ECE 333 – Renewable Energy Systems

2. Power System Basics – *AC* Analysis

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SINUSOIDAL VARIABLES

- In *AC* circuits, the basic current and voltage variables are considered to be *sinusoidal functions*

- A general expression for a sinusoidal function is

$$x(t) = X_m \cos(\omega t + \theta)$$

time — t

magnitude or amplitude — X_m

angular frequency — ω

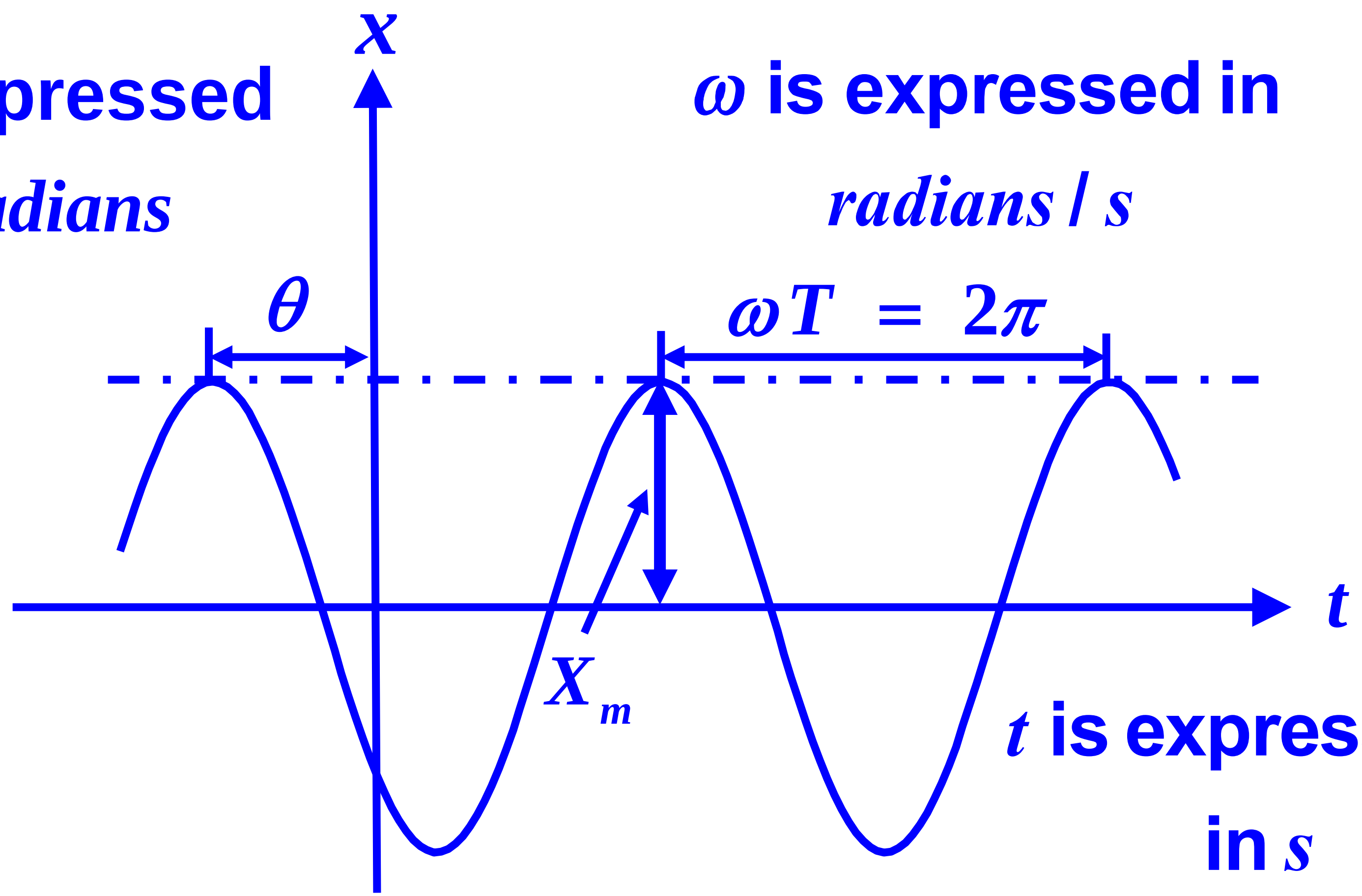
phase angle — θ

SINUSOIDAL VARIABLES

- In our work, we express the argument of the sinusoidal function in *radians* and so:

