ECE 333 – GREEN ELECTRIC ENERGY

Lecture 7: Wind Data Analysis and Its Application

George Gross

Department of Electrical and Computer Engineering

University of Illinois at Urbana–Champaign

WIND POWER MEASUREMENT AND DATA

- **The collection of sufficient wind data to allow the generation estimation is an essential task in the assessment of any wind project at a specified site**
- **Various measurement devices – cup, sonic detection and ranging (*SODAR*) and light detection and ranging (*LIDAR*) *anemometers* – are able to measure *wind speed*, its *direction* and additional *relevant metrics of interest***

WIND POWER MEASUREMENT AND DATA

- ☐ Wind is a highly uncertain phenomenon with high variability and wide-ranging changes over a brief period; as a result, wind speed exhibits much *volatility* and considerable *randomness*
- ☐ While wind speed is a continuous variable, wind speed data are collected on a sampled basis: values are measured on a periodic basis, such as hourly, every 10 minutes or every minute

WIND POWER MEASUREMENT AND DATA

- ❑ **Wind data for wind analysis requires the *collection around–the–clock of wind speed measurements at the altitude(s) of interest* at a frequency commensurate with the nature and scope of the analysis**
- ❑ **Any measurement scheme requires the specification of the smallest indecomposable unit of time:**

WIND POWER MEASUREMENT AND DATA

- **for planning evaluation and assessment, the collection of data on an *hourly or half–hourly basis* is, typically, adequate**
- **for the analysis of dynamic phenomena such as stability, the collection must deploy a finer resolution than hourly to enable capture of the short-time constants of such phenomena**

WIND POWER DATA

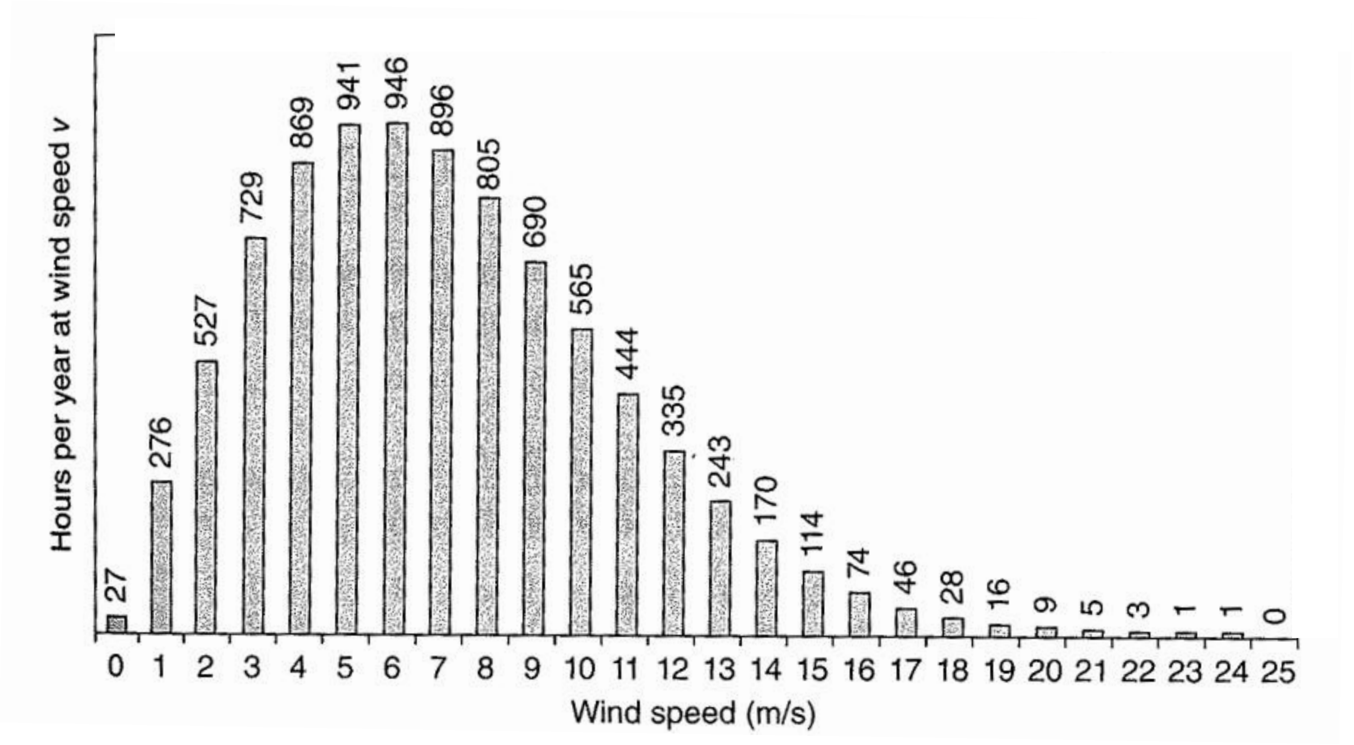
- **We use the wind data collected to approximate the probability distribution of wind at a specified site at the desired altitude**
- **We make use of such approximations under the implicit assumption that natural phenomena, such as wind, continue to behave in the future in a way similar to their past behavior**

WIND SPEED HISTOGRAM

- **Suppose we wish to *probabilistically* characterize the wind speed at a given site and at its specified altitude: for this purpose, we collect, say, *hourly wind speed measurements* over a long period of time and construct a *histogram* of the measured values**
- **We discretize the wind speed axis, *e.g.*, we use the integer values of wind speed, say from 0 to 25 *m/s* to create 26 "buckets" of wind speed values**

WIND SPEED HISTOGRAM

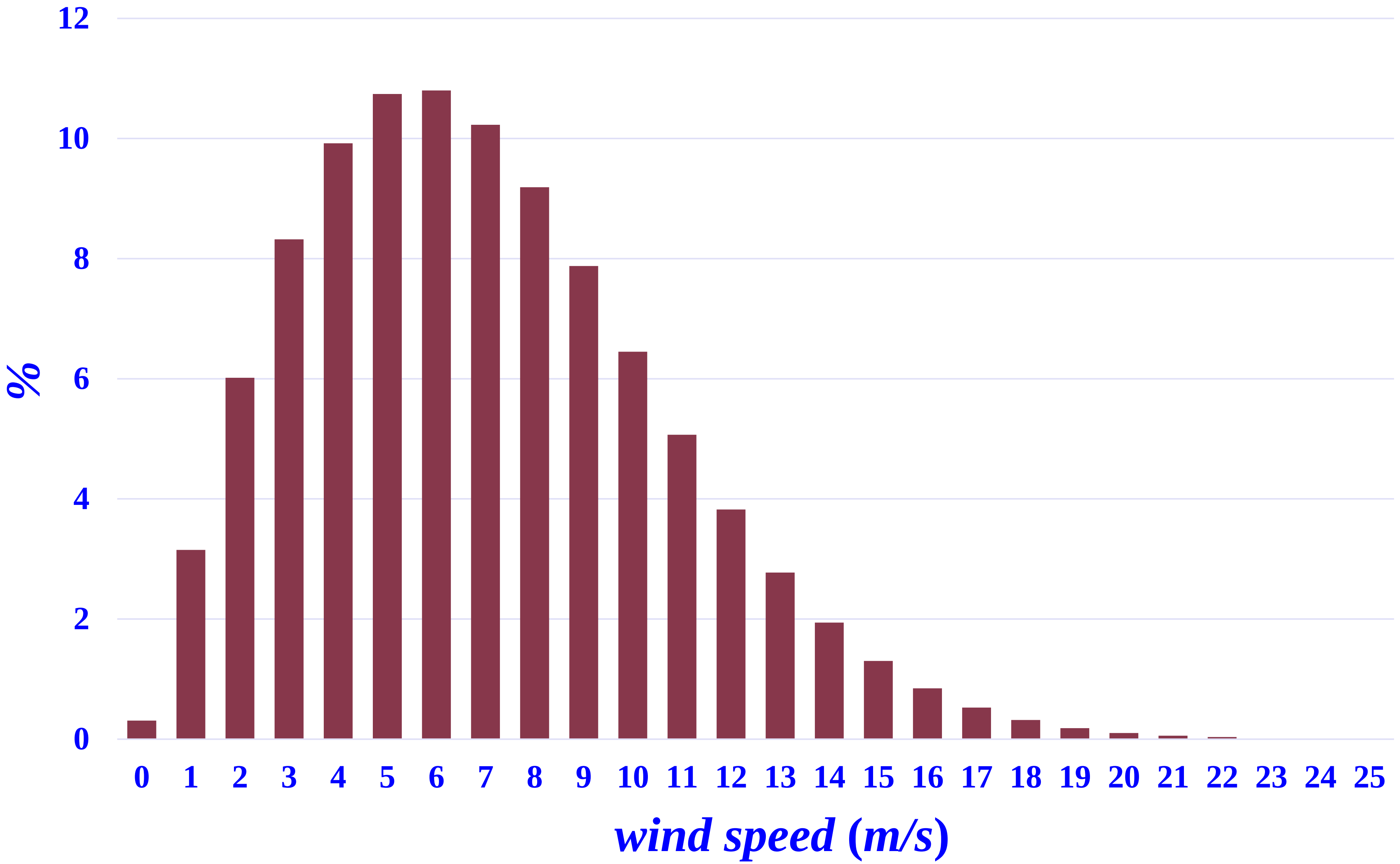
- **We place each hourly measured value in the appropriate "bucket" and we construct a *histogram* of the collected to obtain the *bar* plot below**



