



# **ECE 333 – Green Electric Energy**

## **3. Energy Conservation Principle**

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# ENERGY CONSERVATION PRINCIPLE

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□ Energy is, to some extent, an **abstract term**; for

our purposes, we may view energy as *work*

□ An unalterable characteristic of energy is its

*invariance*: the total energy in the universe remains

**unchanged over time**

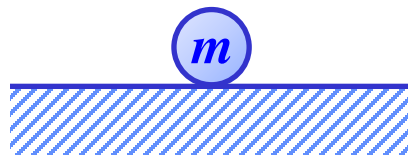
# ENERGY CONSERVATION PRINCIPLE

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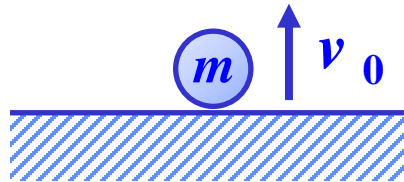
- ❑ The *principle of energy conservation* underlies all of nature's *physical, chemical or biological processes*; the principle is, essentially, a very general physical law that is based on the work of the British physicist Joule
- ❑ Indeed, for a purely mechanical system, we can derive the *energy conservation principle* directly from the application of *Newton's laws of motion*

# ENERGY CONSERVATION PRINCIPLE

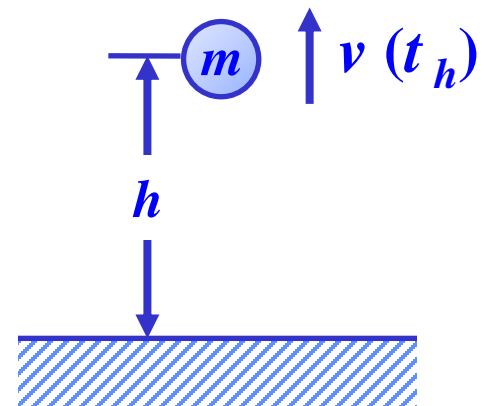
- We examine a very simple example in which a mass  $m$ , initially on the ground at time  $0^-$ , is thrown vertically upwards with a speed  $v_0$  at  $t = 0$
- We determine the relationship between  $v_0$  and the speed  $v(t_h)$  at the time  $t_h$ , when the mass attains the height  $h$



$t = 0^-$



$t = 0$

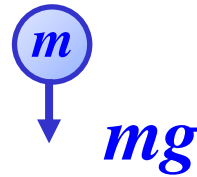


$t = t_h$

# ENERGY CONSERVATION PRINCIPLE

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- We start with Newton's Second Law to consider the relationship  $F = ma$  on our simple system



- Clearly,

$$F = -mg$$

- Let  $z$  be the vertical distance traveled by mass  $m$

and so its velocity is  $\frac{dz}{dt}$  and acceleration is  $\frac{d^2z}{dt^2}$

# ENERGY CONSERVATION PRINCIPLE

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□ At each instant  $t$ ,

$$m \frac{d^2 z}{d t^2} = F = - m g$$

so that

$$\frac{d^2 z}{d t^2} = - g \quad (*)$$

□ The second order differential equation  $(*)$  has initial conditions

$$z(0) = 0 \quad \text{and} \quad \left. \frac{d z}{d t} \right|_{t=0} = v_0 \quad (**)$$

# ENERGY CONSERVATION PRINCIPLE

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□ Since

$$\frac{dz}{dt} = v(t)$$

then

$$\frac{dv}{dt} = \frac{d}{dt} \left( \frac{dz}{dt} \right) = \frac{d^2z}{dt^2} = -g \quad (**)$$

□ We obtain the solution for  $v(t)$  by integrating

$$\int_0^t \left( \frac{dv}{dt} \right) dt = \int_{v(0)}^{v(t)} dv = v(t) - v(0) = -gt \quad (***)$$

