Review of Periodic Signals

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$$

$$\omega_0 = \frac{2\pi}{T}$$

$$F_n \longrightarrow \boxed{H(\omega)} \longrightarrow Y_n = H(n\omega_0) F_n$$

* What if $f(t)$ is no loger periodic?

— Fourier Transform

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$$

$$F_n = F(n\omega_0)$$

$$\frac{\omega_0}{2\pi} \hat{F}(n\omega_0) = F(n\omega_0)$$

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{\omega_0}{2\pi} \hat{F}(n\omega_0) e^{jn\omega_0 t}$$

$$f(t) e^{-jm\omega_0 t} = \sum_{n=-\infty}^{\infty} \frac{\omega_0}{2\pi} \hat{F}(n\omega_0) e^{j(n-m)\omega_0 t}$$

$$\int_{-T/2}^{T/2} f(t) e^{-jm\omega_0 t} dt = \sum_{n=-\infty}^{\infty} \frac{\omega_0}{2\pi} \hat{F}(n\omega_0) \int_{-T/2}^{T/2} e^{j(n-m)\omega_0 t} dt$$

$n = m$ for $\int_{-T/2}^{T/2} e^{j(n-m)\omega_0 t} dt \neq 0$

$$= \frac{\omega_0}{2\pi} \hat{F}(m\omega_0) T$$

$$\boxed{\hat{F}(m\omega_0) = \int_{-T/2}^{T/2} f(t) e^{-jm\omega_0 t} dt} \qquad T = \frac{2\pi}{\omega_0}$$

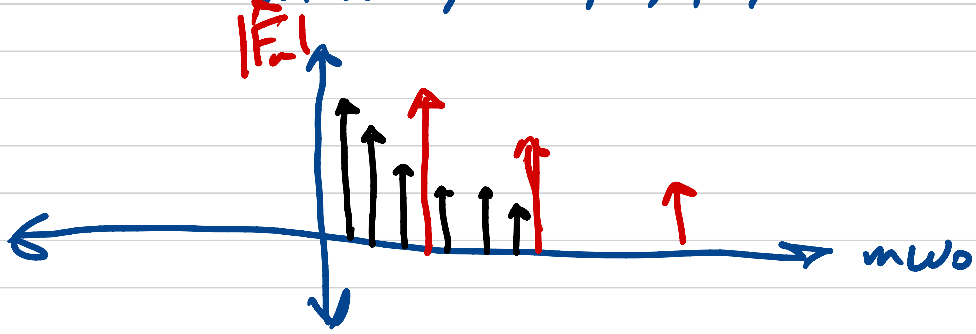
For Non-periodic Signals

$$T \to \infty$$

$$\omega_0 = \frac{2\pi}{T} \to 0$$

$m\omega_0$

m ....., $-2, -1, 0, 1, 2, \ldots$



$m\omega_0 \to \omega$

discrete

continuous

$$\boxed{\hat{F}(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt}$$

Standard Notation

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

Fourier Transform

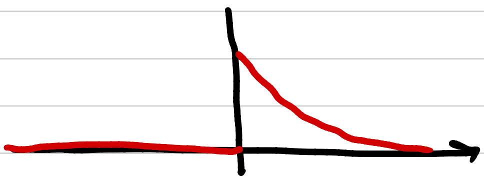
"Forward Fourier Transform"

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega$$ ← Inverse Fourier Transform

Ex] Unit step function

$$u_s(t) = \begin{cases} 0 & t<0 \\ 1 & t>0 \end{cases}$$

$$f(t) = e^{-t} u_s(t)$$



$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt = \cancelto{0}{\int_{-\infty}^{0} f(t) e^{-j\omega t} dt} + \int_{0}^{\infty} f(t) e^{-j\omega t} dt$$

$$= \int_{0}^{\infty} e^{-t}(1) e^{-j\omega t} dt$$

$$= \int_{0}^{\infty} e^{-(1+j\omega)t} dt$$

$$= \frac{1}{-(1+j\omega)} e^{-(1+j\omega)t} \Big|_{0}^{\infty}$$

$$= \frac{1}{-(1+j\omega)} (0-1)$$

$$\boxed{F(\omega) = \frac{1}{1+j\omega}}$$

Ex] $f(t) = e^{-|t|}$

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t}\,dt = \int_{-\infty}^{0} f(t) e^{-j\omega t}\,dt + \int_{0}^{\infty} f(t) e^{-j\omega t}\,dt$$

$$= \int_{-\infty}^{0} e^{t} e^{-j\omega t}\,dt + \int_{0}^{\infty} e^{-t} e^{-j\omega t}\,dt$$

$$= \int_{-\infty}^{0} e^{(1-j\omega)t}\,dt + \int_{0}^{\infty} e^{-(1+j\omega)t}\,dt$$

$$= \frac{1}{1-j\omega} e^{(1-j\omega)t}\Big|_{-\infty}^{0} + \frac{1}{1+j\omega}$$

$$= \frac{1}{1-j\omega}(1-0) + \frac{1}{1+j\omega}$$

$$= \frac{1}{1-j\omega} + \frac{1}{1+j\omega}$$

$$= \frac{1+j\omega + 1-j\omega}{1+\omega^2}$$

$$\boxed{F(\omega) = \frac{2}{1+\omega^2}}$$