ECE 333 – GREEN ELECTRIC ENERGY

6. Limits on Conversion of Wind Into Electricity

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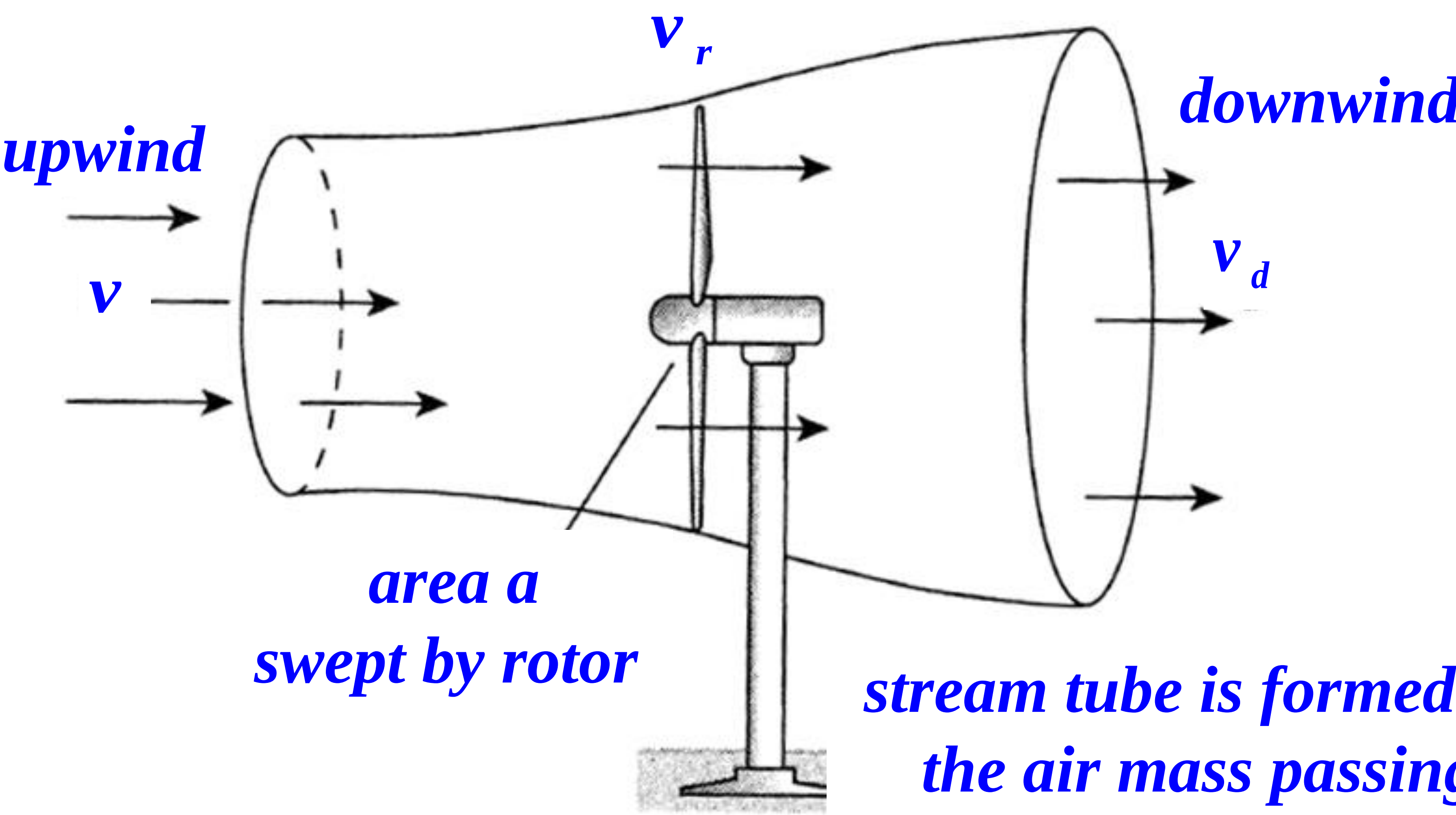
CONVERSION OF WIND INTO ELECTRICITY