# ENERGY CONSERVATION PRINCIPLE

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□ But

$$v(t) = \frac{dz}{dt} = v_0 - gt$$

and upon integration

$$z(t) - z(0) = v_0 t - \frac{1}{2} g t^2$$

□ At  $t = t_h$ ,  $z(t_h) = h$  and so

$$h = v_0 t_h - \frac{1}{2} g [t_h]^2 \quad (****)$$

$$v(t_h) = v_0 - g t_h \quad (*****)$$



# ENERGY CONSERVATION PRINCIPLE

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□ From (\*\*\*\*\*)

$$t_h = \frac{v_0 - v(t_h)}{g}$$

so that (\*\*\*\*) obtains

# ENERGY CONSERVATION PRINCIPLE

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$$\begin{aligned}h &= v_0 \frac{v_0 - v(t_h)}{g} - \frac{1}{2} g \cdot \frac{[v_0 - v(t_h)][v_0 - v(t_h)]}{g \cdot g} \\&= \frac{v_0 - v(t_h)}{2g} \{2v_0 - [v_0 - v(t_h)]\} \\&= \frac{v_0 - v(t_h)}{2g} [v_0 + v(t_h)] \\&= \frac{1}{2g} [v_0^2 - [v(t_h)]^2]\end{aligned}$$

# ENERGY CONSERVATION PRINCIPLE

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□ We rearrange and obtain

$$gh = \frac{1}{2}v_0^2 - \frac{1}{2}[v(t_h)]^2$$

and multiply by  $m$  to express

$$\frac{1}{2}m[v(t_h)]^2 = \frac{1}{2}mv_0^2 - mgh \quad (\dagger)$$

# ENERGY CONSERVATION PRINCIPLE

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□ We associate the relationship with an energy

interpretation since the kinetic energy of mass  $m$

at speed  $v(t)$  at time  $t$  is given by  $\frac{1}{2}m[v(t)]^2$ ; also,

the potential energy of mass  $m$  at height  $h$  is  $mgh$

□ Therefore, we can restate (\*) for height  $h$  as

$$\frac{1}{2}m v_0^2 = \frac{1}{2}m [v(t_h)]^2 + mgh \quad (*\dagger)$$

# ENERGY CONSERVATION PRINCIPLE

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- In words, at an arbitrary height  $h$

$$\textit{kinetic energy}|_h + \textit{potential energy}|_h = \underbrace{\textit{kinetic energy}|_0}_{\textit{constant}}$$

- For two arbitrary values of  $h$ , say  $h'$  and  $h''$

$$\textit{kinetic energy}|_{h'} + \textit{potential energy}|_{h'} = \textit{kinetic energy}|_{h''} + \textit{potential energy}|_{h''}$$

