ECE 333 – Green Electric Energy

8. Energy Economics Concepts

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ENERGY ECONOMICS CONCEPTS

- ☐ **The economic evaluation of a renewable energy resource requires a meaningful quantification of the two key cost elements**
  - ○ **fixed costs**
  - ○ **variable costs**
- ☐ **We use** ***engineering economics*** **notions for this purpose since they provide practical and useful means to compare on a consistent basis**
  - ○ **two different projects; or,**
  - ○ **the costs with and without a given project**

TIME VALUE OF MONEY

- ☐ The fundamental underlying notion is that a dollar today is not the same as a dollar next year
- ☐ We represent the time value of money by the conventional approach of *discounted cash flows*
- ☐ The notation is

P = *principal in $*

i = *interest rate in % per year*

- ☐ We adopt the convention that every payment in our analysis occurs at the *end of a period*

EXAMPLE: A LOAN FOR n YEARS

loan P for 1 *year*

repay $P + iP = P(1+i)$ at the end of 1 *year*

year 0	P	
year 1	$P(1+i)$	

loan P for n *years*

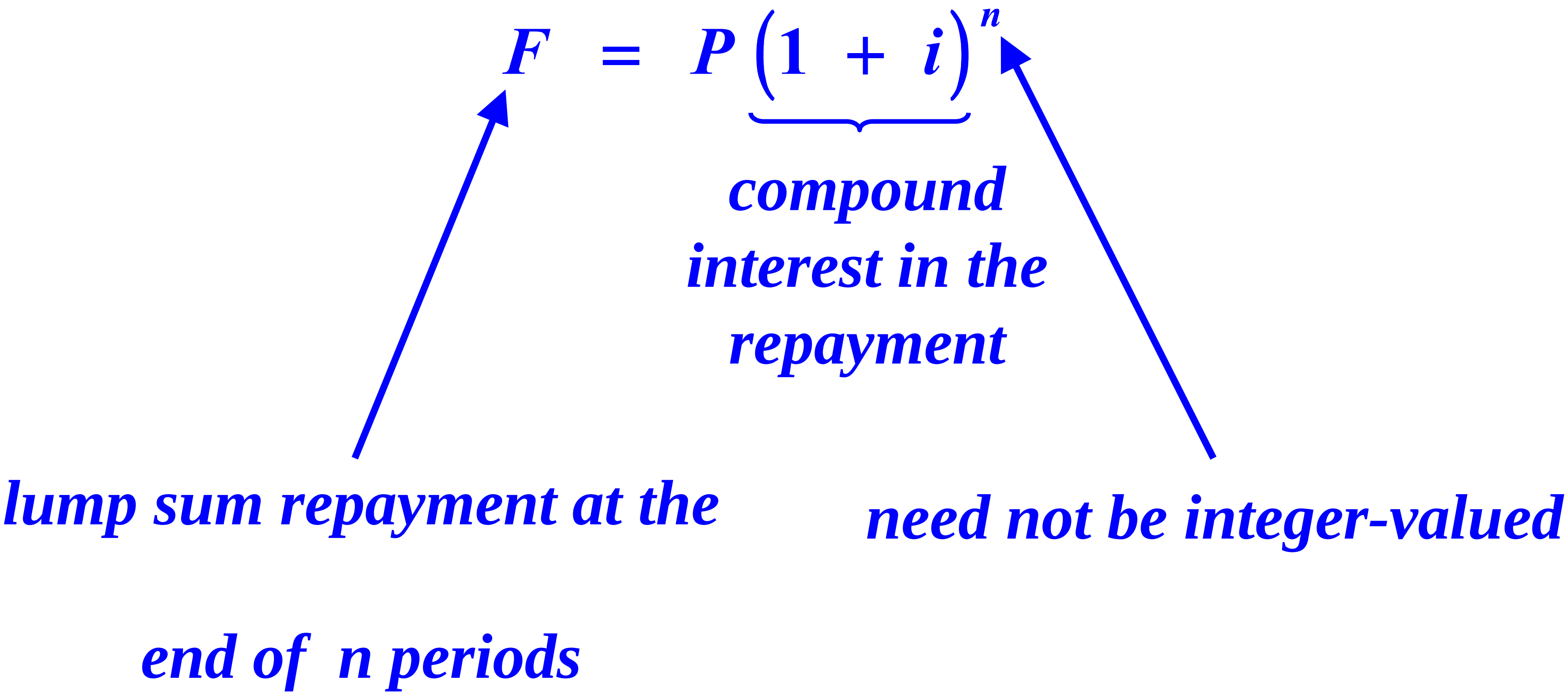
year 0	P	
year 1	$(1+i)\ P$	**repay/reborrow**
year 2	$(1+i)^2 P$	**repay/reborrow**
year 3	$(1+i)^3 P$	**repay/reborrow**
$\vdots$	$\vdots$	$\vdots$
year n	$(1+i)^n P$	**repay**

COMPOUND INTEREST

end of period	*amount owed*	*interest for that period*	*amount owed at the beginning of the next period*
0	P	0	P
1	$P(1+i)$	Pi	$P+Pi=P(1+i)$
2	$P(1+i)^2$	$P(1+i)i$	$P(1+i)+P(1+i)i=P(1+i)^2$
3	$P(1+i)^3$	$P(1+i)^2i$	$P(1+i)^2+P(1+i)^2i=P(1+i)^3$
⋮	⋮	⋮	⋮
n–1	$P(1+i)^{n-1}$	$P(1+i)^{n-2}i$	$P(1+i)^{n-2}+P(1+i)^{n-2}i=P(1+i)^{n-1}$
n	$P(1+i)^n$	$P(1+i)^{n-1}i$	0

the value in the last column at the *e.o.p.* (*k*-1) provides the amount owed (the second column) at the *start of period k*

TERMINOLOGY



$$F = P\underbrace{\left(1 + i\right)}_{\text{compound interest in the repayment}}{}^{n}$$

lump sum repayment at the

end of *n* periods

need not be integer-valued

TERMINOLOGY

- F denotes the *future worth*; P denotes the *present worth* or *present value* at interest i of a future sum F
- We call $(1+i)^n$ the **single payment compound amount factor**
- We define

