,t)}{\partial t} = \frac{\partial \delta n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} - \frac{\delta n}{\tau_n}$$

$$L = \sqrt{D\tau} \qquad J_{total} = J_n + J_p$$

Continuity: $\dfrac{\partial p(x,t)}{\partial t} = \dfrac{\partial \delta p}{\partial t} = -\dfrac{1}{q}\dfrac{\partial J_p}{\partial x} - \dfrac{\delta p}{\tau_p}$ $\qquad \dfrac{\partial \delta n}{\partial t} = \dfrac{1}{q}\dfrac{\partial J_n}{\partial x} - \dfrac{\delta n}{\tau_n}$ $\qquad \boldsymbol{\tau_p = \dfrac{1}{\alpha_r(p_o+n_o)} \approx \dfrac{1}{\alpha_r n_o}}$

Steady State Diffusion: $\dfrac{d^2\delta n}{dx^2} = \dfrac{\delta n}{D_n\tau_n} = \dfrac{\delta n}{L_n^2}$ $\qquad \dfrac{d^2\delta p}{dx^2} = \dfrac{\delta p}{D_p\tau_p} = \dfrac{\delta p}{L_p^2}$ $\qquad \boldsymbol{\tau_n = \dfrac{1}{\alpha_r(p_o+n_o)} \approx \dfrac{1}{\alpha_r p_o}}$

$$V_0 = \frac{kT}{q}\ln\frac{p_p}{p_n} = \frac{kT}{q}\ln\frac{N_a}{(n_i^2/N_d)} = \frac{kT}{q}\ln\frac{N_aN_d}{n_i^2} \qquad \frac{p_p}{p_n} = \frac{n_n}{n_p} = e^{qV_0/kT}$$

$$qV_0 = E_{vp} - E_{vn} \qquad -V_0 = \int_{-x_{p0}}^{x_{n0}} \mathcal{E}(x)\,dx$$

Short Base Diode: $I_p(x_n) = AJ_p(diff) = -AqD_p\dfrac{dp(x_n)}{dx_n} \approx qAD_p\dfrac{\Delta p_n}{\ell}$ $\qquad Q_p = \dfrac{1}{2}qA\ell\Delta p_n$

$$W = \left[\frac{2\varepsilon(V_0 - V)}{q}\left(\frac{N_a + N_d}{N_aN_d}\right)\right]^{1/2} \qquad qAx_{p0}N_a = qAx_{n0}N_d$$

$$\Delta p_n = p(x_{n0}) - p_n = p_n\left(e^{qV/kT} - 1\right) \qquad \Delta n_p = n(-x_{p0}) - n_p = n_p\left(e^{qV/kT} - 1\right)$$

$\delta p(x) = \Delta p_n e^{-x/L_p}$ $\qquad \delta n(x) = \Delta n_p e^{-x/L_n}$ $\qquad$ (referenced to depletion region edge)

$$I = qA\left(\frac{D_p}{L_p}p_n + \frac{D_n}{L_n}n_p\right)\left(e^{qV/kT} - 1\right) = I_0\left(e^{qV/kT} - 1\right) \qquad C = \left|\frac{dQ}{dV}\right|$$

$$I_{op} = qAg_{op}(L_p + L_n + W) \qquad I = I_0\left(e^{qV/kT} - 1\right) - I_{op} \qquad f.f. = \frac{I_mV_m}{I_{sc}V_{oc}}$$

$$C_j = \varepsilon A\left[\frac{q}{2\varepsilon(V_0 - V)}\frac{N_aN_d}{N_a + N_d}\right]^{1/2} = \frac{\varepsilon A}{W}$$

$$Q_p = qA\int_0^\infty \delta p(x_n)\,dx_n = qA\Delta p_n\int_0^\infty e^{-x_n/L_p}\,dx_n = qAL_p\Delta p_n$$

$$I_p(x_n = 0) = \frac{Q_p}{\tau_p} = qA\frac{L_p}{\tau_p}\Delta p_n = qA\frac{D_p}{L_p}p_n\left(e^{qV/kT} - 1\right) \qquad G_s = \frac{dI}{dV} = \frac{qAL_pp_n}{\tau_p}\frac{d}{dV}\left(e^{qV/kT}\right) = \frac{q}{kT}I$$