- **We analytically characterize the power in wind as a cubic function of the wind speed *v***
- **The wind energy is in the form of kinetic energy, whose extraction from the wind is used to rotate the generator shaft mounted in the nacelle**
- **We examine the constraint – the so–called *Betz limit* – that limits the ability of a wind turbine to convert the wind kinetic energy into mechanical energy to rotate the turbine generator shaft**

THE BETZ LIMIT

- The limit was derived in 1919 by Albert Betz, a German physicist
- We consider the wind as it passes through a wind turbine rotor and we examine the wind stream
- We explain the conceptual basis on the limit of the conversion of wind into electricity by means of the diagram below

THE BETZ LIMIT



stream tube is formed by the air mass passing through the turbine rotor

THE BETZ LIMIT

- Clearly, the turbine cannot extract all the kinetic energy in the wind because that implies that the air would have to stop completely after passing the turbine – an impossible situation since it would prevent all the continuing wind to pass through the rotor
- Furthermore, the downwind velocity v_d *cannot equal* v since that would imply that no energy is extracted by the turbine

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THE BETZ LIMIT

- Betz formulated the relationship to determine the maximum mechanical power obtainable from wind
- We focus on what happens with the wind as it passes through the plane of the rotor blades at velocity v_r, with

$$v_d < v_r < v \quad ,$$

where, we explicitly take into account that, as the wind mass of air goes through the stream tube and kinetic energy is extracted, the downwind speed must be lower than the upwind speed

THE BETZ LIMIT

- **The conservation of energy implies that**

$$\begin{array}{c} \textit{kinetic energy} \\ \textit{upwind} \end{array} = \begin{array}{c} \textit{kinetic energy} \\ \textit{downwind} \end{array} + \begin{array}{c} \textit{energy extracted} \\ \textit{by blade rotor} \end{array}$$

- **Therefore, as the mass flow rate $\frac{dm}{dt}$ throughout the stream tube remains unchanged, the power extracted by the rotor blades is**

$$p_r = \frac{d}{dt}\left(\begin{array}{c} \textit{kinetic} \\ \textit{energy} \\ \textit{upwind} \end{array} - \begin{array}{c} \textit{kinetic} \\ \textit{energy} \\ \textit{downwind} \end{array} \right)$$

$$= \frac{1}{2}\frac{dm}{dt}\left(v^2 - v_d^2 \right)$$

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THE BETZ LIMIT

- Now, we can determine $\dfrac{dm}{dt}$ anywhere in the stream tube and at the rotor blade plane since

$$\frac{dm}{dt} = \rho a v_r$$

- We assume that

$$v_r = \frac{v + v_d}{2}$$

and therefore

$$p_r = \frac{1}{2} \rho a \left(\frac{v + v_d}{2} \right) \left(v^2 - v_d^2 \right)$$

THE BETZ LIMIT

- We introduce the ratio λ defined by

$$v_d = \lambda v$$

so that the expression for p_r becomes

$$p_r = \frac{1}{2} \rho a v \left(\frac{1+\lambda}{2} \right) v^2 \left(1 - \lambda^2 \right)$$

$$= \underbrace{\frac{1}{2} \rho a v^3}_{\textit{power in the wind}} \; \underbrace{\frac{1}{2} \left(1 + \lambda \right) \left(1 - \lambda^2 \right)}_{\textit{fraction extracted}}$$