$$\beta \triangleq (1+i)^{-1}$$

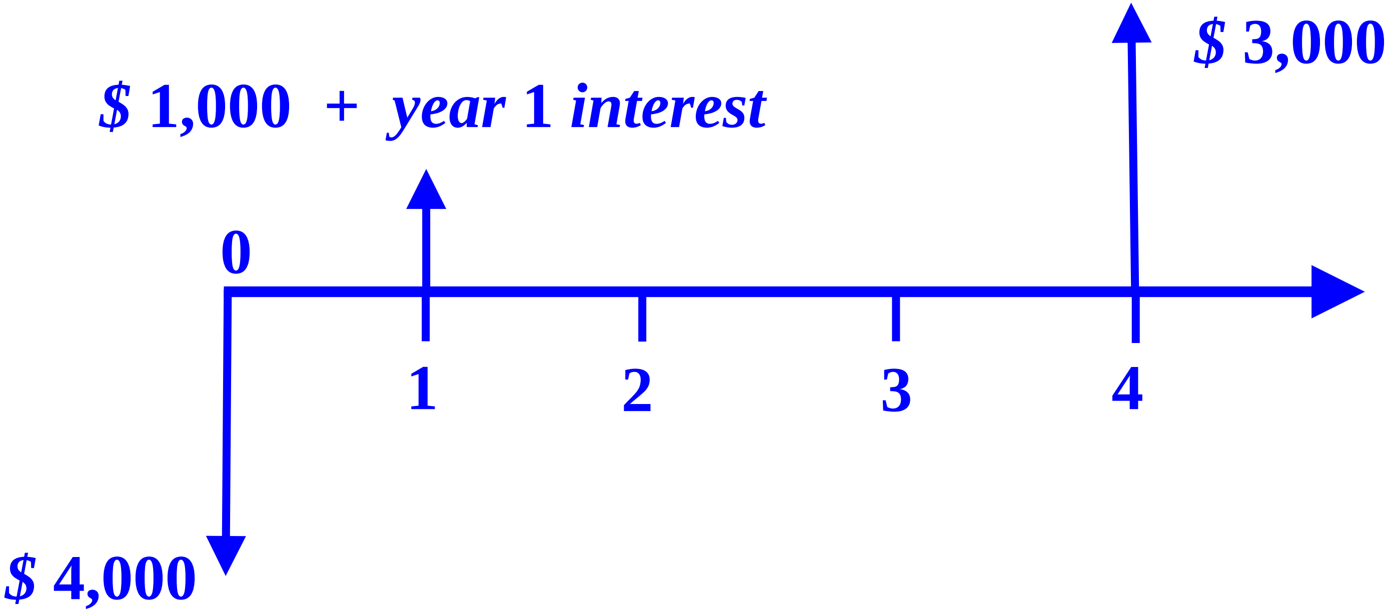
- Then,

$$\beta^{n} = (1+i)^{-n}$$

is the **single payment present worth factor**

EXAMPLE 1

- **Consider a loan of *$* 4,000 at 8 % interest to be repaid in two installments**
  - ○ ***$* 1,000 and interest for year 1 at the *e.o.y.* 1**
  - ○ ***$* 3,000 and interest at the *e.o.y.* 4**



EXAMPLE 1

- **The cash flows are**

  - *e.o.y.* 1: 1,000 + 4,000 (.08) = $ 1,320.00

  - *e.o.y.* 4: $3{,}000\,(1 + .08)^3$ = $ 3,779.14

- **Note that the loan is made at the *e.o.y.* 0 *in present $,* with repayments at the *e.o.y.* 1 and *e.o.y.* 4 *in future $***

EXAMPLE 2

- ☐ Given

$$P = \$\,1{,}000 \qquad \text{and} \qquad i = .12$$

then

$$P\left(1+i\right)^{5} = \$\,1{,}000\left(1+.12\right)^{5} = \$\,1{,}762.34 = F$$

- ☐ We say that with the cost of money of 12 %, P and F are *equivalent* in the sense that $ 1,000 today has the identical worth as $ 1,762.34 in 5 years

EXAMPLE 3

- ❑ Consider an investment that returns

$$\$\,1{,}000 \text{ at the } e.o.y.\ 1$$

$$\$\,2{,}000 \text{ at the } e.o.y.\ 2$$

$$i \;=\; 10\%$$

rate at which money can be freely lent or borrowed

- ❑ We evaluate *P* of the returns

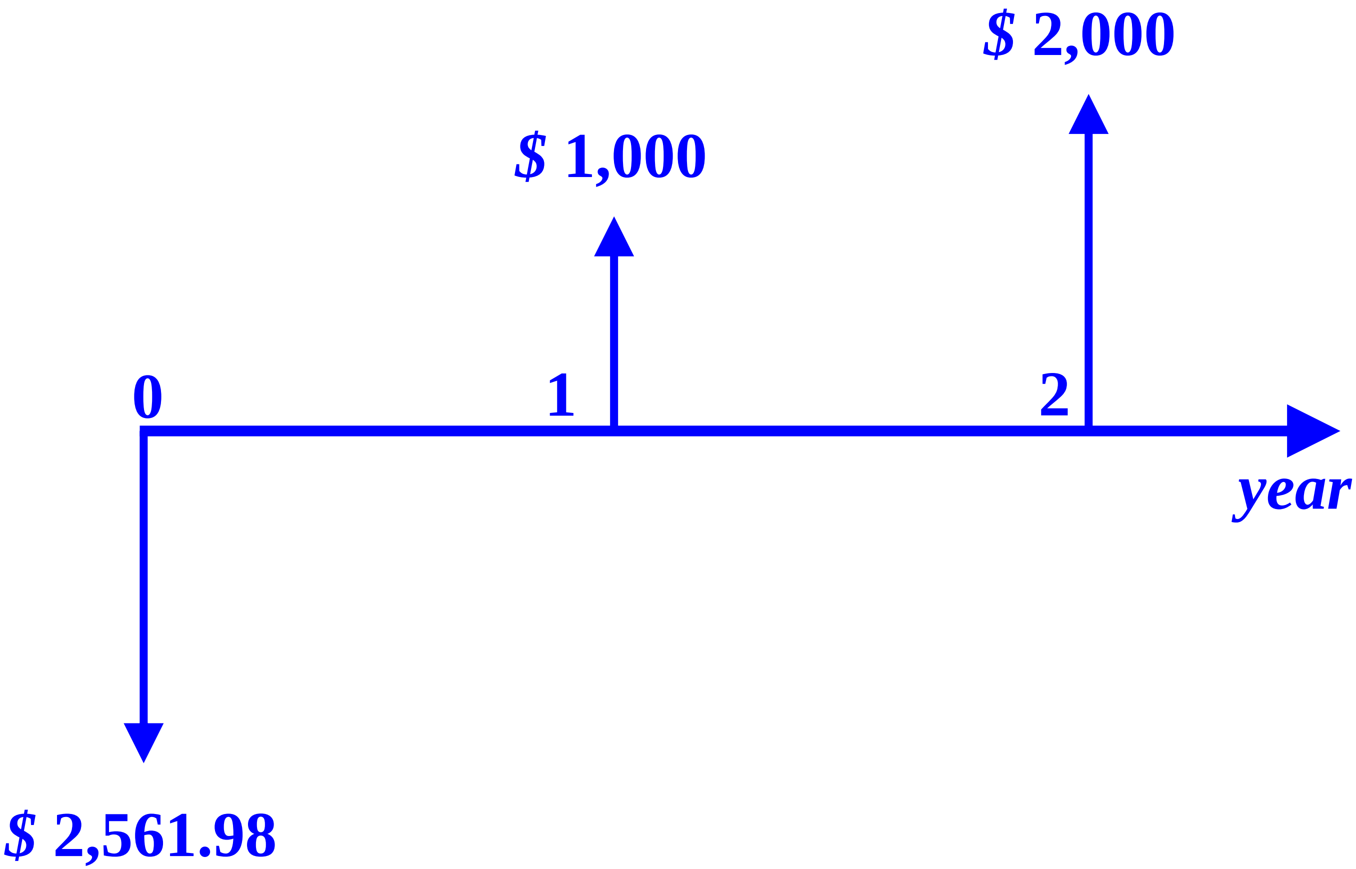
$$P \;=\; \$\,1{,}000\underbrace{\left(1+.1\right)^{-1}}_{\beta} + \$\,2{,}000\underbrace{\left(1+.1\right)^{-2}}_{\beta^{2}}$$