n-Channel MOS:

$$C_i = \frac{\varepsilon_i}{d} \qquad C_d = \frac{\varepsilon_s}{W} \qquad C = \frac{C_i C_d}{C_i + C_d} \qquad V_T = \Phi_{ms} - \frac{Q_i}{C_i} - \frac{Q_d}{C_i} + 2\phi_F$$

$$\phi_s(inv.) = 2\phi_F = 2\frac{kT}{q}\ln\frac{N_a}{n_i} \qquad W = \left[\frac{2\varepsilon_s\phi_s}{qN_a}\right]^{1/2} \qquad Q_d = -qN_a W_m = -2\left(\varepsilon_s q N_a \phi_F\right)^{1/2}$$

At V_{FB}:

$$C_{FB} = \frac{C_i C_{debye}}{C_i + C_{debye}} \qquad L_D = \sqrt{\frac{\varepsilon_s kT}{q^2 p_0}} \qquad C_{debye} = \frac{\varepsilon_s}{L_D}$$

$$I_D = \frac{\bar{\mu}_n Z C_i}{L}\left\{\left(V_G - V_{FB} - 2\phi_F - \frac{1}{2}V_D\right)V_D - \frac{2}{3}\frac{\sqrt{2\varepsilon_s q N_a}}{C_i}\left[\left(V_D + 2\phi_F\right)^{3/2} - \left(2\phi_F\right)^{3/2}\right]\right\}$$

$$I_D \simeq \frac{\bar{\mu}_n Z C_i}{L}\left\{\left(V_G - V_T\right)V_D - \frac{1}{2}V_D^2\right\} \qquad g_m = \frac{\partial I_D}{\partial V_G}$$

$$g_m(sat.) = \frac{\partial I_D(sat.)}{\partial V_G} \simeq \frac{Z}{L}\bar{\mu}_n C_i\left(V_G - V_T\right)$$

Saturation:

$$I_D(sat.) \simeq \frac{1}{2}\bar{\mu}_n C_i \frac{Z}{L}\left(V_G - V_T\right)^2 = \frac{Z}{2L}\bar{\mu}_n C_i V_D^2(sat.)$$

$$V_T = \Phi_{mn} - \frac{Q_i}{C_i} - \frac{Q_d}{C_i} + 2\phi_F = V_{FB} - \frac{Q_d}{C_i} + 2\phi_F$$

$V_T =$	Φ_{ms}	$-\frac{Q_i}{C_i}$	$-\frac{Q_d}{C_i}$	$+2\phi_F$
	(−)	(−)	(+) n channel (−) p channel	(+) n channel (−) p channel

Note: The first two terms on the right-hand side of this expression correspond to the flatband voltatge.

p-Channel MOS: For a p-channel device ϕ_F is negative and Q_d is positive.

p-n-p BJT:

$$I_{Ep} = qA\frac{D_p}{L_p}\left(\Delta p_E ctnh\frac{W_b}{L_p} - \Delta p_C \csc h\frac{W_b}{L_p}\right) \qquad I_C = qA\frac{D_p}{L_p}\left(\Delta p_E \csc h\frac{W_b}{L_p} - \Delta p_C\, ctnh\frac{W_b}{L_p}\right)$$

$$I_B = qA\frac{D_p}{L_p}\left[\left(\Delta p_E + \Delta p_C\right)\tanh\frac{W_b}{2L_p}\right]$$