INTERPRETATION OF THE HISTOGRAM

- ❑ The height of each bar at wind speed value *v* in the histogram is the number of hours with wind speed value *v* in the data collection hours
- ❑ We *normalize* the vertical axis values by dividing the number of hours of each bar by the total number of hours to obtain the fraction of the total hours at a particular wind speed *v*
- ❑ Clearly, each bar has a value < 1 and the sum of all the bars must be exactly 1

WIND SPEED HISTOGRAM



INTERPRETATION OF THE HISTOGRAM

- **In this way, we obtain a *probability mass function* of the wind speed**
- **To understand the probability interpretation, we view wind speed as a *random variable* (*r.v.*) $\underset{\sim}{V}$ whose realizations are given by the *normalized histogram***
- **The normalized histogram provides the probability associated with each of the possible discrete–valued realizations**

INTERPRETATION OF HISTOGRAM

- ☐ **The bar of the mass density function at the wind speed v provides**

$$\mathbb{P}\left\{ \underset{\sim}{V} = v \right\} = \textit{probability of wind speed at } v \ m/s$$

- ☐ **We discretized the values of $\underset{\sim}{V}$ by creating the 26 discrete buckets 0, 1, 2, … , 25, but, in reality, wind speed does not take discrete values since it is a continuously–valued variable**

- ☐ **Alternatively, we may consider to make use of an increasingly finer resolution of the v values in order to capture the fact that $\underset{\sim}{V}$ is a continuous *r.v.***

PROBABILITY DENSITY

☐ **We associate with the continuous** ***r.v.*** $\underset{\sim}{V}$ **a**

probability density function **(*p.d.f.*)** $f_{\underset{\sim}{V}}(v)$ **with the**

following properties

- ○ $f_{\underset{\sim}{V}}(v) \geq 0 \qquad \forall\, v \geq 0$

- ○ $\int_{0}^{\infty} f_{\underset{\sim}{V}}(v)\, dv = 1$

PROBABILITY DENSITY

- ○ for an infinitesimally small $\delta > 0$

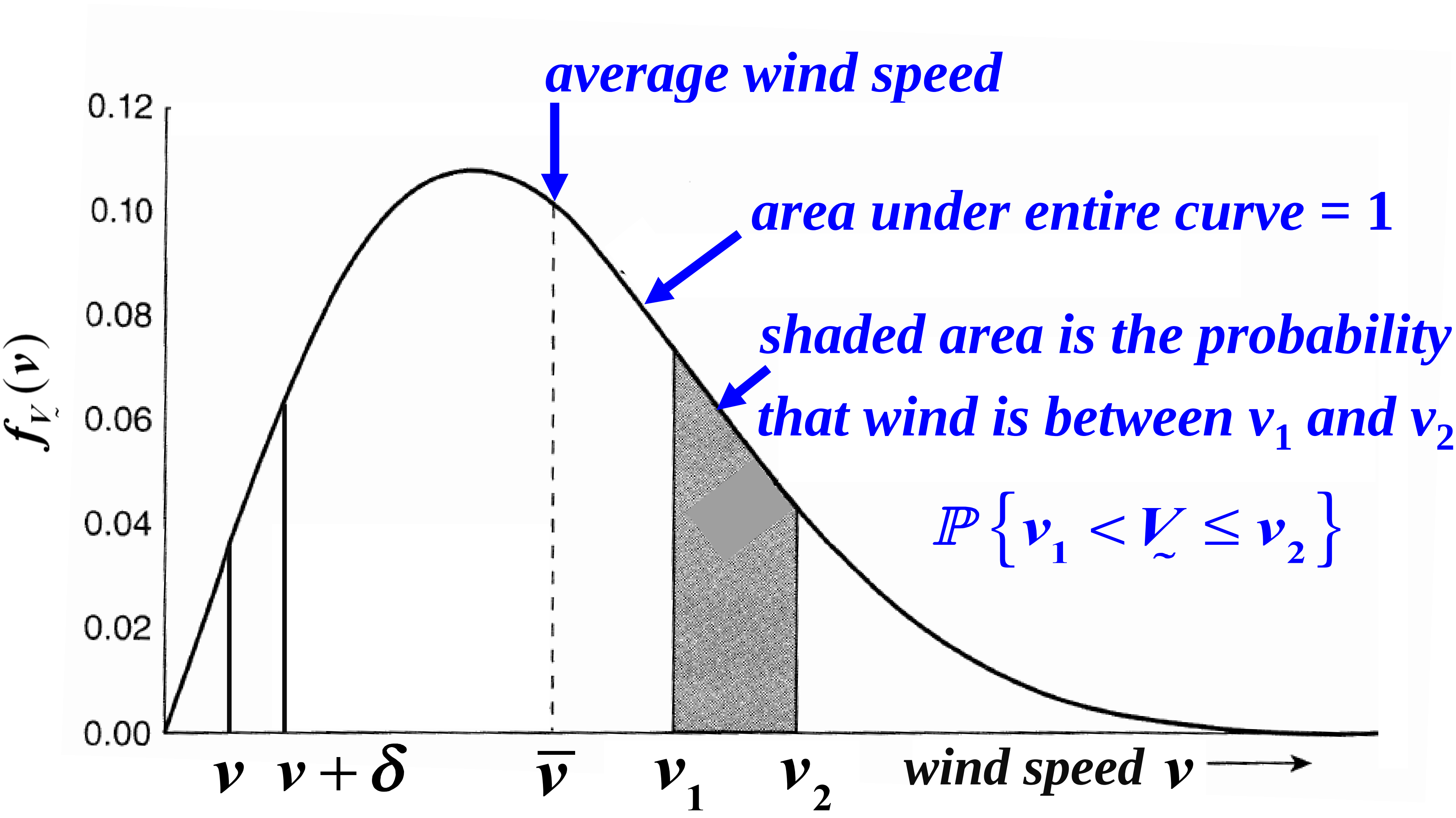
$$\mathbb{P}\left\{ v < \underset{\sim}{V} \leq v + \delta \right\} \approx f_{\underset{\sim}{V}}(v)\,\delta$$

$$\mathbb{P}\left\{ v_1 < \underset{\sim}{V} \leq v_2 \right\} = \int_{v_1}^{v_2} f_{\underset{\sim}{V}}(v)\, dv$$

- ☐ The *p.d.f.* $f_{\underset{\sim}{V}}(\cdot)$ provides the complete analytic

  characterization of the continuous *r.v.* $\underset{\sim}{V}$

PROBABILITY DENSITY



PROBABILITY DENSITY

- ☐ We may readily compute any function of $\underset{\sim}{V}$,
  - ○ average wind speed:

$$\bar{v} = \int_0^\infty v \; f_{\underset{\sim}{V}}(v) \, dv$$

  - ○ wind speed cubed:

$$E\left\{\underset{\sim}{V}^3\right\} = \int_0^\infty v^3 \; f_{\underset{\sim}{V}}(v) \, dv$$

PROBABILITY DENSITY

- number of annual hours $v_1 < \underset{\sim}{V} \leq v_2$: we define the indicator function $i(x)$ by

$$i(x) = \begin{cases} 1 & v_1 < x \leq v_2 \\ 0 & otherwise \end{cases}$$

and compute

$$8{,}760 \int_0^{\infty} i(v)\, f_{\underset{\sim}{V}}(v)\, dv = 8{,}760 \int_{v_1}^{v_2} (1)\, f_{\underset{\sim}{V}}(v)\, dv$$