$$= \; \$\,909.9 + \$\,1{,}652.09$$

$$= \; \$\,2{,}561.98$$

EXAMPLE 3

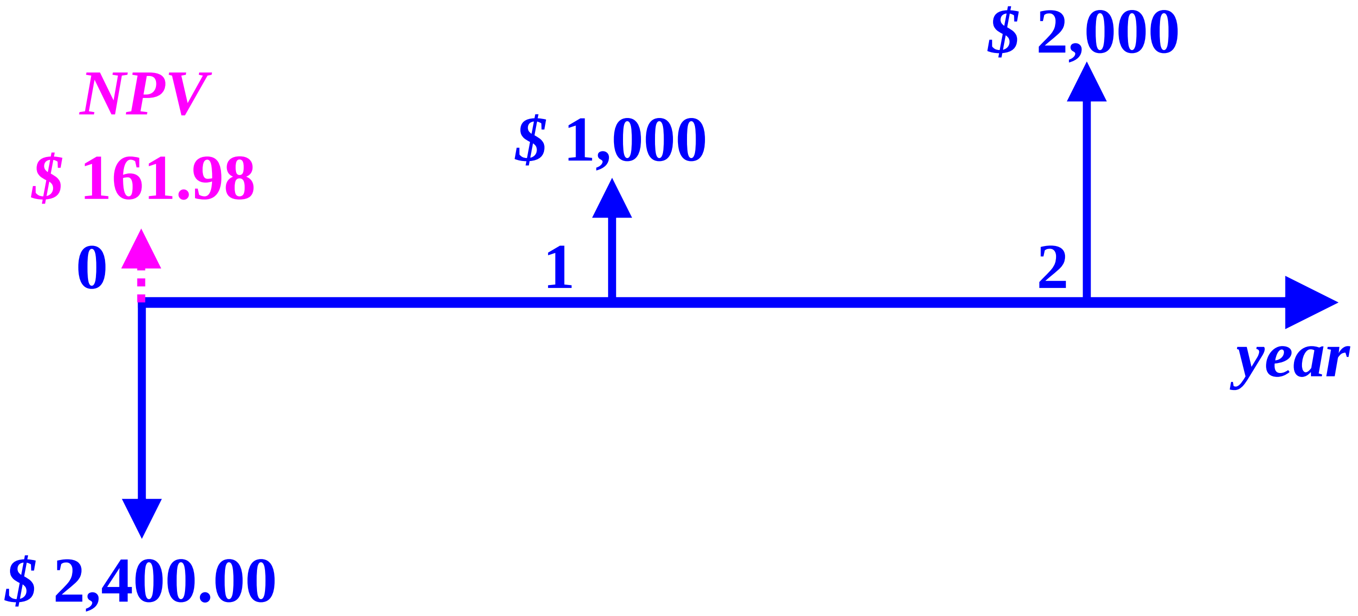
- **We restate this example using a *cash–flow diagram*:**



EXAMPLE 3

- Next, suppose that this investment requires that we invest $ 2,400 now and so at 10 % we say that the investment has a *net present value* given by

$$NPV = \$ 2{,}561.98 - \$ 2{,}400 = \$ 161.98$$



CASH FLOWS

- A *cash–flow* entails a transfer of the amounts A_t from one entity to another at the *e.o.p.* t, $t = 0, 1, \ldots$
- We consider the cash–flow set $\left\{ A_0, A_1, A_2, \ldots , A_n \right\}$
- This set consists of the transfers at the end of the periods in the set $\left\{ 0, 1, 2, \ldots , n \right\}$

CASH FLOWS

- ☐ We associate the transfer A_t at the *e.o.p. t*, for $t = 0, 1, 2, \ldots, n$

- ☐ We use the *sign convention* for cash flows as:

  $+$ *for inflow*

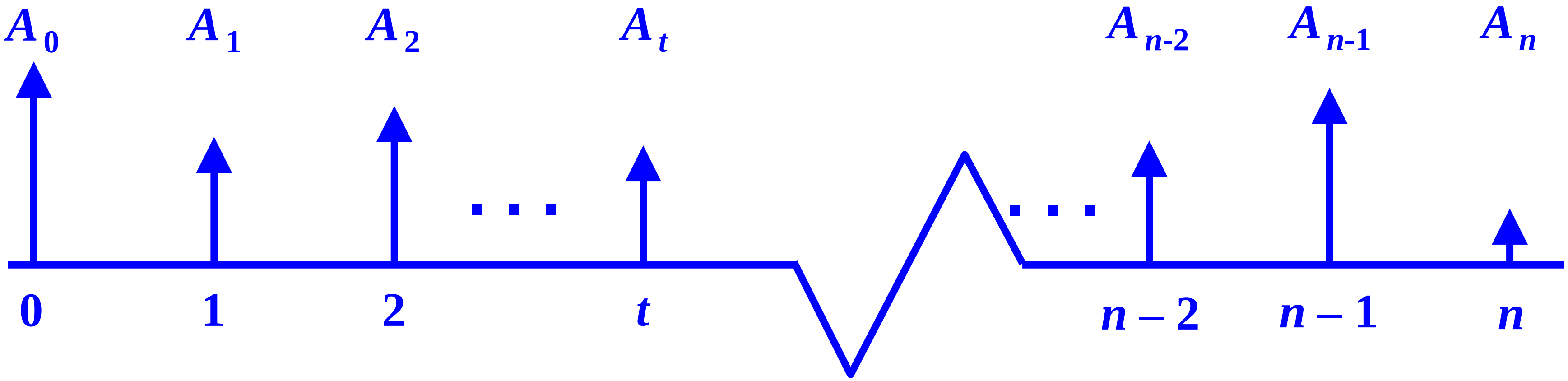
  $-$ *for outflow*

- ☐ The specification of a cash flow entity requires:
  - ○ the amount;
  - ○ the time; and,
  - ○ the sign

CASH FLOWS: FUTURE WORTH

- ☐ Given a cash–flow set $\left\{ A_0, A_1, A_2, \ldots, A_t, \ldots, A_n \right\}$, we define the future worth F_n of the cash flow set at the *e.o.y. n* as

$$F_n = \sum_{t=0}^{n} A_t \left(1 + i\right)^{n-t}$$



CASH FLOWS : FUTURE WORTH

- ☐ Note that each cash flow A_t in the $(n + 1)$–period set makes a specific contribution to F_n:

$$
\begin{array}{ccc}
A_0 & \rightarrow & A_0 \left(1+i\right)^{n} \\
A_1 & \rightarrow & A_1 \left(1+i\right)^{n-1} \\
A_2 & \rightarrow & A_2 \left(1+i\right)^{n-2} \\
\vdots & & \vdots \\
A_t & \rightarrow & A_t \left(1+i\right)^{n-t} \\
\vdots & & \vdots \\
A_n & \rightarrow & A_n
\end{array}
$$

CASH FLOWS : PRESENT WORTH

- ☐ We define the present worth *P* of the cash–flow set as

$$P = \sum_{t=0}^{n} A_t \beta^t = \sum_{t=0}^{n} A_t (1+i)^{-t}$$

- ☐ Note that

$$P = \sum_{t=0}^{n} A_t (1+i)^{-t}$$

$$= \sum_{t=0}^{n} A_t (1+i)^{-t} \underbrace{(1+i)^n (1+i)^{-n}}_{1}$$

CASH FLOWS

$$= \underbrace{(1+i)^{-n}}_{\beta^n} \underbrace{\sum_{t=0}^{n} A_t (1+i)^{n-t}}_{F_n}$$

$$= \beta^n F_n$$

or, equivalently,

$$F_n = (1+i)^n P$$

UNIFORM CASH – FLOW SET

- ❑ **Consider the cash–flow set** $\left\{ A_1, A_2, \ldots, A_n \right\}$ **with**

$$A_t = A \qquad t = 1, 2, \ldots, n$$

- ❑ **Such a set is called an** ***equal payment cash flow set***

- ❑ **We compute the present worth at** $t = 0$:

$$P = \sum_{t=1}^{n} A_t \beta^t = A \sum_{t=1}^{n} \beta^t = A\beta \left[1 + \beta + \beta^2 + \ldots + \beta^{n-1} \right]$$