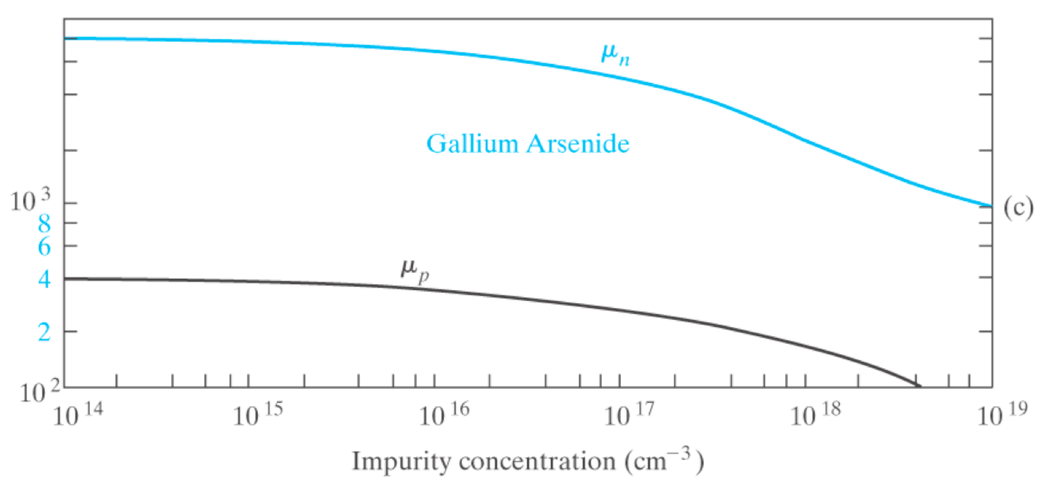
$$\Delta p_E = p_n\left(e^{qV/kT} - 1\right) \qquad \Delta p_C = p_n\left(e^{qV_{CB}/kT} - 1\right) \qquad I_B(recomb) = \frac{Q}{\tau}$$

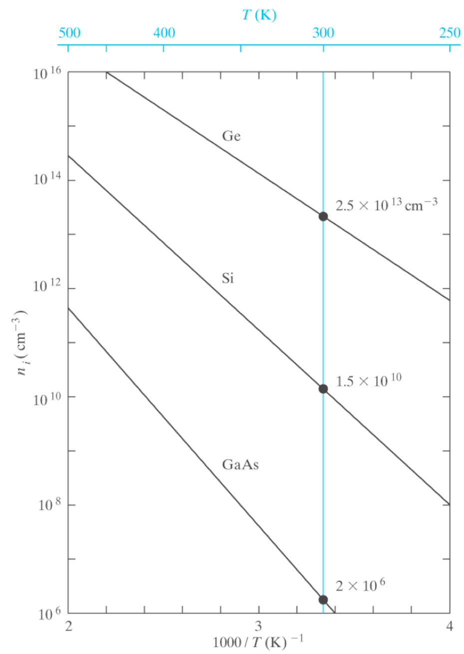
$$B = \frac{I_C}{I_{Ep}} = \frac{\csc h\frac{W_b}{L_p}}{ctnh\frac{W_b}{L_p}} = \sec h\frac{W_b}{L_p} \simeq 1 - \left(\frac{W_b^2}{2L_p^2}\right) \qquad \frac{i_C}{i_E} = B\gamma \equiv \alpha$$

$$\gamma = \frac{I_{Ep}}{I_{En} + I_{Ep}} = \left[1 + \frac{L_p^n n_n \mu_n^p}{L_n^p p_p \mu_p^n}\tanh\frac{W_b}{L_p^n}\right]^{-1} \simeq \left[1 + \frac{W_b n_n \mu_n^p}{L_n^p p_p \mu_p^n}\right]^{-1}$$

$$\frac{i_C}{i_B} = \frac{B\gamma}{1 - B\gamma} = \frac{\alpha}{1-\alpha} \equiv \beta \qquad \frac{i_C}{i_B} = \beta = \frac{\tau_p}{\tau_t} \quad \left(\text{for } \gamma = 1\right)$$