THE BETZ LIMIT

- We can think of the fraction of power extracted by the rotor as the rotor efficiency η_r

$$\eta_r = \frac{1}{2}(1+\lambda)(1-\lambda^2)$$

so that we may write

$$p_r = \frac{1}{2}\rho a v^3 \eta_r$$

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THE BETZ LIMIT

- To determine the maximum rotor efficiency, we evaluate the derivative of p_r $w.r.t.$ λ

$$\frac{dp_r}{d\lambda} = \frac{1}{2}\rho a v^3 \ \frac{1}{2}\left[\left(1+\lambda\right)\left(-2\lambda\right)+\left(1\right)\left(1-\lambda^2\right)\right]$$

$$= \frac{1}{4}\rho a v^3 \left[-2\lambda^2 - 2\lambda + 1 - \lambda^2\right]$$

$$= \frac{1}{4}\rho a v^3 \left[1 - 2\lambda - 3\lambda^2\right]$$

$$= \frac{1}{4}\rho a v^3 \left[\left(1+\lambda\right)\left(1-3\lambda\right)\right]$$

THE BETZ LIMIT

- We set $\dfrac{dp_r}{d\lambda}$ to be 0 and we solve for λ

- The only physically meaningful solution is

$$\lambda = \frac{1}{3} ,$$

i.e., the efficiency is maximized when the ratio of

v_d to v is 1/3 so that

THE BETZ LIMIT

$$\eta_r = \frac{1}{2}\left(1+\frac{1}{3}\right)\left(1-\frac{1}{3^2}\right) = \frac{16}{27} = 59.3\ \%$$

- **This optimal theoretical efficiency – better known as the *Betz efficiency* – cannot be higher than 59.3 %; this value is the essence of the *Betz limit***

THE BETZ LIMIT

- The *Betz* limit implies that even under ideal conditions less than 60 % of the power in wind can be extracted; indeed, in actual systems, the best that is attainable is, typically, below 50 % – in other words, at most half of the energy in wind can be converted into mechanical energy to rotate the generator shaft

TIP SPEED RATIO

- The tip speed of the rotor is a function of the rate of rotation of the rotor specified by its *r.p.m.*: in each revolution of the rotor, the tip traverses a distance πd and so the tip speed is $(\pi d)(r.p.m.)$
- A convenient way to express rotor efficiency is in terms of the tip speed ratio τ , where

$$\tau = \frac{rotor\ tip\ speed}{v} = (r.p.m.) \cdot \frac{min}{60\ sec} \cdot \frac{\pi d}{v}$$

TIP SPEED RATIO

- **Studies indicate that modern turbines attain maxi-mum efficiency for $4 \leq \tau \leq 6$: the tip of the blade moves 4 – 6 times faster than the wind speed**
- **It follows that for maximum efficiency it is desirable that turbine blades change their speed as wind speed changes – as is the case in the so–called *variable speed generators***

INTERPRETATION OF η_r

- Betz's law states that the maximum power that we can extract from wind is

$$p_r = \left(\frac{1}{2} \rho a v^3 \right) 0.593$$

- Engineers define the efficiency as the ratio of the output to the input quantity

$$\eta_r = \frac{p_{out}}{p_{in}}$$

and so the natural question is what is p_{in}

INTERPRETATION OF η_r

- A convenient way to think about p_{in} is that p_{in} is the power in the wind prior to the installation of a turbine: absent the turbine, $v_r = v$ and so

$$p_{in} = \frac{1}{2}\rho a v^{3}$$

- The Betz efficiency determines the limit on the conversion of p_{in} into mechanical power to rotate the generator shaft