UNIFORM CASH – FLOW SET

- ☐ **Now, for $0 < \beta < 1$, we have the identity**

$$\sum_{j=0}^{\infty} \beta^{j} = \frac{1}{1 - \beta}$$

- ☐ **It follows that**

$$1+\beta+\ldots+\beta^{n-1} = \sum_{j=0}^{\infty}\beta^{j} - \beta^{n}\left[\overbrace{1+\beta+\beta^{2}+\ldots+\beta^{n-1}+\ldots}^{\sum_{j=0}^{\infty}\beta^{j}}\right]$$

$$= \left(1-\beta^{n}\right)\sum_{j=0}^{\infty}\beta^{j}$$

UNIFORM CASH – FLOW SET

$$= \frac{1-\beta^{n}}{1-\beta}$$

- ☐ Therefore

$$P = A\beta \frac{1-\beta^{n}}{1-\beta}$$

- ☐ But

$$\beta = \left(1+d\right)^{-1} ,$$

where d is the interest or discount rate and so

UNIFORM CASH – FLOW SET

$$1 - \beta = 1 - \frac{1}{1+d} = \frac{d}{1+d} = \beta d$$

- ☐ **We write**

$$P = A \frac{1 - \beta^{n}}{d}$$

and we call $\frac{1 - \beta^{n}}{d}$ the *equal payment series present worth factor for an n-period set*

EQUIVALENCE

- We consider two cash – flow sets

$$\left\{ A_t^a : t = 0,1,2, \ldots ,n \right\} \; and \; \left\{ A_t^b : t = 0,1,2, \ldots ,n \right\}$$

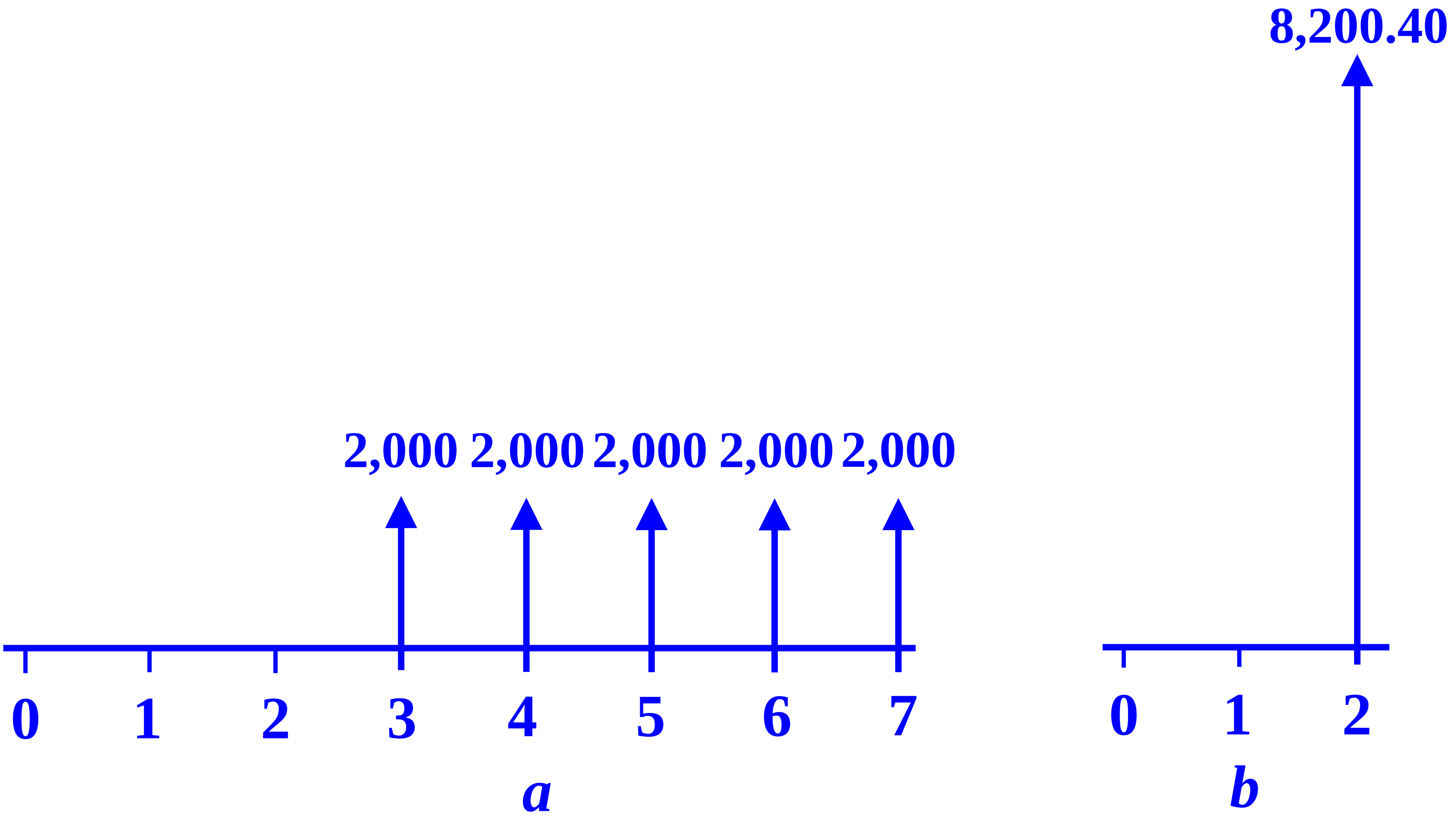
under a given discount rate *d*

- We say $\left\{ A_t^a \right\}$ and $\left\{ A_t^b \right\}$ are *equivalent* cash – flow sets under a specified *d* if and only if

$$F_m \; of \; \left\{ A_t^a \right\} = F_m \; of \; \left\{ A_t^b \right\} \; for \; every \; value \; of \; m$$