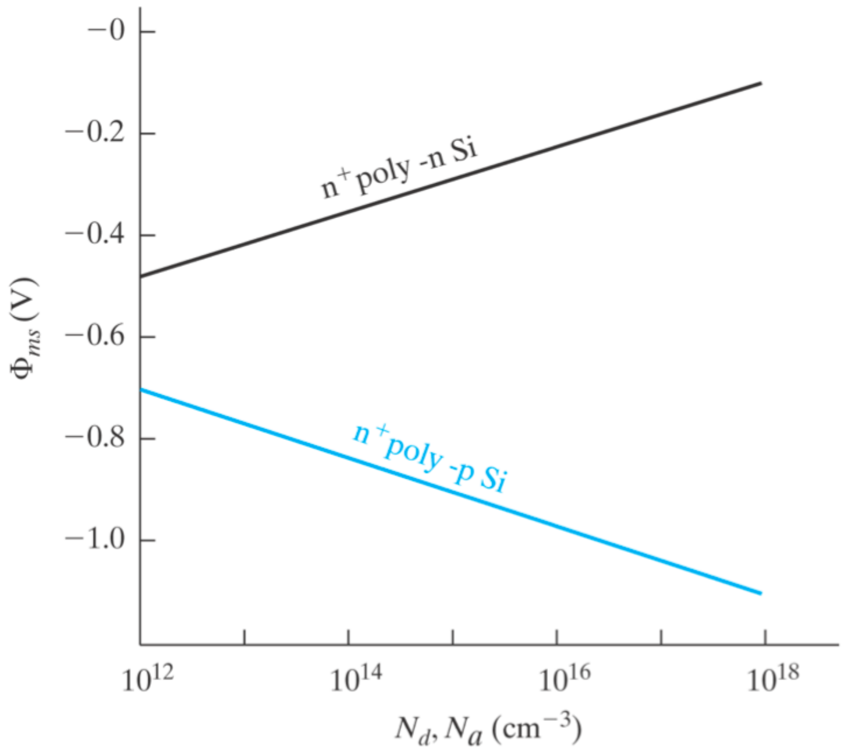
$$\frac{1}{r_\pi} = \frac{dI_B}{dV_{EB}} \approx \frac{q}{kT}I_B \qquad i_C = \beta i_B = -\beta\frac{v_{in}}{r_\pi} \qquad v_{out} = i_C R_C = -\beta\frac{R_C}{r_\pi}v_{in}$$