WEIBULL DISTRIBUTION

- The *general Weibull distribution* function is given by

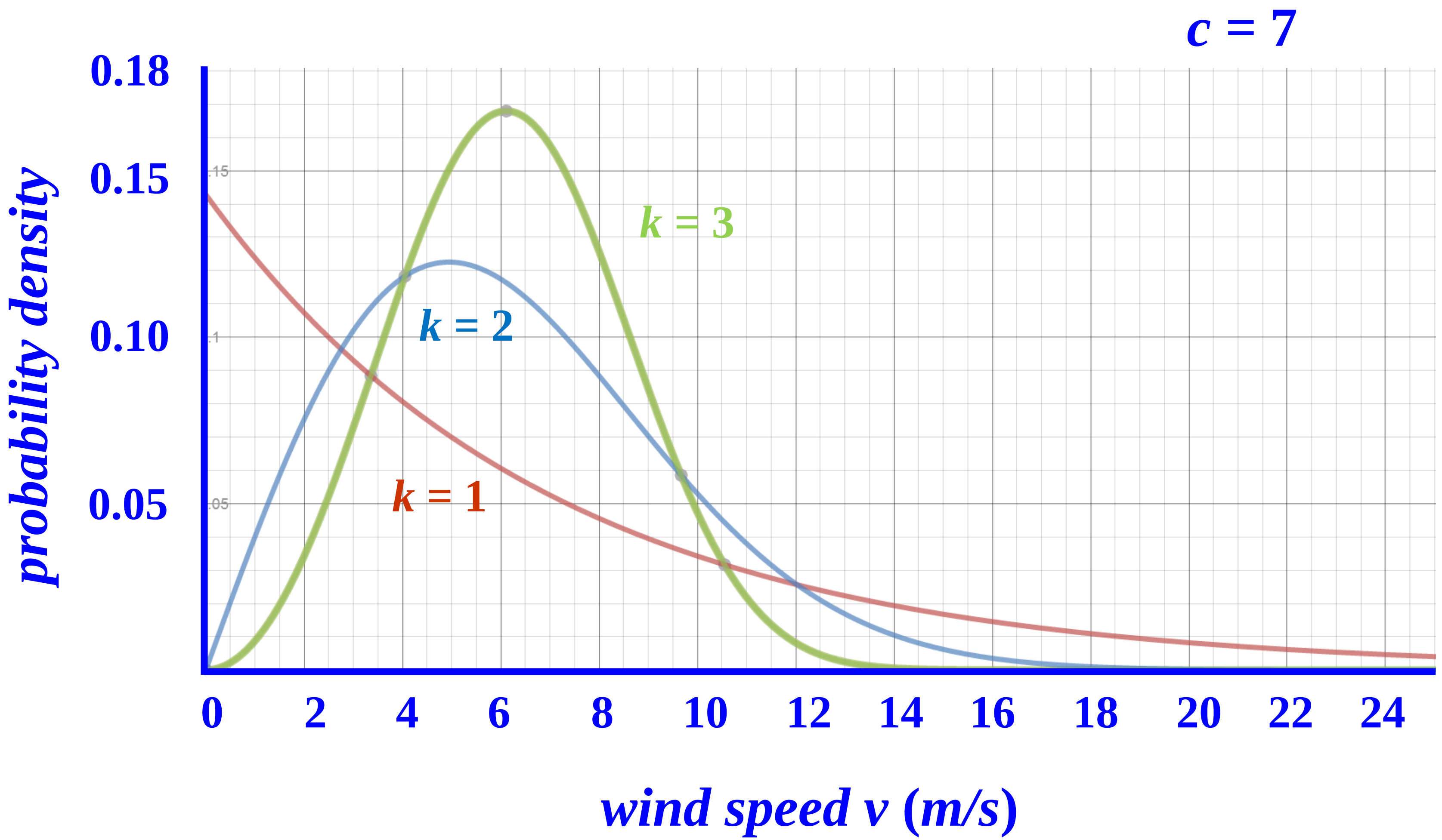
$$f\left(v\right)=\frac{k}{c}\left(\frac{v}{c}\right)^{k-1} e^{-\left(\frac{v}{c}\right)^{k}}$$

k = *shape parameter*

c = *scale parameter*

and is widely used to approximate the *p.d.f.* of $\underset{\sim}{V}$

WEIBULL DISTRIBUTIONS



WEIBULL DISTRIBUTION

- ❑ **For $k = 2$, the *Weibull distribution* is called the**

  Rayleigh p.d.f.

$$f(v) = \frac{2v}{c^2} e^{-\left(\frac{v}{c}\right)^2}$$

Rayleigh p.d.f.

- ❑ **The Rayleigh distribution is very widely used in the analytic characterization of wind**

WEIBULL DISTRIBUTION

- ☐ Note that for $\underset{\sim}{V} \sim Rayleigh$, the mean is given by

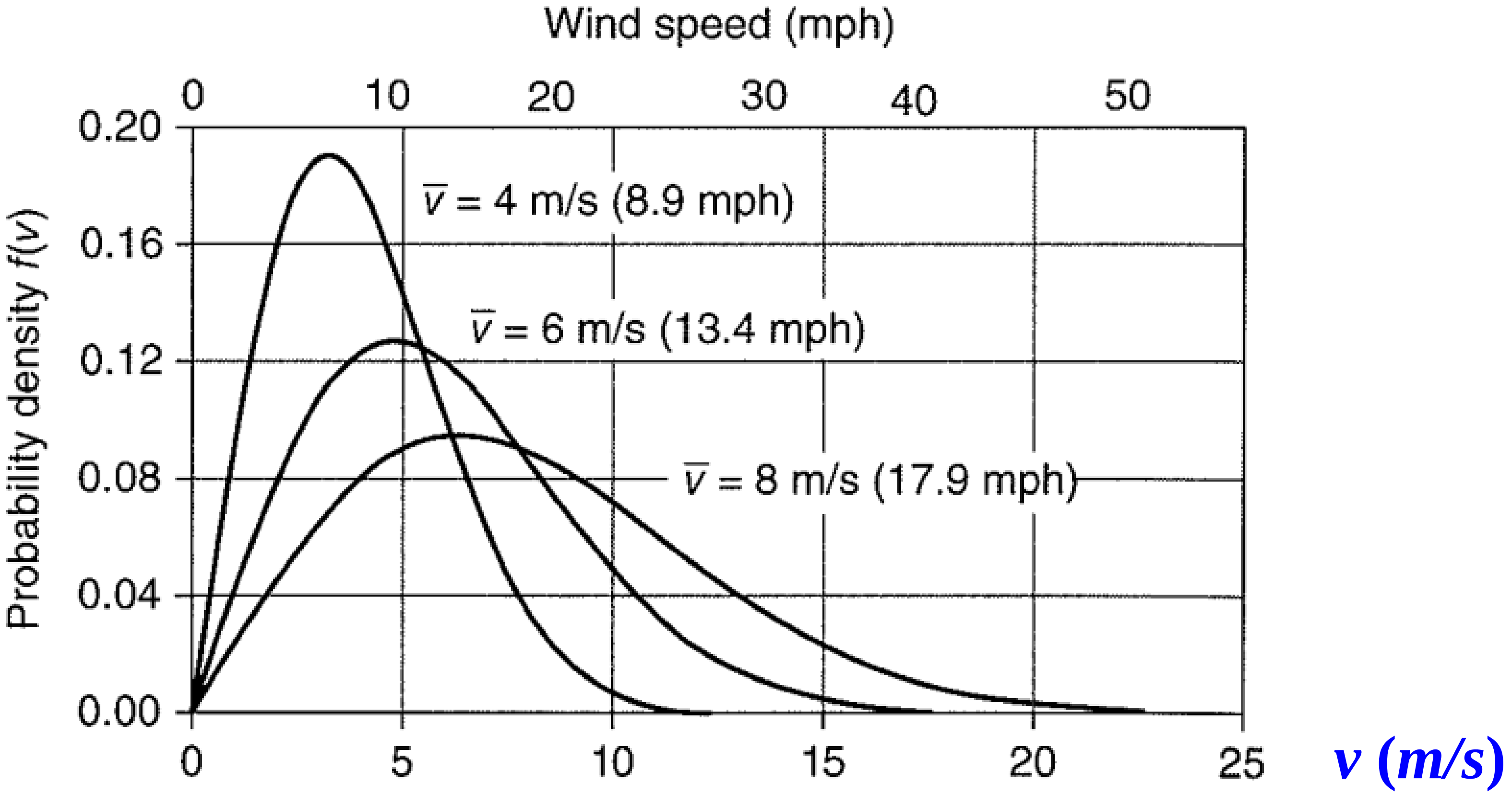
$$\bar{v} = \int_0^\infty v \, f_{\underset{\sim}{V}}(v) dv = 2\int_0^\infty \left(\frac{v}{c}\right)^2 e^{-\left(\frac{v}{c}\right)^2} dv = \frac{\sqrt{\pi}}{2} c$$

and, so we may restate the expression for $f_{\underset{\sim}{V}}(\bullet)$ as

$$f_{\underset{\sim}{V}}(v) = \frac{v\,\pi}{2\left(\bar{v}\right)^2} e^{-\frac{\pi}{4}\left(\frac{v}{\bar{v}}\right)^2}$$