EQUIVALENCE EXAMPLE

- Consider the two cash–flow sets under *d* = *7%*



EQUIVALENCE

- ❑ We compute

$$P^{a} = 2,000 \sum_{t=3}^{7} \beta^{t} = 7,162.55$$

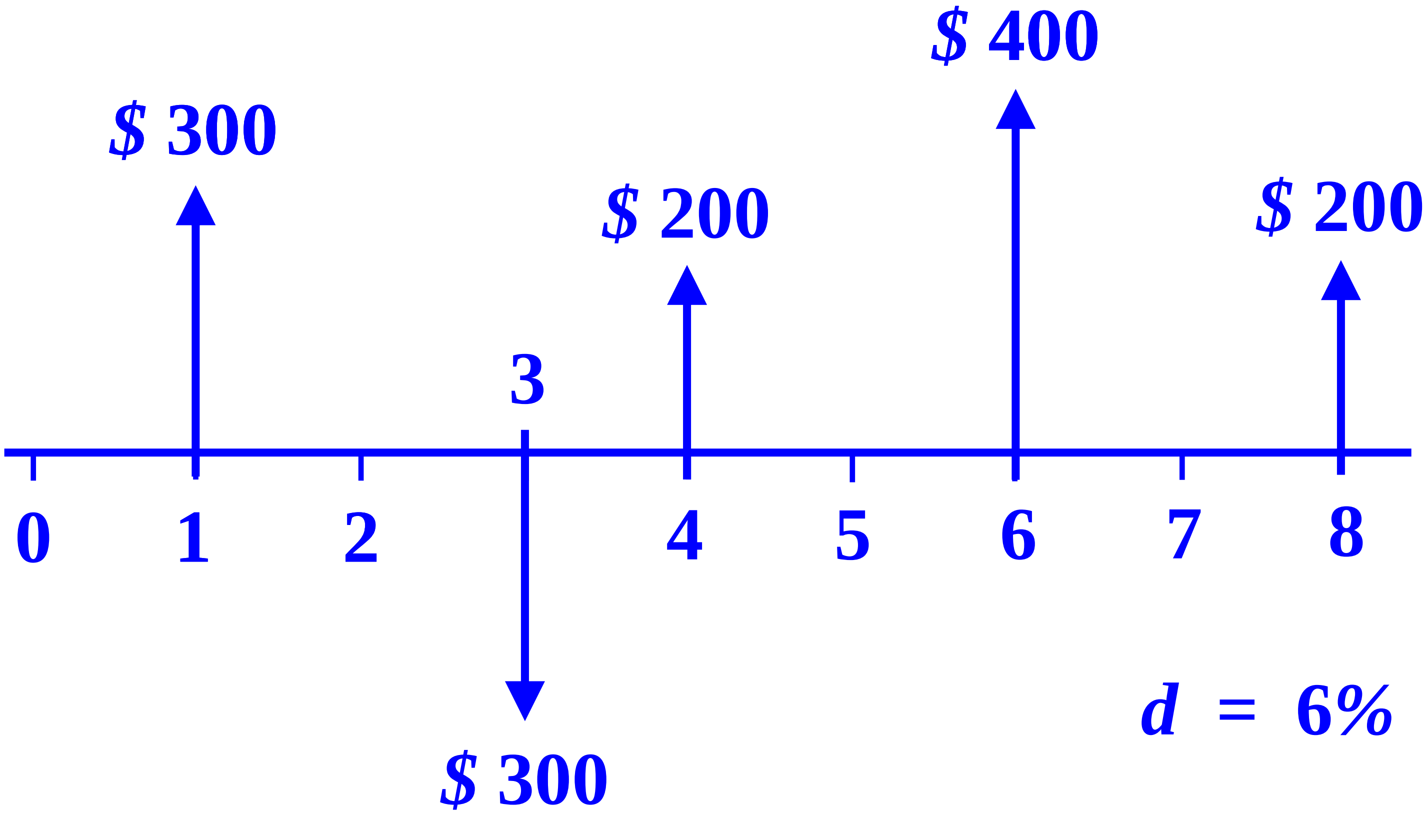
and

$$P^{b} = 8,200.40 \ \beta^{2} = 7,162.55$$

- ❑ Therefore, $\left\{ A_t^a \right\}$ and $\left\{ A_t^b \right\}$ are equivalent cash flow sets under $d = 7\%$

EXAMPLE

- Consider the cash–flow set illustrated below



- We compute F_8 at $t = 8$ with $d = 6\ \%$

EXAMPLE

$$F_8 = 300\left(1+.06\right)^7 - 300\left(1+.06\right)^5 +$$

$$200\left(1+.06\right)^4 + 400\left(1+.06\right)^2 + 200$$

$$= \$\,951.56$$

☐ We also compute P

EXAMPLE

$$P = 300\left(1+.06\right)^{-1} - 300\left(1+.06\right)^{-3} +$$

$$200\left(1+.06\right)^{-4} + 400\left(1+.06\right)^{-6} + 200\left(1+.06\right)^{-8}$$

$$= \$\ 597.04$$

- ☐ **We verify that for** $d = 6\%$

$$F_8 = 597.04\left(1+.06\right)^{8} = \$\ 951.56$$

DISCOUNT RATE

- The interest rate i is, often, referred to as the *discount rate* and is denoted by d
- In the conversion of the future amount F to the present worth P, we view the *discount rate* as the interest rate that may be earned from the best investment alternative
- A postulated savings of $\$10{,}000$ at the end of 5 years in a project has a present worth

$$P = F_5 \beta^5 = 10{,}000(1 + d)^{-5}$$

DISCOUNT RATE

- ☐ For $d = 0.1$

$$P = \$\ 6{,}201,$$

while for $d = 0.2$

$$P = \$\ 4{,}019$$

- ☐ In general, for a specified future worth, the *lower the discount factor, the higher is the present worth*

DISCOUNT RATE

- We may restate this notion slightly differently; the lower the discount factor, the more valuable a future payoff becomes
- The present worth of the set of costs under a given discount rate is called the *life–cycle costs* – an important metric in economic assessment studies

EXAMPLE: MOTOR DECISION

- ☐ **We consider the purchase of two 100–*hp* motors – *a* and *b* – to be used over a 20–*year* period; the discount rate is 10 %**

- ☐ **The relative merits of *a* and *b* are tabulated to be**

motor	*costs* (*$*)	*load* (*kW*)
a	2,400	79.0
b	2,900	77.5

EXAMPLE: MOTOR DECISION

- ☐ The motor is used 1,600 *hours per year*; electricity costs are constant at 0.08 *$/kWh*
- ☐ We evaluate yearly energy costs for *the two motors*

$$A_t^a = (79.0\ kW)(1600\ h)(.08\ \$/kWh) = \$\ 10{,}112$$

$$t = 1, 2, \ldots, 20$$

$$A_t^b = (77.5\ kW)(1600\ h)(.08\ \$/kWh) = \$\ 9{,}920$$

EXAMPLE: MOTOR DECISION

- ☐ **We next evaluate the present worth of *a* and *b***

$$P^{a} = 2,400 + 10,112 \boxed{\sum_{t=1}^{20} (1.1)^{-t}} \longleftarrow 8.5136$$

$$= \$ 88,489$$

$$P^{b} = 2,900 + 9,920 \boxed{\sum_{t=1}^{20} (1.1)^{-t}} \longleftarrow 8.5136$$

$$= \$ 87,354$$

EXAMPLE: MOTOR DECISION

- **The difference**

$$P^{a} - P^{b} = 88,489 - 87,354 = \$1,135$$

- **Therefore, the motor *b* purchase results in *$* 1,135 savings despite its higher purchase price due to its smaller load consumption over the 20–*year* horizon under the 10 % discount rate**

INFINITE HORIZON CASH – FLOW SETS

- ☐ **Consider an infinite, uniform cash–flow set as**

$$\left\{ A_t = A : t = \ 1, 2, \ldots \right\}$$

- ☐ **Then, as $n \to \infty$**

$$P \ = \ A \ \frac{\left(1 - \beta^{\,n}\right)}{d} \ \xrightarrow[n \to \infty]{} \ A \, \frac{1}{d}$$

- ☐ **For an infinite horizon uniform cash–flow set**

INFINITE HORIZON CASH – FLOW SETS

$$\frac{A}{P} = d$$

- We may view d as the *capital recovery factor* and so provide the following interpretation to the term:

  for an initial investment of P, the amount

  $$d * P = A$$

  is recovered annually as the return on the initial investment P

INTERNAL RATE OF RETURN

- **We consider an n–period cash–flow set:**

$$\left\{ A_t = A : t = 1, 2, \ldots , n \right\}$$

- **The value of d for which**

$$P - \sum_{t=1}^{n} A_t \beta^t = 0$$

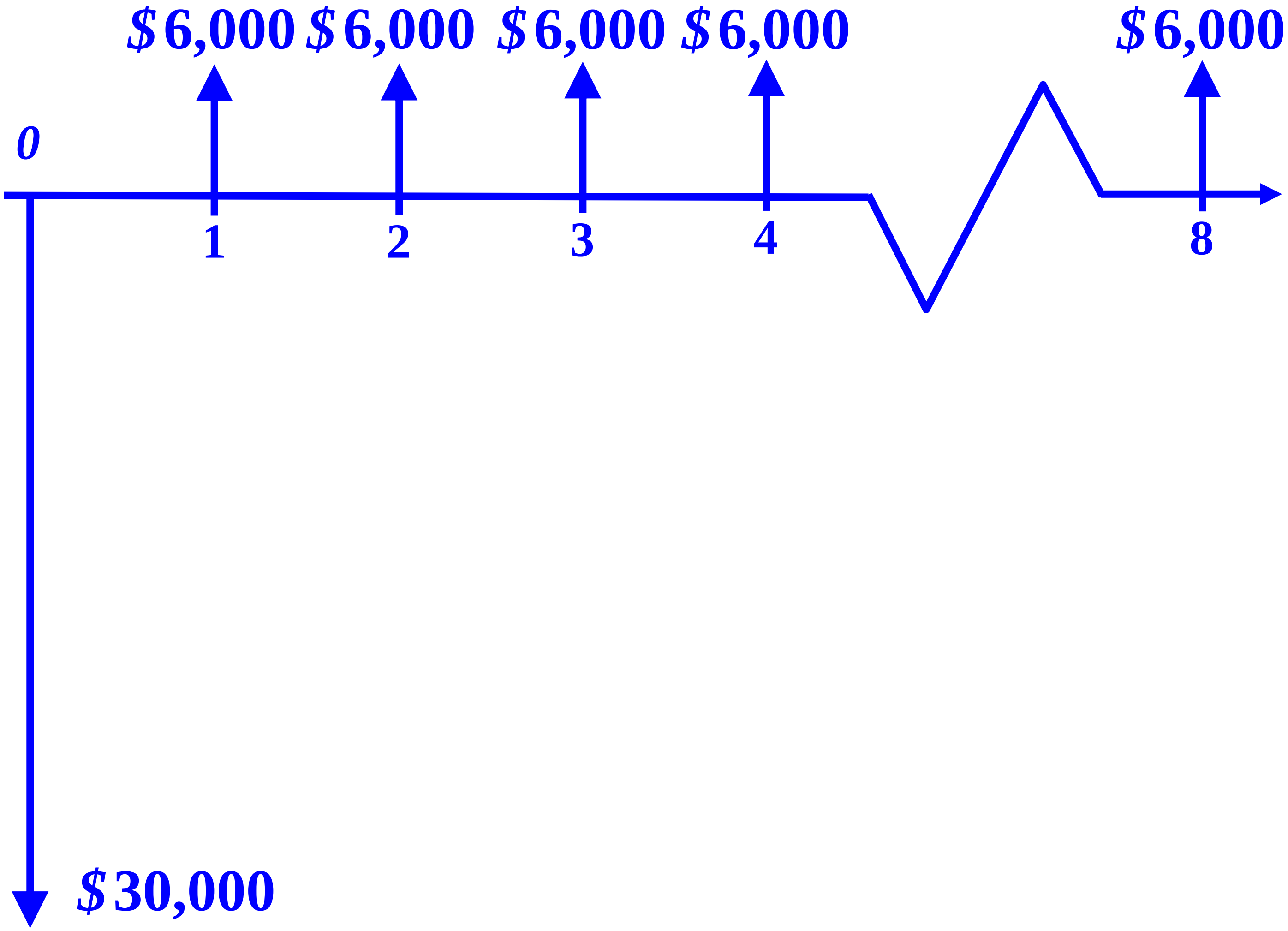
is called the *internal rate of return* (*IRR*)

- **The *IRR* measures how quickly we recover an investment, or stated differently, *IRR* is the *speed* o**

***rate* at which the returns recover an investment**

EXAMPLE: INTERNAL RATE OF RETURN

- Consider the following cash–flow set



INTERNAL RATE OF RETURN

- ☐ **The present value of this cash–flow set is**

$$P \ = \ -30,000 \ + \ 6,000 \frac{1-\beta^{8}}{d} \ = \ 0,$$

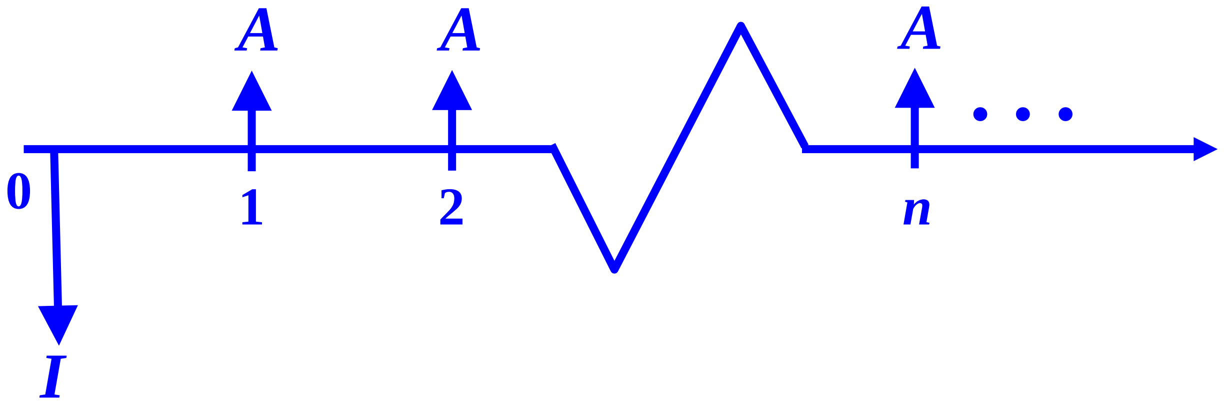
whose solution is

$$d \ \approx \ 12\%$$

- ☐ **We interpret the 12 % *discount rate* under which the cash–flow set *present value* becomes 0 as the *IRR* for the given cash–flow set**

INTERNAL RATE OF RETURN

- ❑ Consider an *infinite horizon* simple investment



- ❑ Therefore

$$d = \frac{A}{I}$$

ratio of annual return to the initial investment I

INTERNAL RATE OF RETURN

- ❑ Consider an infinite, uniform cash–flow set with

$$P = \$\ 1{,}000$$

$$A = \$\ \ 200$$

and

$$d = 20\ \%$$

- ❑ We interpret that the returns capture 20 % of the investment each year, or, equivalently, that we have a *simple payback period of* 5 *years*

EXAMPLE: EFFICIENT REFRIGERATOR

- ☐ **A more efficient refrigerator requires an additional investment of *$* 1,000 but provides *$* 200 of energy savings annually**

- ☐ **For a 10-year lifetime of the refrigerator, the *IRR* of this *$* 1,000 is obtained from the solution of**

$$0 = -1,000 + 200 \frac{1 - \beta^{10}}{d}$$

or

EXAMPLE: EFFICIENT REFRIGERATOR

$$\frac{1-\beta^{10}}{d} = 5$$

- *IRR* tables show that

$$\left.\frac{1-\beta^{10}}{d}\right|_{d = 15\,\%} = 5.02;$$

therefore, the *IRR* is about 15 % – *i.e.*, we recover

in 10 *y* our initial investment at a *c.r.f.* $d = 0.15$

INFLATION IMPACTS

- ☐ **Inflation is a general *increase* in the level of prices in an economy; equivalently, we may view inflation as a general *decline* in the value of the *purchasing power of money***
- ☐ **Inflation is measured using prices: different products may have distinct escalation rates**
- ☐ **Typically, indices such as the *CPI* – the *consumer price index* – use a market basket of goods and**

INFLATION IMPACTS

services as a proxy for the entire *US* economy

- reference basis is the year 1967 with the price of $ 100 for the basket $\longrightarrow L_0$
- in the year 1990, the same basket price costs

$$\$\,374 \longrightarrow L_{23}$$

- the average inflation rate j is estimated from

$$(1+j)^{23} = \frac{374}{100} = 3.74$$

and so

$$j = (3.74)^{\frac{1}{23}} - 1 \approx 0.059$$

INFLATION RATE

- ☐ **The inflation rate contributes to the *overall market interest rate i*, sometimes called the *combined interest rate***

- ☐ **We write, using *d* for *i* for the *combined interest rate***

$$\underbrace{(1+d)}_{\textit{combined interest rate}} = \underbrace{(1+j)}_{\textit{inflation rate}} \underbrace{(1+d')}_{\textit{real interest rate}}$$

INFLATION

- We obtain the following identities

$$d' = \frac{d - j}{1 + j}$$

and

$$j = \frac{d - d'}{1 + d'}$$

CASH – FLOWS WITH INFLATION INCORPORATED

- ❑ **We express the cash flows in *then current dollars* in the set $\left\{ A_t : t = 0,1,2, \ldots ,n \right\}$**

- ❑ **The following are synonymous terms**

$$current \;\equiv\; then\ current \;\equiv\; inflated \;\equiv\; after\ inflation$$

- ❑ **An *indexed* or *constant–worth* cash–flow is one that does not explicitly take inflation into account, i.e.,**