WIND TURBINE GENERATORS

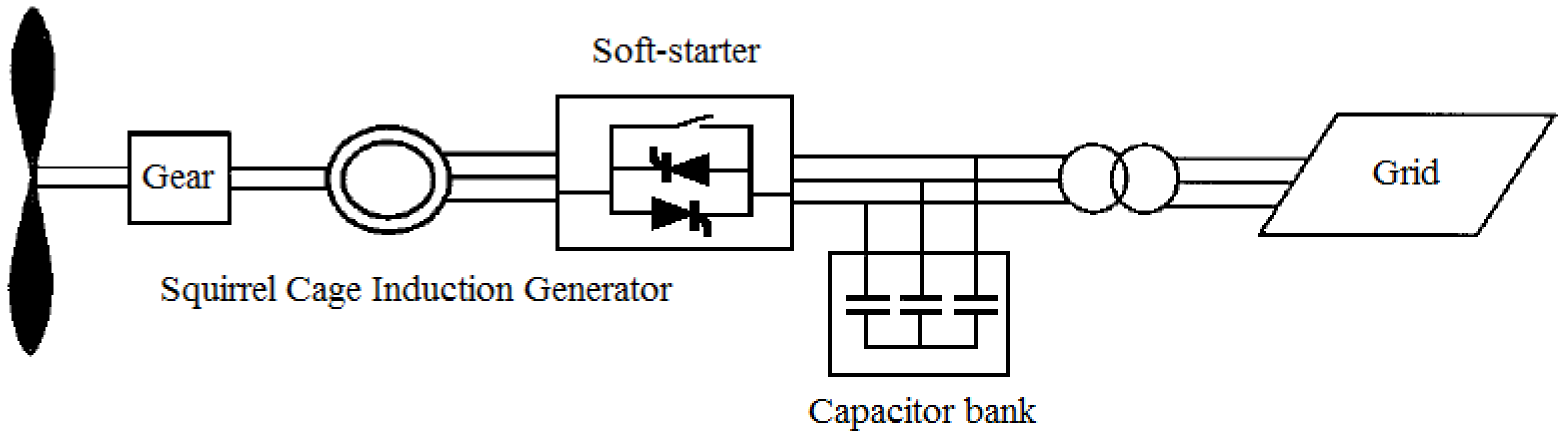
- **The wind turbine generators or wind energy conversion systems may be classified into two principal categories**
  - **variable–speed rotors**
  - **fixed–speed rotors**
- **The variable–speed turbines are able advanta-geously use the fact that wind speed varies to**

WIND TURBINE GENERATORS

adjust the rotor speed in order to *optimally match* wind speed

- **The fixed–speed rotor generators are simpler but do not operate at *optimal efficiency*; moreover, the stresses from the rapidly varying wind speeds, typically, require sturdier design of such turbines**

WECS TYPE A



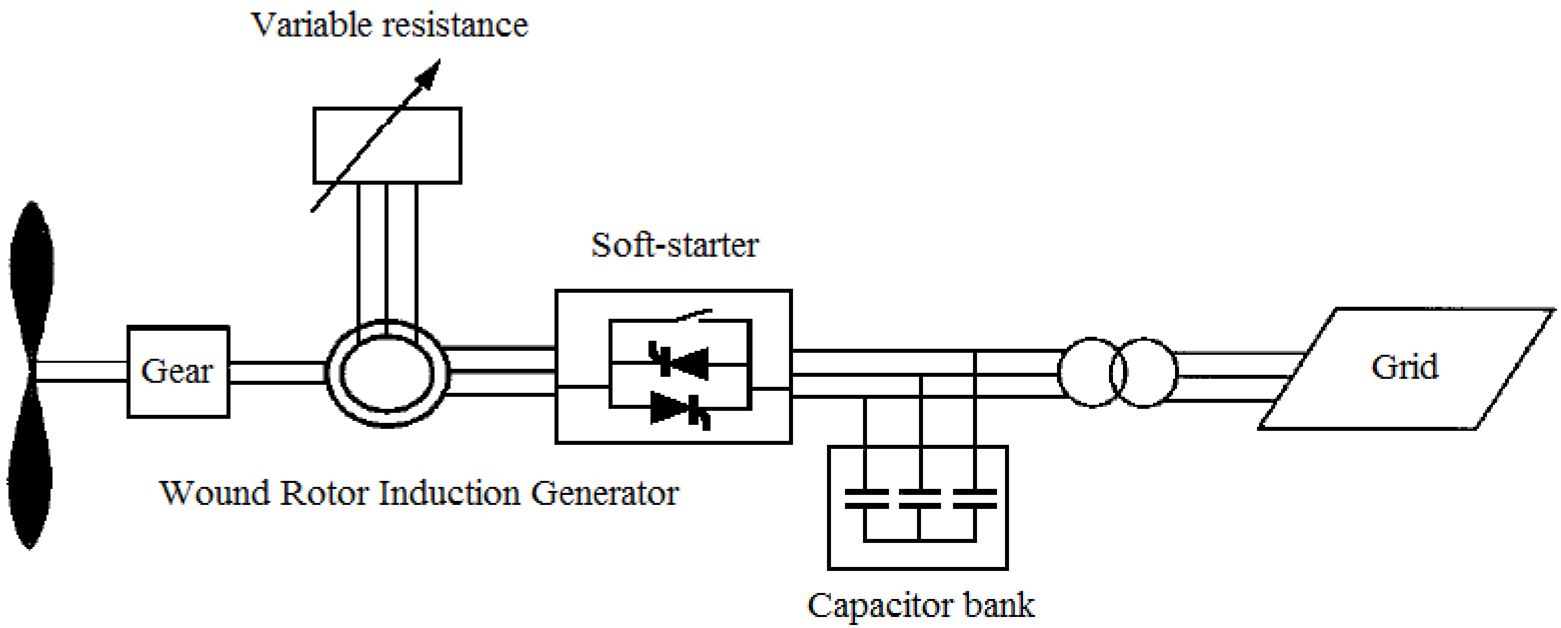
- **The turbine design uses an induction generator connected to a fixed–speed wind turbine**
- **This design needs two additional components for grid connection:**