WEIBULL DISTRIBUTION

- **As $\overline{v}$ increases, $f_{\underset{\sim}{V}}(\cdot)$ becomes flatter and shifts further to the right, as seen in the plots below**



RAYLEIGH–DISTRIBUTION–BASED CALCULATIONS

- **The widely-spread use of Rayleigh distribution is due to the good approximations it provides for the average wind speed $\bar{v}$**
- **Since**

$$\bar{v} = \frac{\sqrt{\pi}}{2} c ,$$

we evaluate

$$E\left(\underset{\sim}{V}^{3} \right) = \int_0^\infty v^3 \frac{\pi\, v}{2\left(\bar{v} \right)^2} e^{-\left[\frac{\pi}{4} \left(\frac{v}{\bar{v}} \right)^2 \right]} dv = \frac{6}{\pi} \left(\bar{v} \right)^3 \approx 1.91 \left(\bar{v} \right)^3$$

RAYLEIGH–DISTRIBUTION–BASED CALCULATIONS

- **A Rayleigh wind is the term we use to indicate a wind whose probability distribution is specified by the Rayleigh distribution**
- **We note that a Rayleigh distribution is completely specified once the average wind speed $\overline{v}$ is given, as all the parameters of this special case Weibull distribution with $k = 2$ are known**

RAYLEIGH–DISTRIBUTION–BASED CALCULATIONS

- ☐ This closed–form expression for Rayleigh–based wind distribution allows us to calculate the average power in wind

  $$\bar{p} = \frac{1}{2}\rho a\left(\bar{v}\right)^{3}\left(1.91\right)$$

  and therefore, it becomes very clear that we cannot simply use $\left(\bar{v}\right)^{3}$ directly to evaluate $\bar{p}$ but must also explicitly include the $\frac{6}{\pi}\approx 1.91$ factor

WIND POWER OUTPUT DISTRIBUTION

- Wind power output is a function of the *r.v.* $\underset{\sim}{V}$ and, therefore, wind power output is itself a *r.v.*, i.e.,

$$\underset{\sim}{P} = g\left(\underset{\sim}{V}\right) = \frac{1}{2}\rho a\left(\underset{\sim}{V}\right)^{3}$$

- For wind *r.v* $\underset{\sim}{V} \sim \textit{Weibull}$ *p.d.f.* with

$$f_{\underset{\sim}{V}}\left(v\right) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} e^{-\left(\frac{v}{c}\right)^{k}}$$

the *cumulative distribution function* is given by

$$F_{\underset{\sim}{V}}\left(v\right) = \mathbb{P}\left\{\underset{\sim}{V} \leq v\right\} = \int_{0}^{v} \frac{k}{c}\left(\frac{\xi}{c}\right)^{k-1} e^{-\left(\frac{\xi}{c}\right)^{k}} d\xi$$

WIND POWER OUTPUT DISTRIBUTION

- ☐ We next introduce a change of variables and we set

$$u = \left(\frac{\xi}{c}\right)^{k} \quad \text{and} \quad du = \frac{k}{c}\left(\frac{\xi}{c}\right)^{k-1} d\xi$$

so that

$$F_{\underset{\sim}{V}}(v) = \int_0^{\left(\frac{v}{c}\right)^k} e^{-u}\, du = 1 - e^{-\left(\frac{v}{c}\right)^k}$$

WIND POWER OUTPUT DISTRIBUTION

- ❑ **For the special case of *Rayleigh p.d.f.***

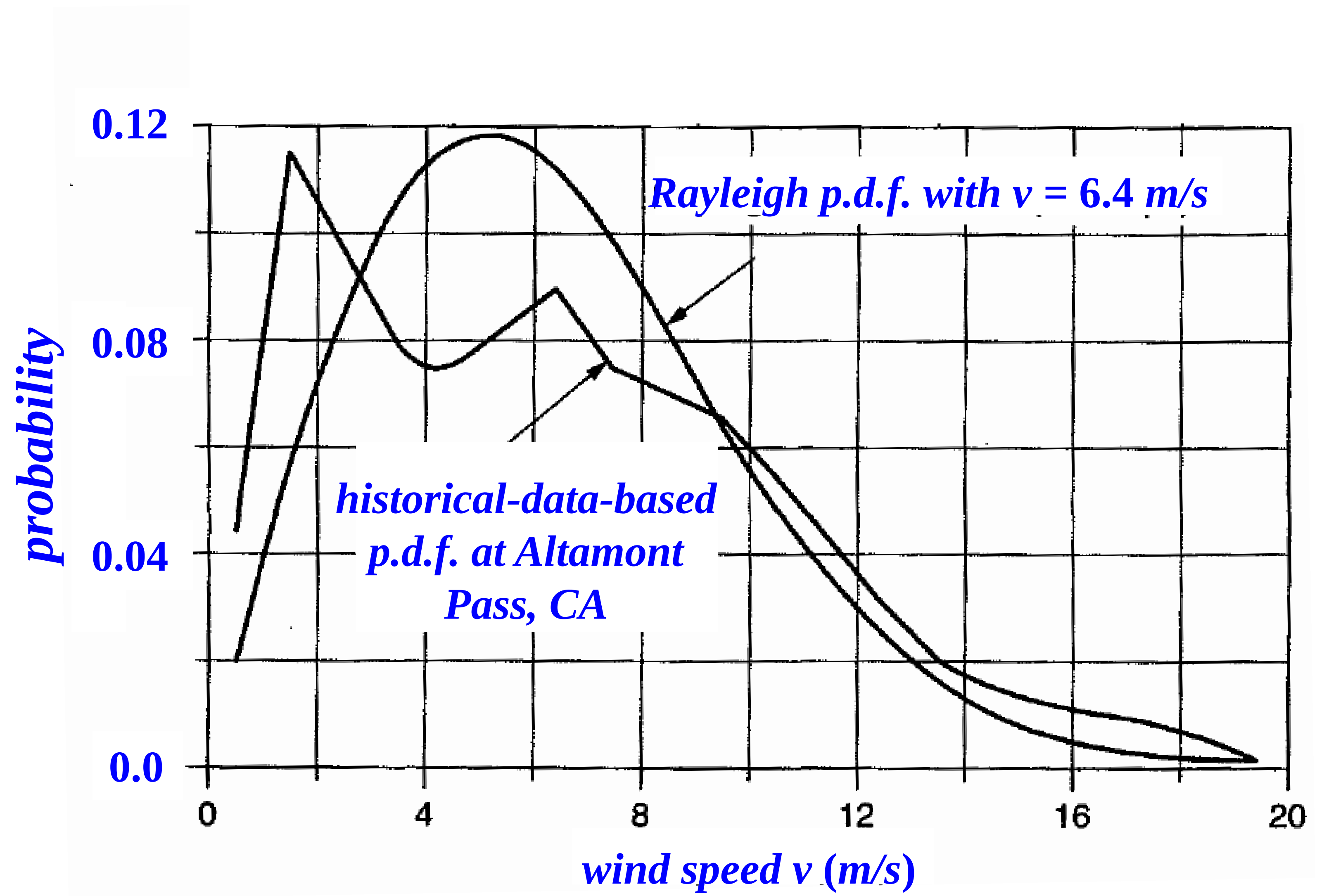
$$F_{\underset{\sim}{V}}(v)\Big|_{Rayleigh} = 1 - e^{-\left[\frac{\pi}{4}\left(\frac{v}{\bar{v}}\right)^{2}\right]}$$

- ❑ **Note that the probability that *Rayleigh* wind**

  exceeds the value *v* is

$$\mathbb{P}\left\{\underset{\sim}{V} > v\right\} = 1 - F_{\underset{\sim}{V}}(v)\Big|_{Rayleigh} = e^{-\left[\frac{\pi}{4}\left(\frac{v}{\bar{v}}\right)^{2}\right]}$$

ALTAMONT PASS, CA: HISTORICAL DATA *p.d.f.* *vs.* RAYLEIGH *p.d.f.*



EXAMPLE: AVERAGE POWER IN THE WIND

- ❑ Based on data from a standard anemometer at a height of $10\ m$, $\bar{v}(10) = 6\ m/s$
- ❑ The plan is to erect a $50\ m$ tower for the nacelle installation; we need to estimate the average power under the following assumptions:
  - ○ Hellman exponent $\alpha = \dfrac{1}{7}$
  - ○ $\rho = 1.225 \dfrac{kg}{m^3}$
  - ○ Rayleigh distribution may be used