CASH – FLOWS INCORPORATING INFLATION

whatever amount in *current inflated dollars* will buy the same goods and services as in the reference year, typically, the year 0

- ❑ **The following terms are also synonymous**

constant* ≡ *indexed* ≡ *inflation free* ≡ *before inflation

and we use them interchangeably

CASH – FLOWS INCORPORATING INFLATION

- **We define the set of constant currency flows**

$$\left\{ W_t : t = 0,1,2, \ldots ,n \right\}$$

corresponding to the set

$$\left\{ A_t : t = 0,1,2, \ldots ,n \right\}$$

with each element A_t given in period *t dollars*

CASH – FLOWS INCORPORATING INFLATION

- **We use the relationship**

$$A_t = W_t \left(1 + j\right)^t$$

or equivalently

$$W_t = A_t \left(1 + j\right)^{-t}$$

with W_t expressed in reference ***year 0 – today's or present – dollars***

CASH – FLOWS INCORPORATING INFLATION

- ☐ **We have**

$$P = \sum_{t=0}^{n} A_t \beta^t$$

$$= \sum_{t=0}^{n} W_t (1+j)^t (1+d)^{-t}$$

$$= \sum_{t=0}^{n} W_t (1+j)^t (1+j)^{-t} (1+d')^{-t}$$

$$= \sum_{t=0}^{n} W_t (1+d')^{-t}$$

CASH – FLOWS INCORPORATING INFLATION

- ❑ Therefore, the *real interest rate* d' is used to discount the indexed cash flows
- ❑ In summary,

we discount current *dollar* cash flow at d

we discount indexed *dollar* cash flow at d'

CASH – FLOWS INCORPORATING INFLATION

- ❑ **Whenever we take inflation into account, it is convenient to carry out the analysis in *present worth* rather than future worth or on a *cash–flow basis***
- ❑ **Under inflation $\left(j > 0 \right)$, a uniform set of cash flows $\left\{ A_t = A : t = 1, 2, \ldots , n \right\}$ indicates a real decline in the cash flow amounts year by year**

EXAMPLE: INFLATION CALCULATIONS

- ☐ We consider an annual inflation rate of $j = 4\ \%$;

  the cost for a piece of equipment is assumed

  constant for the next 3 years in terms of today's $\$$

$$W_0 = W_1 = W_2 = W_3 = \$\,1{,}000$$

- ☐ The corresponding cash flows in current $\$$ are

$$A_0 \qquad\qquad = \$\,1{,}000$$

$$A_1 = 1{,}000\left(1+.04\right) = \$\,1{,}040$$

EXAMPLE: INFLATION CALCULATIONS

$$A_2 = 1,000\left(1+.04\right)^2 = \$\,1,081.60$$

$$A_3 = 1,000\left(1+.04\right)^3 = \$\,1,124.86$$

- ☐ The interpretation of A_3 is that under 4 % inflation, $ 1,125 in 3 years will have the *identical value – same purchasing power –* as $ 1,000 today; it **must not** be confused with the present worth calculation

MOTOR SELECTION EXAMPLE

- ☐ **For the motor *a* or *b* purchase example, we consider the escalation of the electricity rate at an annual rate of $j = 5\ \%$**
- ☐ **We compute the *NPV*, explicitly accounting for *price escalation of* 5 % – inflation – and, with $d = 10\ \%$**
- ☐ **Then, the *real interest rate***

$$d' = \frac{d - j}{1 + j} = \frac{.10 - .05}{1 + .05} = \frac{.05}{1.05} = 0.04762$$

MOTOR SELECTION EXAMPLE

- The savings of $ 192 per year are in constant dollars

$$P_{savings} = \sum_{t=1}^{20} W_t \left(1 + d'\right)^{-t} \longleftarrow\!\!\longleftarrow\!\!\longleftarrow \; 0.04762$$

and so

$$P_{savings} = \$\,2,442$$

- The total savings are

$$P = -500 + P_{savings} = \$\,1,942$$

- These savings are markedly larger than those of $ 1,135, without the electricity price escalation

EXAMPLE: *IRR* FOR *HVAC* RETROFIT WITH INFLATION

- ☐ **An energy efficiency retrofit of a commercial site reduces the *HVAC* load consumption to $0.8\ GWh$ from $2.3\ GWh$ and the peak demand by $0.15\ MW$**
- ☐ **Electricity costs are $60\ \$/MWh$ and demand charges are $7{,}000\ \$/(MW\text{–}mo)$ and each rate escalates at an annual rate of $j = 5\ \%$**
- ☐ **The retrofit requires a $\$\ 500{,}000$ investment today and is planned to have a $15-year$ lifetime**

EXAMPLE: *IRR* FOR *HVAC* RETROFIT WITH INFLATION

- ☐ **We evaluate the *IRR* for this project**

- ☐ **The annual savings are**

$$energy \ : \left(2.3 - 0.8\right) GWh \left(60 \ \$ / MWh\right) \qquad = \ \$ \ 90,000$$

$$demand : \left(.15 \, MW\right)\left(7,000 \ \$ / (MWh - mo)\right) 12 mo \ = \ \$ \ 12,600$$