- We derived in a straightforward way that the total

**energy of mass  $m$  is invariant over time**

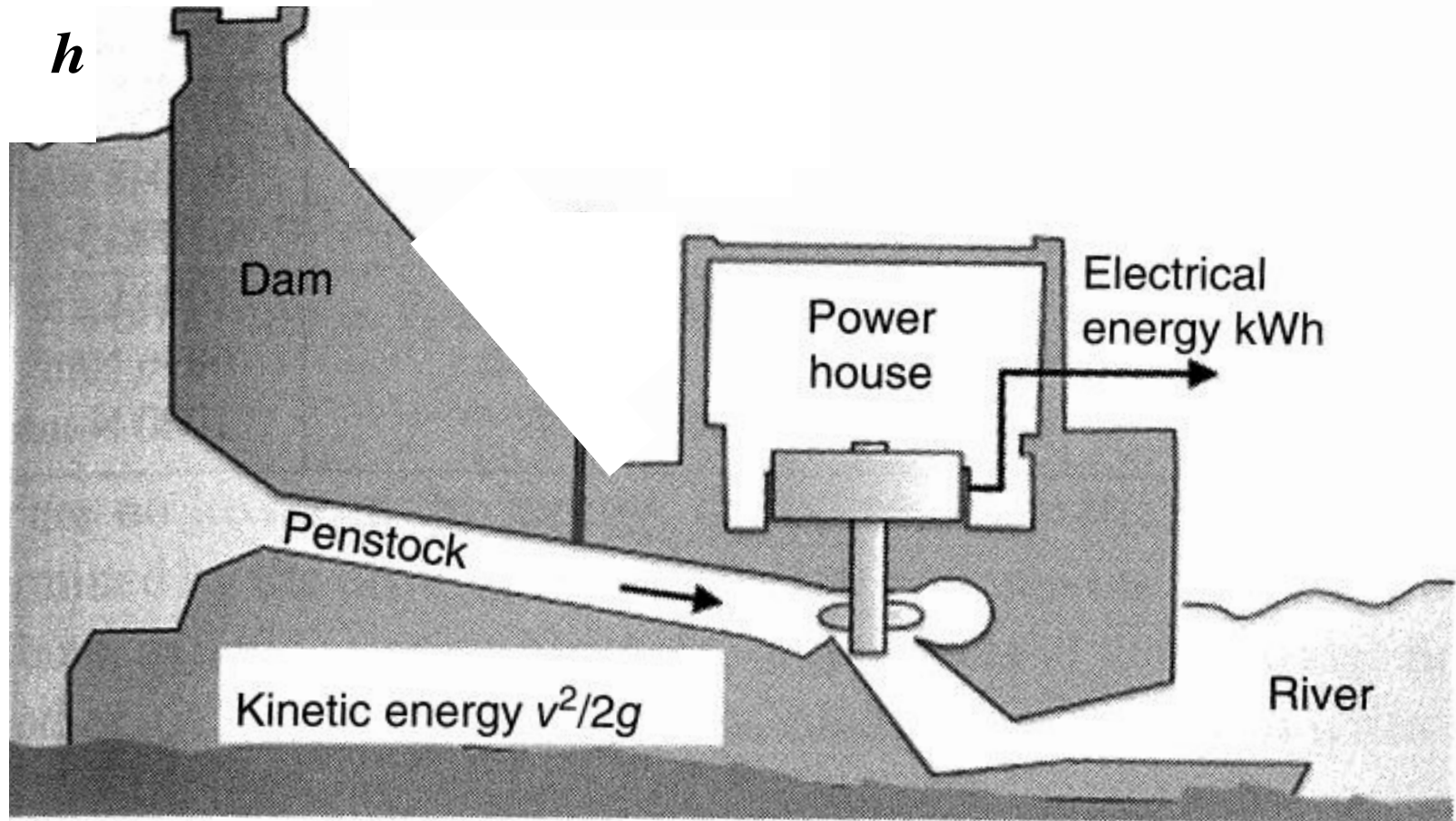
# APPLICATION: ENERGY CONVERSION

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- ❑ The *energy conservation principle* holds whenever we convert energy from one form into another form
- ❑ We consider a hydroelectric system where a reservoir stores water behind a dam at some height  $h$ : the water flows through a penstock and drives a turbine, whose rotor is connected through a mechanical shaft to the electrical generator

# HYDROELECTRIC ENERGY GENERATION

Potential energy



# HYDROELECTRIC ENERGY GENERATION

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- The water at the height  $h$  – typically, called the head  $h$  – has potential energy, which is converted into mechanical energy as the water flows through the penstock; the mechanical energy drives the turbine and is converted into electric energy by the generator
- Each unit volume of water in the reservoir has mass  $\rho$ , where  $\rho$  is the density of water, and so has potential energy  $\rho gh$



# HYDROELECTRIC ENERGY GENERATION

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- As each unit traverses the penstock, its *potential energy* is converted into *kinetic energy* related to its speed  $v$  ; upon arrival at the turbine, each unit of volume of water has kinetic energy  $\frac{1}{2} \rho v^2$
- We assume that the pressure energy is negligibly small and there are no losses in the system – in effect, a frictionless penstock – so that the energy conservation law for the mass of the unit volume of fluid results in

# HYDROELECTRIC ENERGY GENERATION

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$$\rho g h = \frac{1}{2} \rho v^2$$

- **The energy conservation law applies to all energy conversion processes; as a specific process has losses due to inefficiencies of the process, some of the energy is converted into such losses and as a result the process efficiency is reduced**

# HYDROELECTRIC ENERGY GENERATION

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- ❑ **As we shall see, the wind speed air mass has kinetic energy which rotates the wind turbine, which connects to the rotor of an electric generator to convert that kinetic energy into electricity**
- ❑ **Similar notions hold, for example, for a steam generation plant**

# FOSSIL – FUEL FIRED STEAM GENERATION PLANT