EXAMPLE: AVERAGE POWER IN THE WIND

- ☐ The first step is to compute $\bar{v}(50)$

$$\bar{v}(50) = \bar{v}(10)\left(\frac{50}{10}\right)^{\frac{1}{7}} = 7.55\ \frac{m}{s}$$

- ☐ Since the Rayleigh distribution holds

$$\frac{\bar{p}(50)}{a} = \frac{6}{\pi}\cdot\frac{1}{2}\rho\left[\bar{v}(50)\right]^3 = 1.91\cdot\frac{1}{2}\cdot 1.225\cdot(7.55)^3 = 504\,\frac{W}{m^2}$$

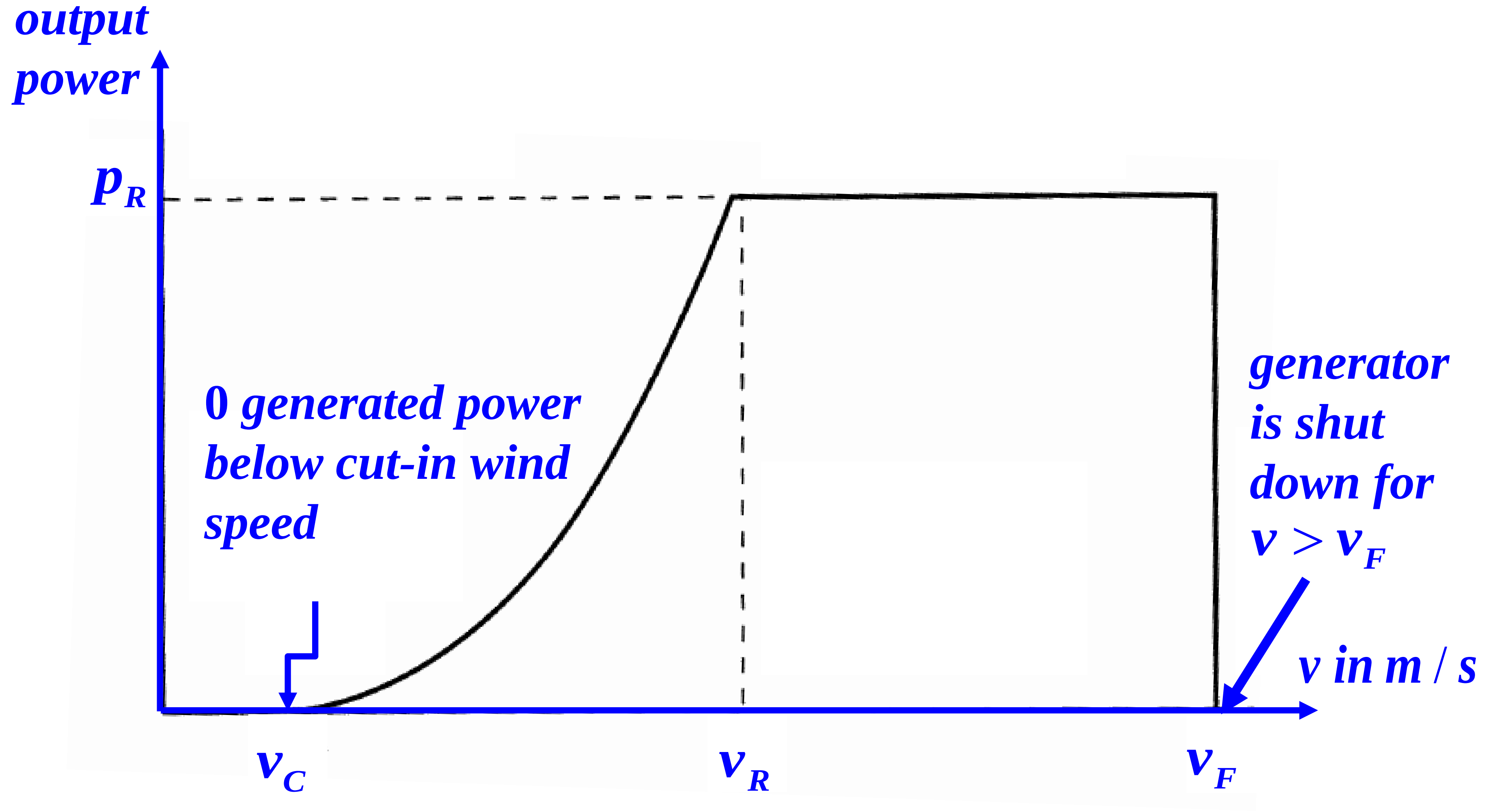
- ☐ Sensitivity case for an 80–m tower:

$$\frac{\dfrac{\bar{p}(80)}{a}}{\dfrac{\bar{p}(50)}{a}} = \left(\frac{80}{50}\right)^{\frac{3}{7}} \qquad \frac{\bar{p}(80)}{a} = 504\left(\frac{80}{50}\right)^{\frac{3}{7}} = 616\,\frac{W}{m^2}$$

THE *IDEALIZED WIND TURBINE POWER CURVE*

- Each turbine manufacturer provides a plot of the electrical power output of the entire system – the blades, the gearbox, the generator and the other components – as a function of wind speed
- Such a plot is called an *idealized wind turbine power curve*
- The typical shape of an idealized wind turbine power curve is illustrated below

THE *IDEALIZED WIND TURBINE POWER CURVE*



0 generated power below cut-in wind speed

generator is shut down for $v > v_F$

THE IDEALIZED WIND TURBINE POWER CURVE

- ❑ **At low speeds, the wind has insufficient energy to overcome friction in the turbine drive train, even when the generator rotor is spinning: below the *cut–in wind speed* v_C , the power output is 0**
- ❑ **Above v_C, the power output is a cubic function of v ; at the *rated wind speed* v_R , the generator output is at its *rated power* p_R**

THE IDEALIZED WIND TURBINE POWER CURVE

- ❑ **At $v > v_R$, controls are deployed to shed some of the wind so as not to exceed p_R**
- ❑ **When wind speed reaches the *cut–out value* v_F – sometimes called as the furling wind speed – a sailing term – the machine is shut down and the mechanical brakes lock down the rotor shaft above v_F wind speeds and the power output is 0**

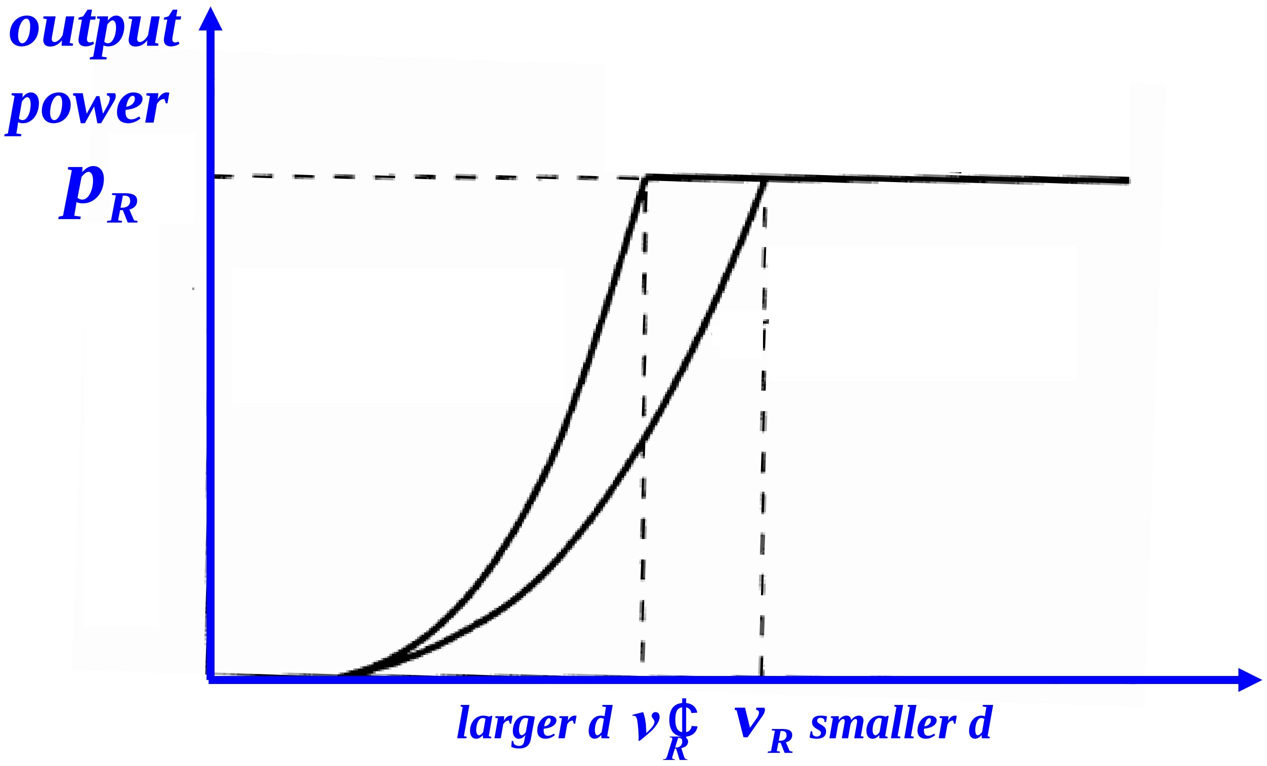
IMPACTS OF DESIGN PARAMETERS

- ☐ We can assess the impact of two key design parameters
  - ○ the diameter d of the blade rotor
  - ○ the rated generator capacity