θ is expressed in *radians*

ω is expressed in *radians / s*



t is expressed in s

ANGULAR FREQUENCY ω

- **We may also express the argument of the sinusoidal function in terms of the *frequency f* stated in *Hz* or *cycles per second* with**

$$\omega = \underbrace{2\pi}\, f$$

radians/s — *radians / cycle* — *Hz*

ANGULAR FREQUENCY ω

- The sinusoidal function is periodic with a period of T s, where

s/cycle → $$T = \frac{1}{f}$$ ← cycles/s

that is each cycle (or period) duration is T s

- We may express $x(t)$ therefore as

$$x(t) = X_m \cos(\omega t + \theta) = X_m \cos(2\pi f t + \theta)$$

$$= X_m \cos\left(\frac{2\pi}{T} t + \theta\right)$$

AC SYSTEM

- The current in the *AC* system is specified by

$$i(t) = I_m \cos(\omega t + \theta_i)$$

- The use of the cosine function is entirely arbitrary since for any arbitrary angle ϕ

$$\sin(\phi) = \cos\left(\frac{\pi}{2} - \phi\right)$$

or, equivalently,

$$\cos(\phi) = \sin\left(\frac{\pi}{2} - \phi\right)$$

AC SYSTEM

- The voltage is also sinusoidal

$$v(t) = V_m \cos(\omega t + \theta_v)$$

- The power is

$$p(t) = v(t)i(t) = V_m I_m \cos(\omega t + \theta_i)\cos(\omega t + \theta_v)$$

- Recall that

$$\cos\phi \cos\xi = \frac{1}{2}\left[\cos(\phi + \xi) + \cos(\phi - \xi)\right]$$

POWER EXPRESSION: *AC* NETWORK

- **We are interested to evaluate the *average value of the power* $p(t)$**
- **As we consider two periodic functions, each with the identical period T, the average value is given by the average value over any single period**
- **Therefore,**

AC SYSTEM

$$p_{avg} = \frac{1}{T}\int_{0}^{T} p(t)\, dt$$

$$= \frac{V_m \, I_m}{2T}\int_{0}^{T} \left[cos\left(2\omega t + \theta_i + \theta_v\right) + cos\left(\theta_i - \theta_v\right)\right] dt$$

$$= \frac{V_m \, I_m}{2T}\int_{0}^{T} \left[cos\left(2\omega t + \theta_i + \theta_v\right) + cos\left(\theta_v - \theta_i\right)\right] dt$$

$$= \frac{V_m \, I_m}{2\,T} \; cos\left(\underbrace{\theta_v - \theta_i}_{\theta} \right) \cdot T \quad ,$$

AC SYSTEM

where, we use the fact that the average value of a sinusoid is *0*, as the positive and negative areas under the curve cancel each other out

- We use the standard definition for the angle θ

$$\boxed{\theta = \theta_v - \theta_i}$$

- Therefore,

$$p_{avg} = \frac{1}{2} V_m I_m \cos\theta$$

EFFECTIVE VALUE DEFINITION

- The **effective value** of a periodic variable is the square root of the average of the squared value of the variable

- For the current

$$I \triangleq \left[\frac{1}{T} \int_{0}^{T} i^{2}(t)\, dt \right]^{\frac{1}{2}}$$

EFFECTIVE VALUE CALCULATION

- We next evaluate I

$$I = \left[\frac{1}{T} \int_0^T I_m^2 \underbrace{\cos^2\left(\omega t + \theta_i\right)}dt \right]^{\frac{1}{2}}$$

$$\underbrace{\frac{1}{2}\left[\cos 2\left(\omega t + \theta_i\right) + \cos\left(0\right) \right]}$$

$$\frac{1}{2}\left[1 + \cos 2\left(\omega t + \theta_i\right) \right]$$

$$I = \frac{I_m}{\sqrt{2}}$$

EFFECTIVE VALUE

- We refer to I is as the ***r.m.s. value***
- The *r.m.s.* value of a sinusoid is equal to its amplitude divided by $\sqrt{2}$
- The 240–*V*, 60–*Hz* voltage at which electricity is supplied to a dryer means that the *effective voltage*

$$V = 240\ V$$

and so we compute V_m from the *r.m.s.* value, to get the value of the amplitude V_m to be