WECS TYPE A

- a soft–starter to decrease current transients during startup phase
- a capacitor bank to supply reactive power to the generator

- As a result of the capacitor bank, the generator can operate essentially as a zero-value generation source or consumption sink of reactive power
- However, such capacitive compensation unable to provide flexible reactive power control by the wind turbine

WECS TYPE B



- **The *Vestas*–developed *type B WECS* generator is designed to work with a limited variable speed**

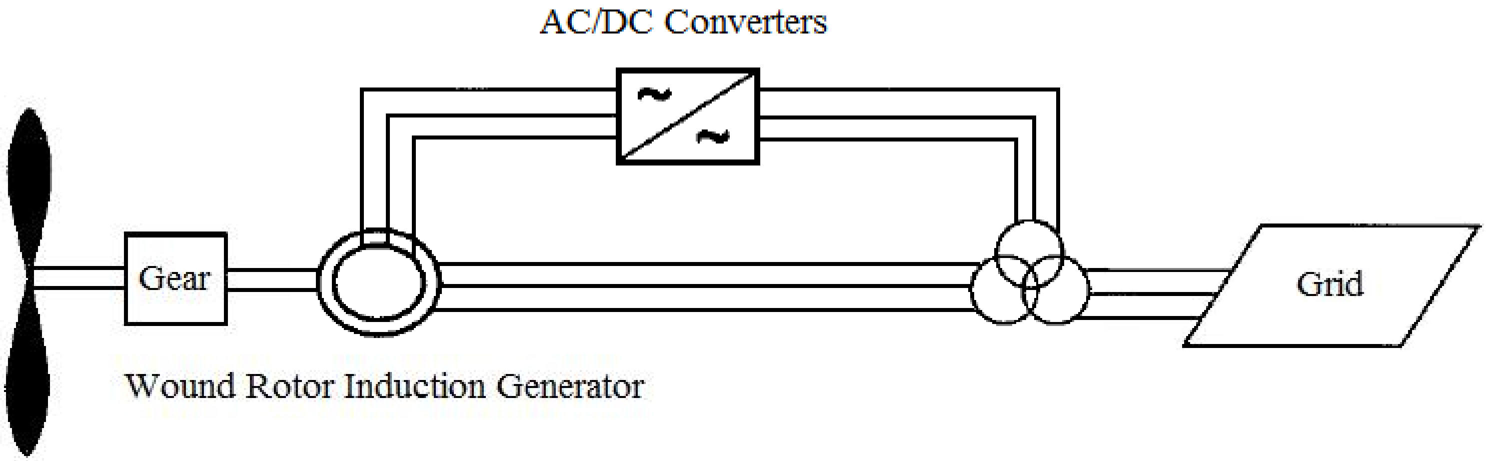
WECS TYPE B

wind turbine

- **The turbine uses the variable resistor in the rotor to control the real power output**