  on the power output determined by means of the idealized power curve
- ☐ The power output $p \propto d^2$ since d^2 determines the area swept by the blades

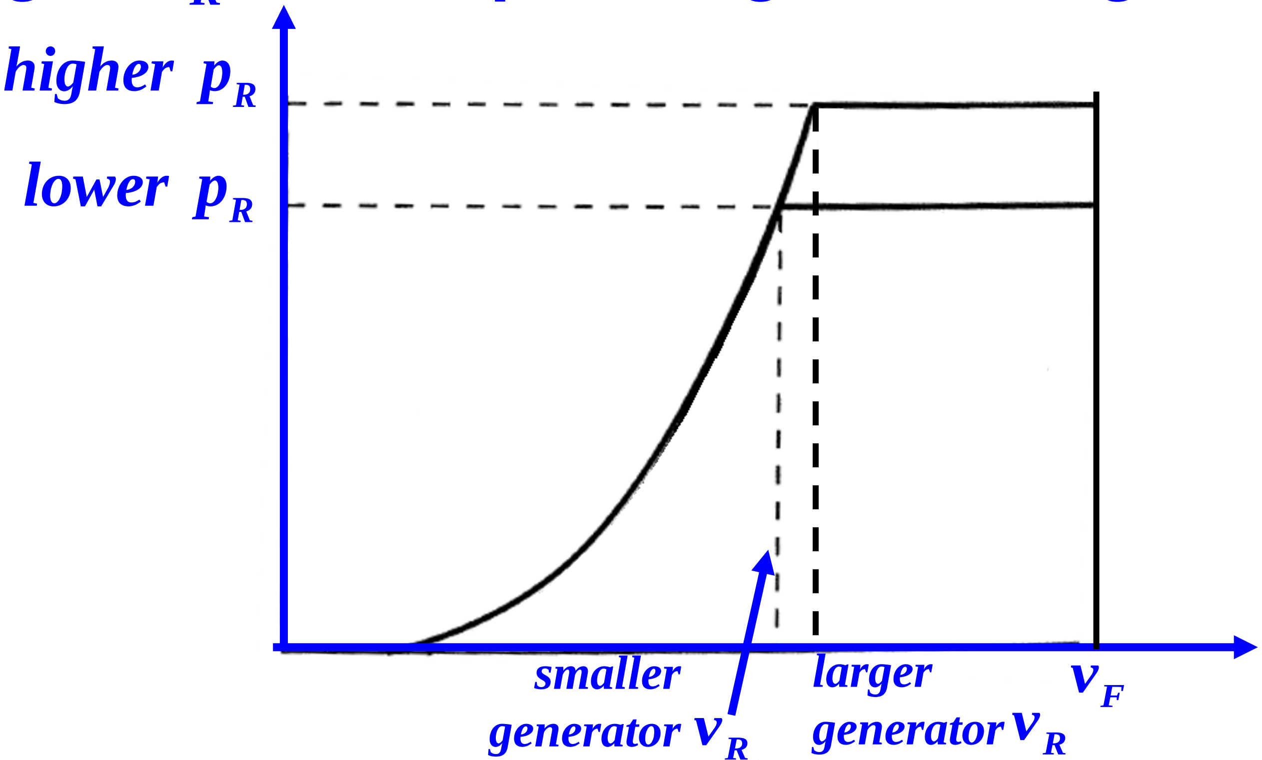
IMPACTS OF DESIGN PARAMETERS

- **For a generator with rated power p_R, an increase in *d* produces a shift in the power curve to the left and the output p_R is thus reached at a *lower speed***



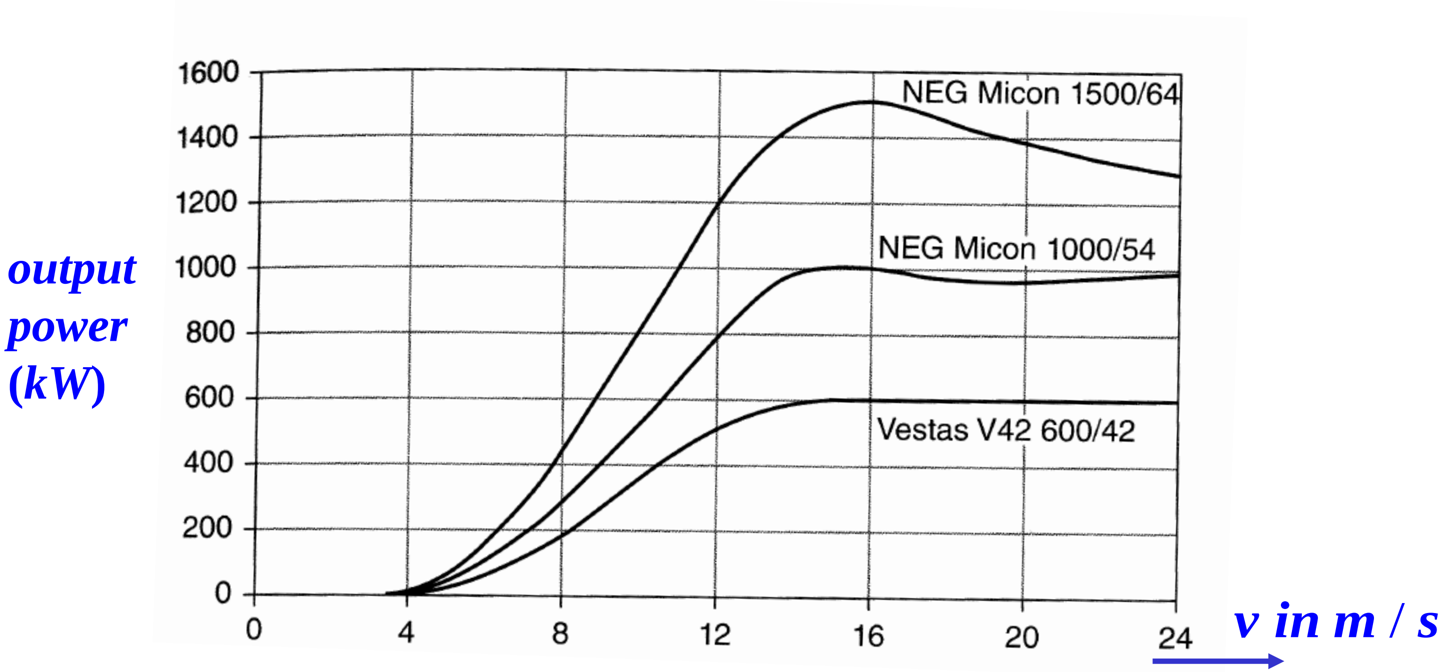
IMPACTS OF DESIGN PARAMETERS

- ☐ **For a fixed rotor diameter d, an increase in the generator rated capacity may be determined by the continuation of the power curve up to the larger v_R corresponding to the higher p_R**



IMPACTS OF DESIGN PARAMETERS

- **Actual power curves do not veer too far from the idealized ones, with much of the variance due to the *inability of wind shedding techniques to control the power outputs at speeds* $v > v_R$; in certain cases, the value of v_R is difficult to determine precisely**