AC SYSTEM

$$V_m = 240\sqrt{2} = 339.41\ V$$

with the angular frequency

$$\omega = 2\pi \cdot 60 = 377\ radians/s$$

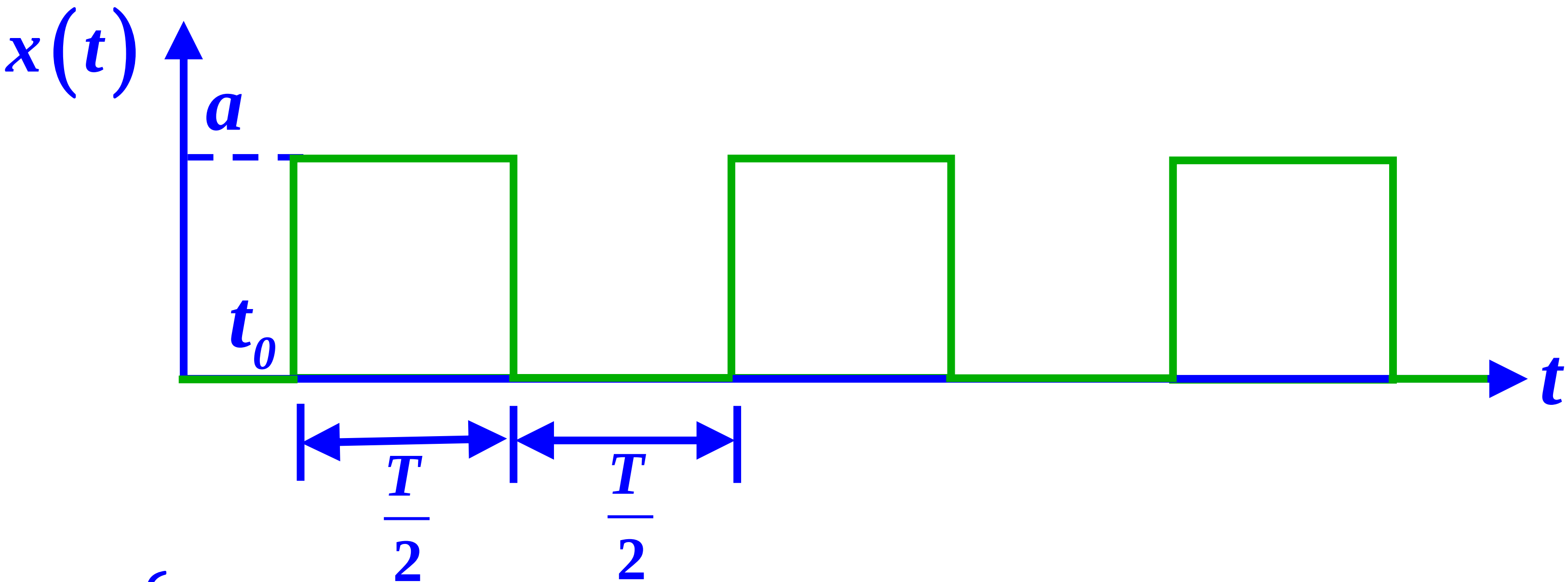
and so we have the voltage sinusoid as

$$v(t) = 339.41\ cos(377t + \theta_v)$$

- We, henceforth, adopt the convention that the input voltage has $\theta_v = 0$ and measure all other variables' phase angles with respect to the phase angle of the input voltage

r.m.s. VALUE OF A SQUARE WAVE

- We consider the *square wave*



$$x(t) = \begin{cases} a & t_0 + (n-1)T \leq t \leq t_0 + (2n-1)\frac{T}{2} \\ & \qquad n = 1, 2, \ldots \\ 0 & t_0 + (2n-1)\frac{T}{2} < t < t_0 + nT \end{cases}$$