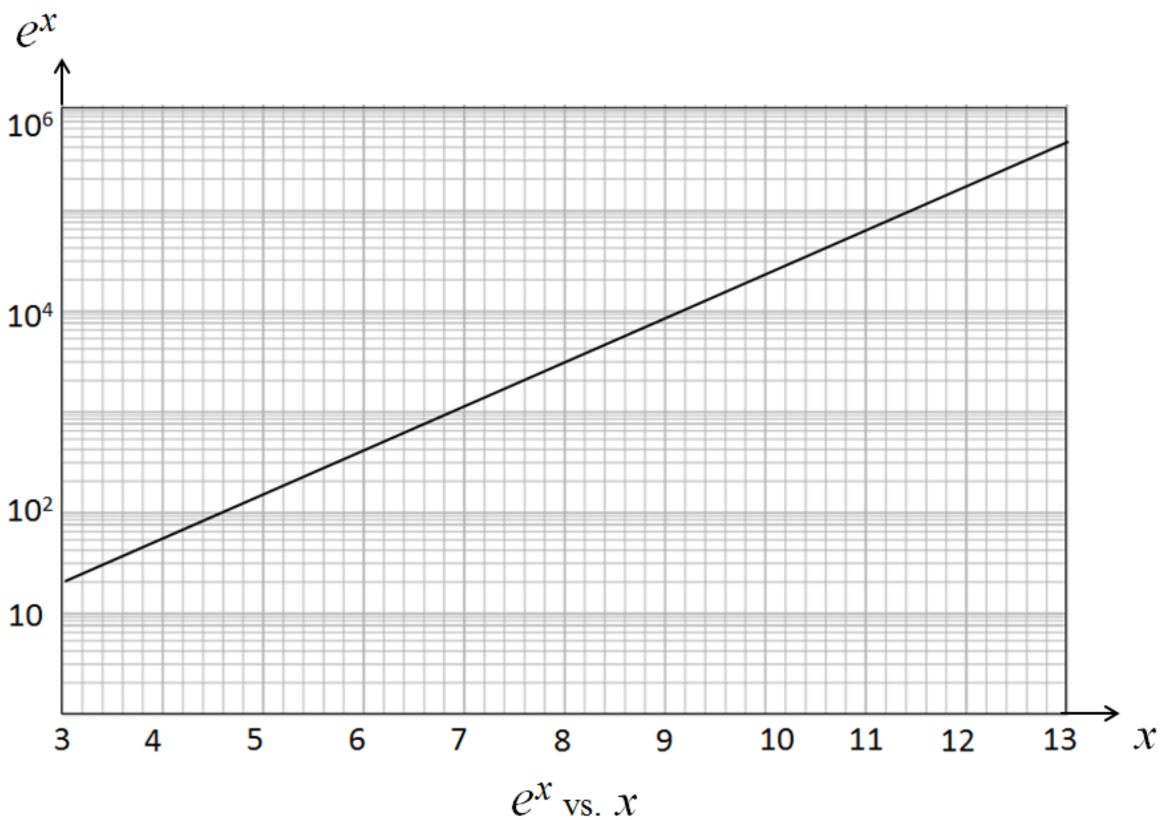




		E_g (eV)	μ_n (cm^2/V-s)	μ_p (cm^2/V-s)	m_n^*/m_0 (m_l, m_t)	m_p^*/m_0 (m_{lh}, m_{hh})	a (Å)	ϵ_r	Density (g/cm^3)	Melting point (°C)
Si	(i/D)	1.11	1350	480	0.98, 0.19	0.16, 0.49	5.43	11.8	2.33	1415
Ge	(i/D)	0.67	3900	1900	1.64, 0.082	0.04, 0.28	5.65	16	5.32	936
SiC (α)	(i/W)	2.86	500	—	0.6	1.0	3.08	10.2	3.21	2830
AlP	(i/Z)	2.45	80	—	—	0.2, 0.63	5.46	9.8	2.40	2000
AlAs	(i/Z)	2.16	1200	420	2.0	0.15, 0.76	5.66	10.9	3.60	1740
AlSb	(i/Z)	1.6	200	300	0.12	0.98	6.14	11.0	4.26	1080
GaP	(i/Z)	2.26	300	150	1.12, 0.22	0.14, 0.79	5.45	11.1	4.13	1467
GaAs	(d/Z)	1.43	8500	400	0.067	0.074, 0.50	5.65	13.2	5.31	1238
GaN	(d/Z, W)	3.4	380	—	0.19	0.60	4.5	12.2	6.1	2530
GaSb	(d/Z)	0.7	5000	1000	0.042	0.06, 0.23	6.09	15.7	5.61	712
InP	(d/Z)	1.35	4000	100	0.077	0.089, 0.85	5.87	12.4	4.79	1070
InAs	(d/Z)	0.36	22600	200	0.023	0.025, 0.41	6.06	14.6	5.67	943
InSb	(d/Z)	0.18	10^5	1700	0.014	0.015, 0.40	6.48	17.7	5.78	525
ZnS	(d/Z, W)	3.6	180	10	0.28	—	5.409	8.9	4.09	1650*
ZnSe	(d/Z)	2.7	600	28	0.14	0.60	5.671	9.2	5.65	1100*
ZnTe	(d/Z)	2.25	530	100	0.18	0.65	6.101	10.4	5.51	1238*
CdS	(d/W, Z)	2.42	250	15	0.21	0.80	4.137	8.9	4.82	1475
CdSe	(d/W)	1.73	800	—	0.13	0.45	4.30	10.2	5.81	1258
CdTe	(d/Z)	1.58	1050	100	0.10	0.37	6.482	10.2	6.20	1098
PbS	(i/H)	0.37	575	200	0.22	0.29	5.936	17.0	7.6	1119
PbSe	(i/H)	0.27	1500	1500	—	—	6.147	23.6	8.73	1081
PbTe	(i/H)	0.29	6000	4000	0.17	0.20	6.452	30	8.16	925

All values at 300 K. *Vaporizes


−0
−0.2
−0.4
−0.6
−0.8
−1.0
Φ_{ms} (V)
n⁺poly -n Si
n⁺poly -p Si
10^{12}
10^{14}
10^{16}
10^{18}
N_d, N_a (cm^{-3})


e^x
10^6
10^4
10^2
10
3
4
5
6
7
8
9
10
11
12
13
x
e^x vs. x