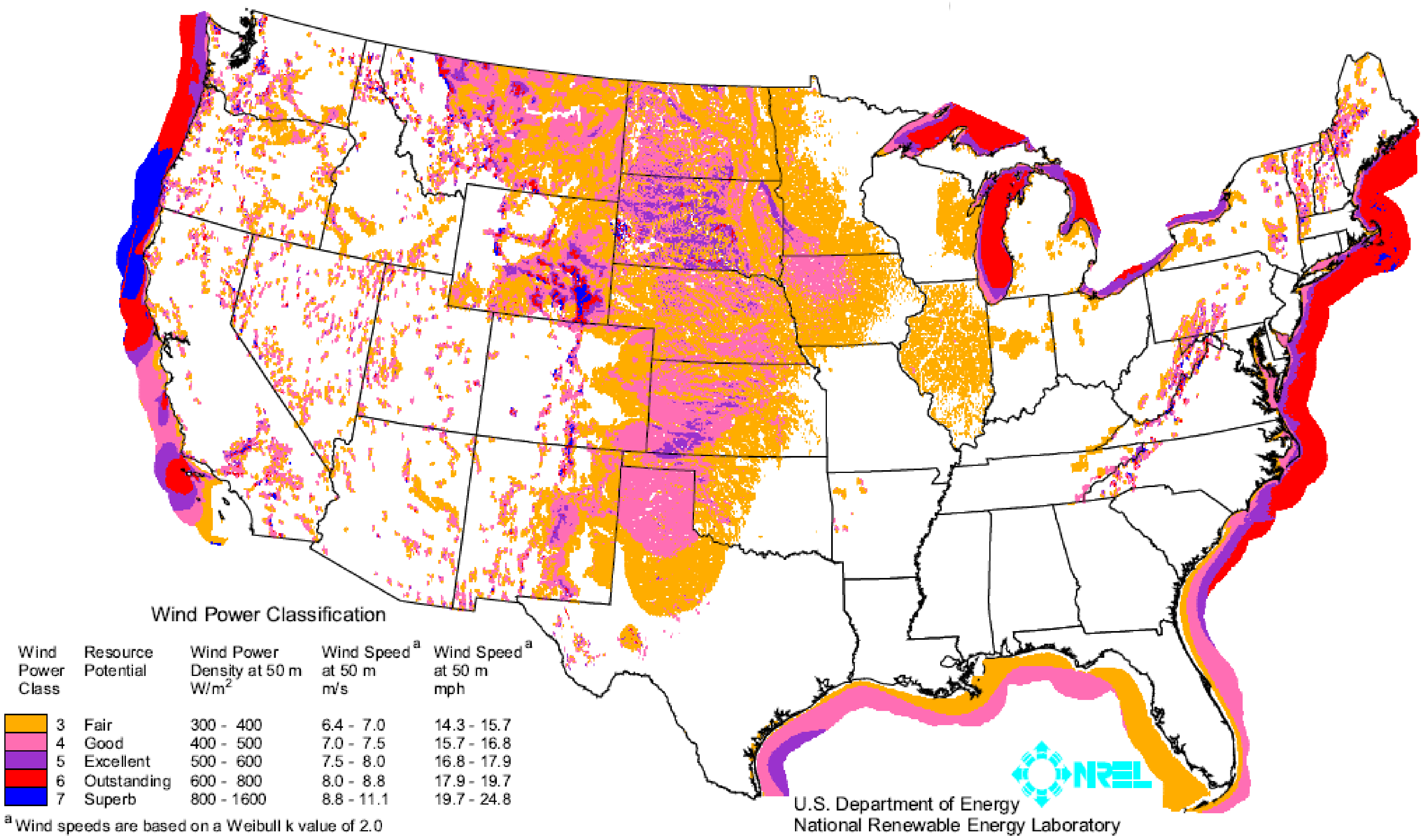


classes of wind power density at 10 m and 50 m

	10 *m* (33 *ft*)		**50 *m* (164 *ft*)**	
wind power class	***wind power density (W/m^2)***	***speed m/s (mph)***	***wind power density (W/m^2)***	***speed m/s (mph)***
1	**< 100**	**< 4.4 (9.8)**	**< 200**	**< 5.6 (12.5)**
2	**100 - 150**	**4.4 (9.8)/5.1 (11.5)**	**200 - 300**	**5.6 (12.5)/6.4 (14.3)**
3	**150 - 200**	**5.1 (11.5)/5.6 (12.5)**	**300 - 400**	**6.4 (14.3)/7.0 (15.7)**
4	**200 - 250**	**5.6 (12.5)/6.0 (13.4)**	**400 - 500**	**7.0 (15.7)/7.5 (16.8)**
5	**250 - 300**	**6.0 (13.4)/6.4 (14.3)**	**500 - 600**	**7.5 (16.8)/8.0 (17.9)**
6	**300 - 400**	**6.4 (14.3)/7.0 (15.7)**	**600 - 800**	**8.0 (17.9)/8.8 (19.7)**
7	**> 400**	**> 7.0 (15.7)**	**> 800**	**> 8.8 (19.7)**

Source*: *http://www.awea.org/faq/basicwr.html

WIND POWER EQUI – DENSITY CONTOURS AT 50 *m*



Wind Power Classification

	Wind Power Class	Resource Potential	Wind Power Density at 50 m W/m²	Wind Speed[a] at 50 m m/s	Wind Speed[a] at 50 m mph
🟧	3	Fair	300 - 400	6.4 - 7.0	14.3 - 15.7
🟪	4	Good	400 - 500	7.0 - 7.5	15.7 - 16.8
🟪	5	Excellent	500 - 600	7.5 - 8.0	16.8 - 17.9
🟥	6	Outstanding	600 - 800	8.0 - 8.8	17.9 - 19.7
🟦	7	Superb	800 - 1600	8.8 - 11.1	19.7 - 24.8

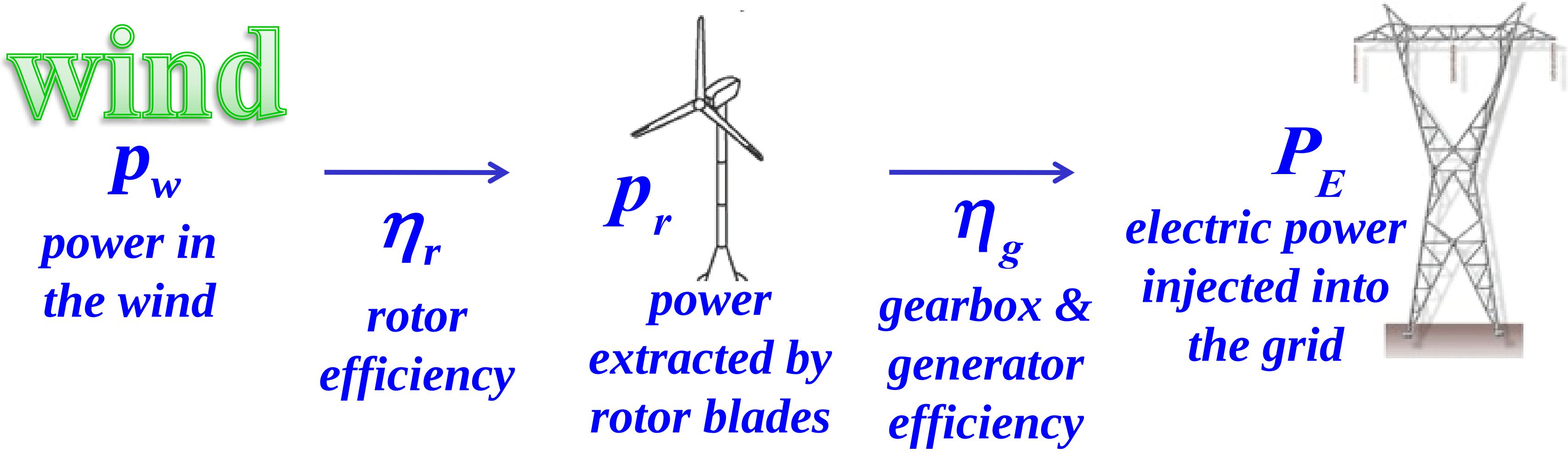
[a] Wind speeds are based on a Weibull k value of 2.0