- **The capacitor bank and soft–starter device roles are analogous to those in the the *type A* design**

WECS TYPE C

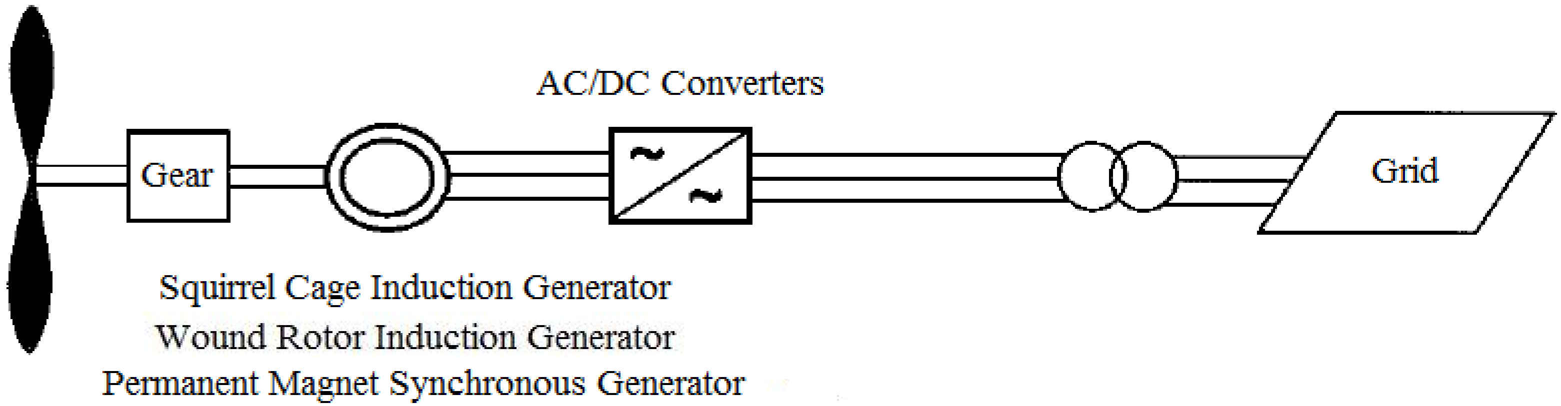


- **This design uses two *AC/DC* converters rated at 25 *%* of total generator power with a capacitor between them to control the *WECS***
- **The wound rotor induction generator topology is also known as a doubly fed induction generator (*DFIG*)**

WECS TYPE C

- The term "doubly" comes from the fact that the generator has two electrical ports – one stator and one rotor; unlike the squirrel cage rotor, the *DFIG* has windings in the rotor, which are accessible via the use of brushes and slip rings
- *DFIG*s can be controlled to provide active and reactive power to the grid
- The *WECS type C* is the most widespread of all wind turbines on the market

WECS TYPE D



- **The *type D* design uses a full–scale frequency converter with different types of generators**
- **The most common design in use is the *permanent magnet synchronous generator* (*PMSG*)**

WECS TYPE D

- **This design allows full control over the active and the reactive power production that results in a high-wind-energy extraction value**
- **Full power control improves power and frequency stability in the grid and reduces the short circuit power**
- **Most *type D* designs do not require a gearbox – a distinct advantage of type *D WECS***

WIND TURBINE CLASSIFICATION

- *wind energy conversion systems*
  - *variable-speed turbines*
    - *indirect drive with gearbox*
      - *wound-rotor synchronous generator*
      - *permanent-magnet synchronous generator*
      - *squirrel-cage induction generator*
      - *doubly-fed induction generator*
      - *wound-rotor induction generator with variable R*
    - *direct drive without gearbox*
      - *wound-rotor synchronous generator*
      - *permanent-magnet synchronous generator*
  - *fixed-speed turbines*
    - *squirrel-cage induction generator*

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