$$total \quad : 90,000 + 12,600 \ = \ \$ \ 102,600$$

- ☐ **The *IRR* is the value of d' that results in**

EXAMPLE: *IRR* FOR *HVAC* RETROFIT WITH INFLATION

$$0 = -500{,}000 + 102{,}600\,\frac{1-\left(\beta'\right)^{15}}{d'}$$

- ☐ The table look up produces the d' of 19 % and

  with inflation factored in, we have

$$\left(1+d\right) = \left(1+j\right)\left(1+d'\right)$$

$$= \left(1.05\right)\left(1.19\right)$$

$$= 1.25$$

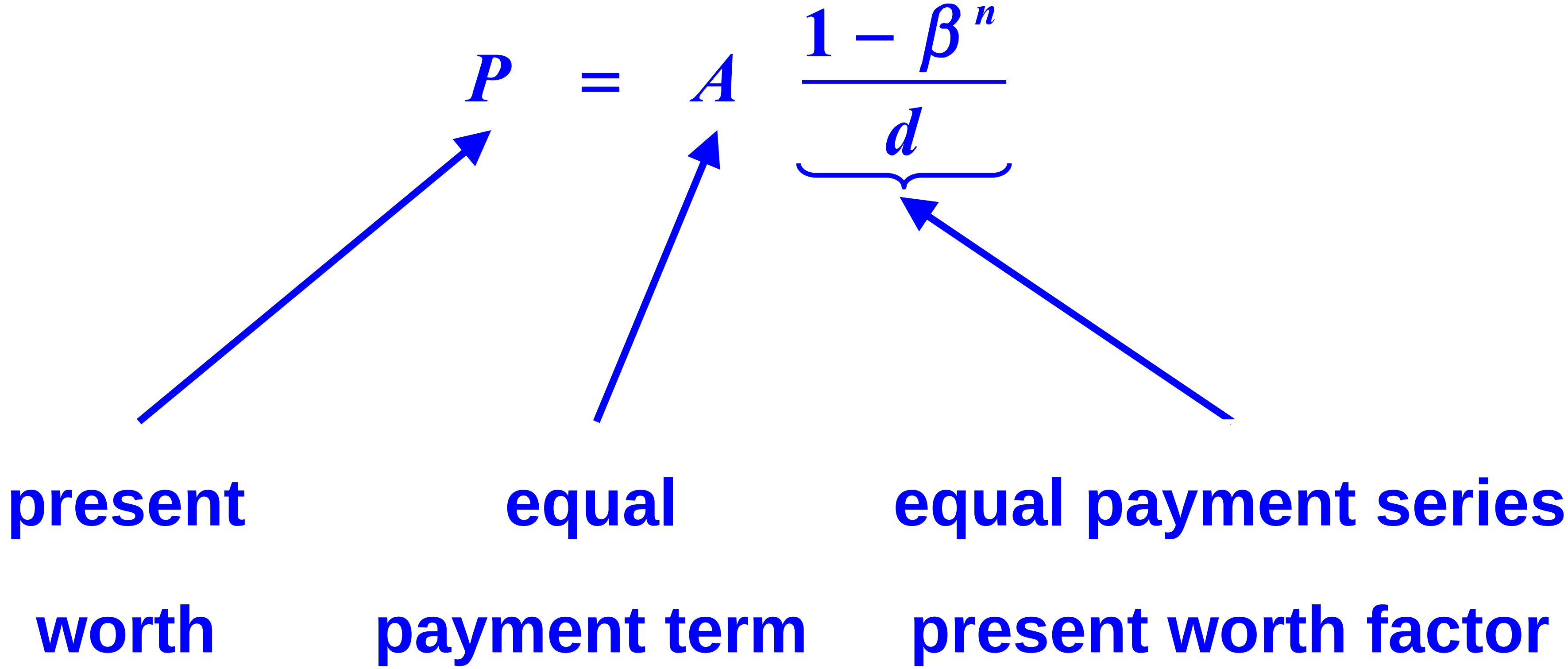
resulting in a *combined IRR* of 25 %

ANNUALIZED INVESTMENT

- ❑ **A capital investment, such as a renewable energy project, requires repayment of funds, either borrowed from a bank or obtained from investors or taken from the owner's own accounts**
- ❑ **We may, conceptually, view the investment costs as a loan whose repayment is done in a series of equal annual payments with the explicit inclusion of the contracted interest**

ANNUALIZED INVESTMENT

- For this purpose, we use a uniform cash – flow set and use the relation



$$P = A \underbrace{\frac{1 - \beta^{n}}{d}}$$

present worth

equal payment term

equal payment series present worth factor

ANNUALIZED INVESTMENT

- **Therefore, the equal payment is given by**

$$A = P \boxed{\frac{d}{1-\beta^{n}}} \longleftarrow \textit{capital recovery factor (c.r.f.)}$$

- **The *capital recovery factor* is the fraction of the initial investment P at which P is repaid each period**

EXAMPLE: EFFICIENT AIR CONDITIONER

- An efficiency upgrade of an air conditioner incurs a *$* 1,000 investment and results in annual savings of *$* 200
- The initial *$* 1,000 investment is obtained as a 10–*year* loan to be repaid at 7 % interest
- The repayment on the loan is done as a uniform cash flow

$$A = 1{,}000 \, \frac{0.07}{1 - \beta^{10}} = \$\, 142.38$$

EXAMPLE: EFFICIENT AIR CONDITIONER

- The net annual savings amount to

$$200 - 142.38 \;=\; \$\,57.62$$

  and not only are the savings sufficient to pay back the loan in 10 *years*, they also result in a yearly surplus of $57.62

- The *benefits/costs ratio* is

$$\frac{200}{142.38} = 1.4$$

EXAMPLE: *PV* SYSTEM

- **We consider a $3\text{–}kW$ *PV* system whose capacity factor $\kappa = 0.25$**
- **The investment incurred a $\$10{,}000$ cost using funds borrowed as a $20\text{–}year$, $6\ \%$ loan**
- **The annual loan repayments are**

$$A = 10{,}000 \frac{0.06}{1 - \beta^{20}} = 10{,}000\left(0.0872\right) = \$\ 872$$

EXAMPLE: *PV* SYSTEM

- ❑ The total annual energy generated is

$$\left(3\right)\left(0.25\right)\left(8,760\right) = 6,570\ kWh$$

- ❑ Under the assumption of 0 *O&M* costs, the unit costs of electricity for break–even operations is

$$\frac{872}{6,570} = 0.133\ \$ / kWh$$

LEVELIZED BUS – BAR COSTS

- **The comparison of various alternatives must be carried out on a consistent basis taking into account the following factors**
  - **inflation impacts**
  - **fixed costs, *i.e.*, investment & other costs**
  - **variable costs**
- **The conventional approach for cost valuation of a project consists of the following steps:**

LEVELIZED BUS – BAR COSTS

  - ○ **present worth evaluation of each cash–flow amount (costs) throughout the project life**
  - ○ **determination of an *equivalent equal amount annual uniform cash–flow set* (costs)**
  - ○ **evaluation of the yearly total generation**

- ☐ **The ratio of the equal amount to the annual generation is called the *levelized bus–bar* costs of energy**

ANNUALIZED POWER PLANT FIXED COSTS

- **The total fixed costs include capital costs, all taxes including property, fixed *O&M expenses* that are required and insurance premiums**
- **Typically, we aggregate all such costs into a *single sum* that is annualized via the use of a yearly *fixed charge rate* or *f.c.r.***
- **The *f.c.r.* accounts for investment financing costs, including acceptable return to investors, all taxes, insurance and fixed *O&M* costs**

ANNUALIZED POWER PLANT FIXED COSTS

- ☐ **Investment financing costs are a function of the capital structure of the plant owner in terms of the relative fractions of equity and debt components, the costs of capital of each component and the resulting weighted average costs of capital**
- ☐ **The annualized payments on a loan use the weighted average costs of capital as the value for *d* to compute the *c.r.f.* of the equal annual payment series for the loan repayment**

ANNUALIZED POWER PLANT FIXED COSTS

- The other fixed costs components for the *f.c.r.* for the taxes, fixed *O&M* and insurance costs are determined as a single term expressed as a fraction of the per unit investment costs and is added to the financing charges *c.r.f.*
- It follows that the *f.c.r.* is given as the sum

$$f.c.r. = financing\ charges\ c.r.f. + added\ term$$

EXAMPLE: MICRO–TURBINE ENGINE

- We consider the economics of a micro–turbine whose information is given in the table below
- We evaluate the
  - annualized fixed costs
  - initial–year variable costs
  - inflation impacts

EXAMPLE: MICRO–TURBINE ENGINE

item	**value**	*unit*
investment costs	**850**	*$/kW*
heat rate	**12,500**	*Btu/kWh*
capacity factor	**0.7**	—
fuel costs (year 0)	**4.00×10^{-6}**	*$/Btu*
variable costs	**0.002**	*$/kWh*
annual fuel & O&M escalation rate	**6**	*%*
annual discount rate	**10**	*%*
fixed charge rate	**12**	*%*
lifetime	**20**	*y*

EXAMPLE: MICRO–TURBINE ENGINE

$$\frac{\left(850\ \$/kW\right)\left(12\ \%\right)}{\left(8760\ \ h\right)\left(0.70\right)} = 0.0166\ \$/kWh$$

$$A_0 = \left(12,500\ Btu\ /\ kWh\right)\left(4\times10^{-6}\ \$\ /\ Btu\right) + 0.002\ \ \$\ /\ kWh$$

$$= 0.052\ \$\ /\ kWh$$

$$d' = \frac{d\ -\ j}{1+\ j} = \frac{0.1\ -\ 0.06}{1\ +\ 0.06} = 0.037736$$

EXAMPLE: MICRO–TURBINE ENGINE

- ☐ The present worth of the constant uniform cash–flow set is

$$A_0 \cdot \frac{1-\left(\beta'\right)^{20}}{d'} = 0.052\left(\frac{1-\left(\frac{1}{1.037736}\right)^{20}}{0.037736}\right)$$

- ☐ The levelized annual costs are stated in *current $*

EXAMPLE: MICRO–TURBINE ENGINE

$$0.052 \left(\frac{1 - \left(\frac{1}{1.037736} \right)^{20}}{0.037736} \right) \left(\frac{0.10}{1 - \left(\frac{1}{1.1} \right)^{20}} \right) = 0.0847 \ \$/kWh$$

- The *levelized bus–bar costs* are, therefore, in *current $*

$$0.0166 + 0.0847 = 0.1013 \ \$/kWh$$