U.S. Department of Energy
National Renewable Energy Laboratory

Source: http://www.windpoweringamerica.gov/pdfs/wind_maps/us_windmap.pdf

ESTIMATES OF WIND TURBINE ENERGY

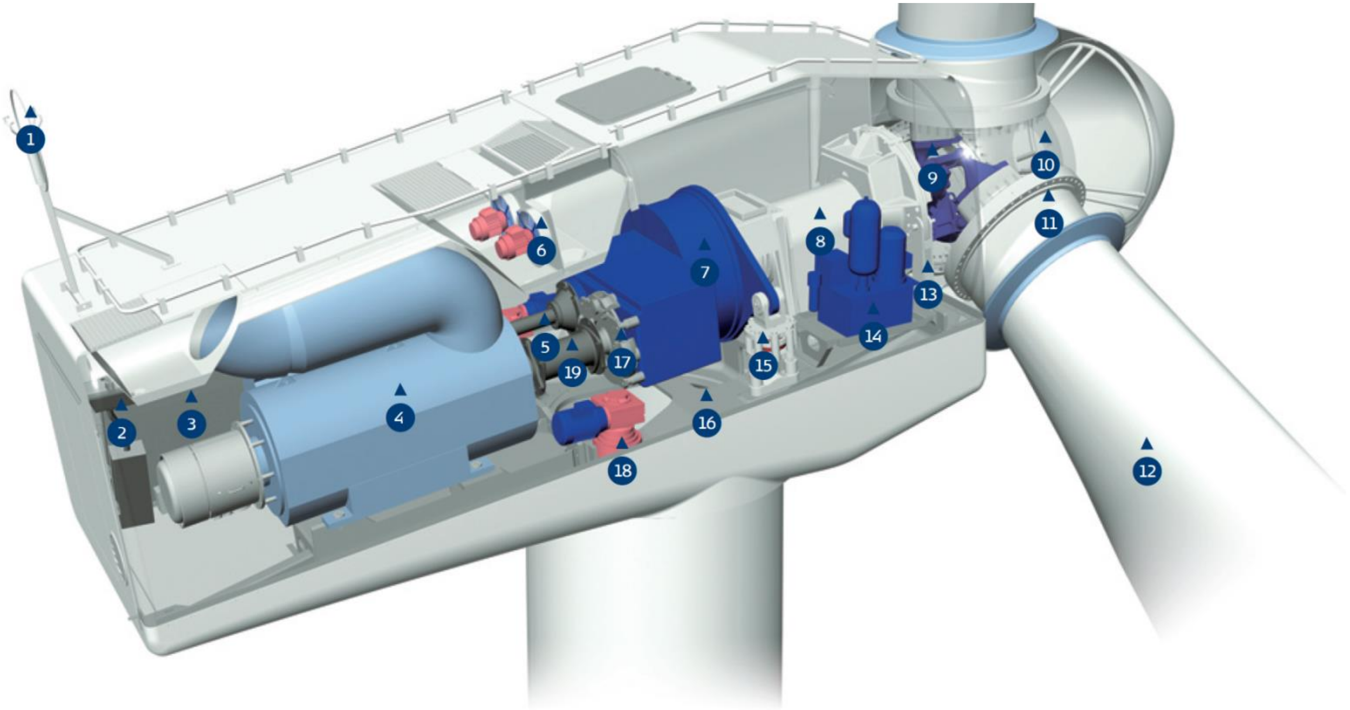
- It is impossible to extract 100 % of the power in the wind as the rotor spills high–speed winds and the limited energy at low–speed winds is lost
- The energy generated depends on rotor, gearbox, generator, tower, controls, terrain, and the wind



p_w *power in the wind* $\xrightarrow{\quad}$ η_r *rotor efficiency* $\to$ p_r *power extracted by rotor blades* $\xrightarrow{\quad}$ η_g *gearbox & generator efficiency* $\to$ P_E *electric power injected into the grid*

- Overall conversion efficiency $\eta_r \eta_g$ is around 30 %

VESTAS *V*52 850 – *kW*
WIND TURBINE COMPONENTS



① Ultrasonic wind sensor

② Service crane

③ VMP-Top controller
with converter

④ OptiSpeed® Generator

⑤ Pitch cylinder

⑥ Oil and water coolers

⑦ Gearbox

⑧ Main shaft

⑨ Pitch system

⑩ Blade hub

⑪ Blade bearing

⑫ Blade

⑬ Rotor lock system

⑭ Hydraulic unit

⑮ Torque arm

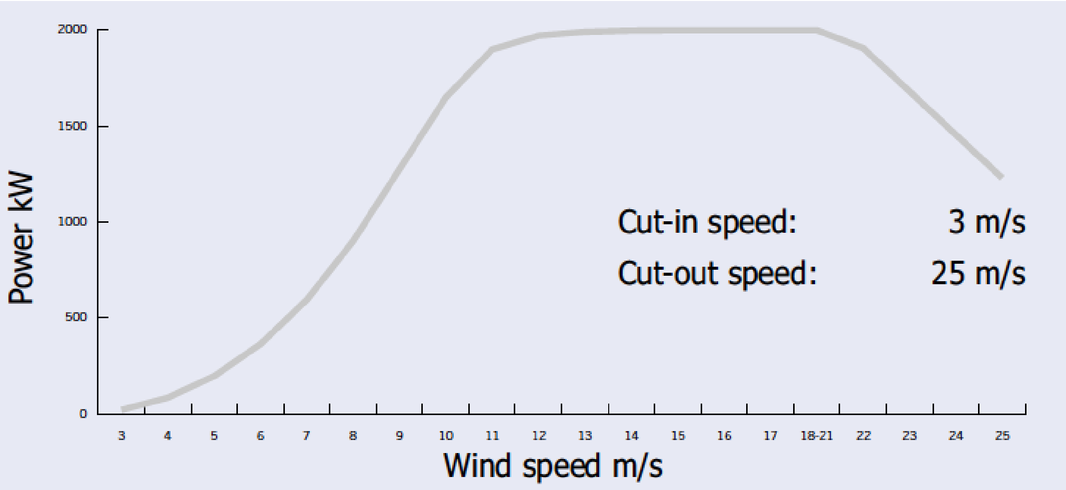
⑯ Machine foundation

⑰ Mechanical disc brake

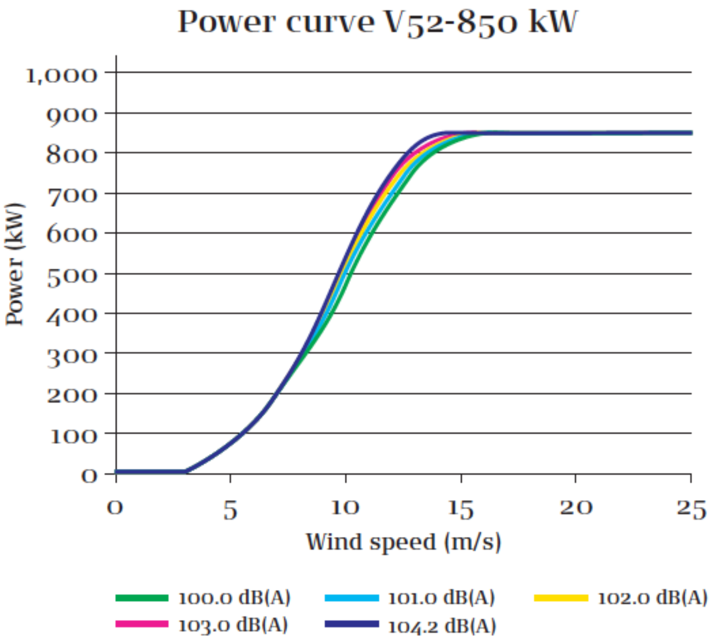
⑱ Yaw gear

⑲ Composite disc coupling

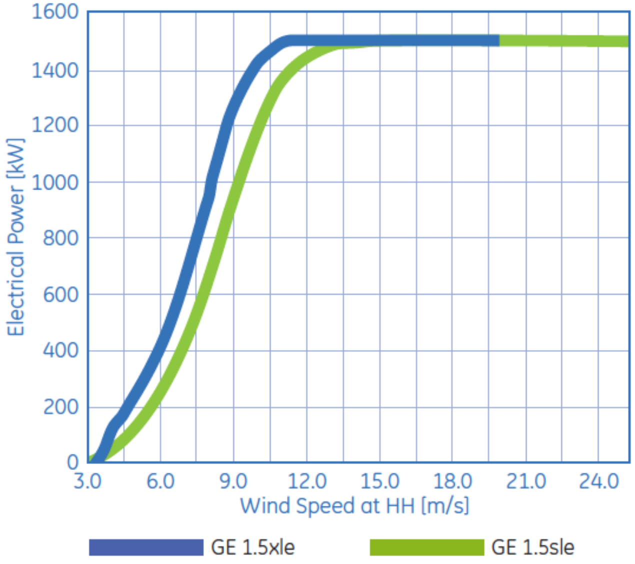
MANUFACTURER POWER CURVES



Gamesa G90*: 2.0 *MW



Vestas V52*: 850 *kW



GE 1.5sle/xle*: 1.5 *MW