r.m.s. VALUE OF A SQUARE WAVE

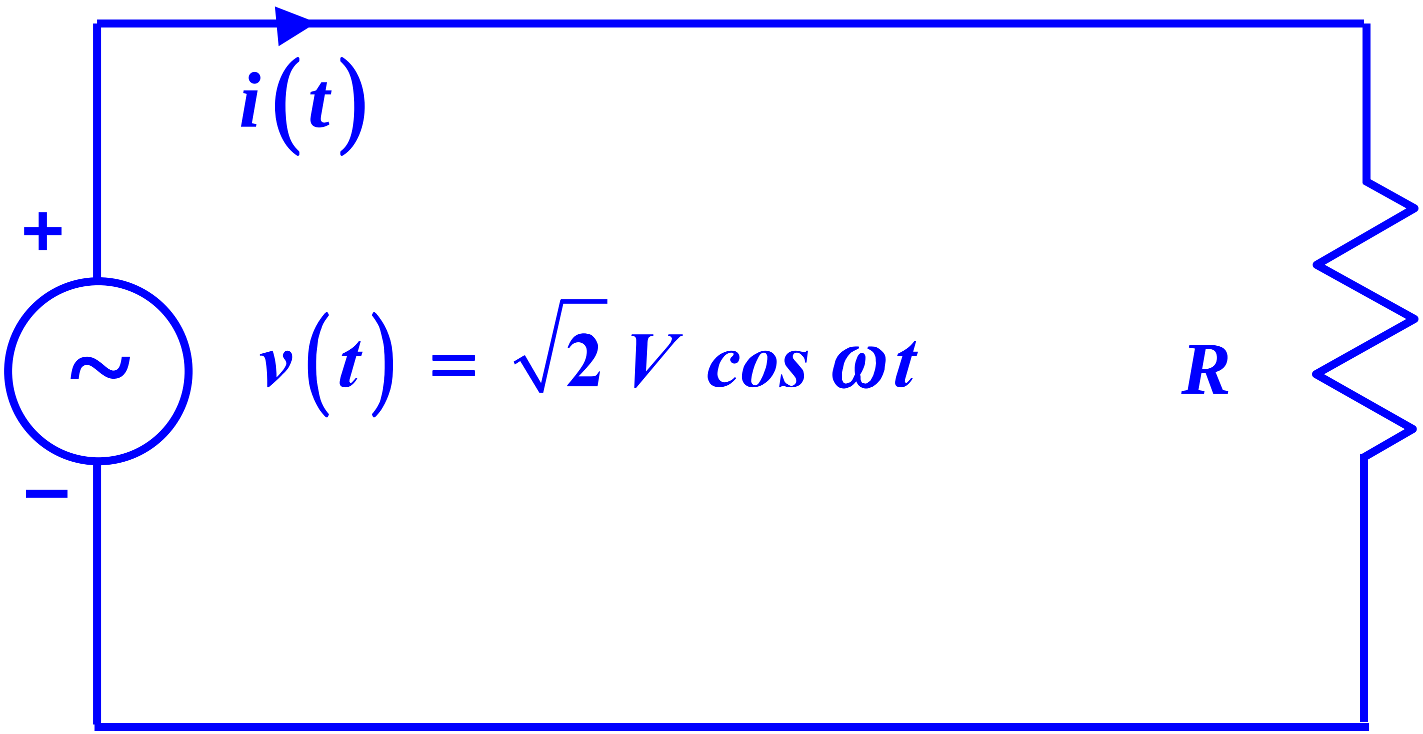
- We compute the *r.m.s.* value of $x(t)$ by evaluating the average value over a cycle

$$X = \left\{ \frac{1}{T} \int_{t_0}^{t_0 + T} \left[x(t) \right]^2 dt \right\}^{\frac{1}{2}} = \left\{ \frac{1}{T} \int_{t_0}^{t_0 + \frac{T}{2}} a^2 \, dt \right\}^{\frac{1}{2}}$$

$$= \left[\frac{a^2}{T} \cdot \frac{T}{2} \right]^{\frac{1}{2}}$$

$$= \frac{a}{\sqrt{2}}$$

IDEAL RESISTOR IN *AC* NETWORKS



IDEAL RESISTOR IN *AC* NETWORKS

- We analyze the behavior of an ideal resistor in a circuit with a sinusoidal voltage source

$$v(t) = V_m \cos \omega t$$

or in the terms of the *r.m.s.* voltage V

$$v(t) = \sqrt{2} V \cos \omega t$$

- Now,

$$i(t) = \frac{v(t)}{R} = \sqrt{2} \frac{V}{R} \cos \omega t$$

IDEAL RESISTOR IN *AC* NETWORKS

is the current through the resistor whose *r.m.s.* value is

$$I = \frac{V}{R}$$

- Since there is a *0* angle phase difference between the $v(t)$ and $i(t)$ sinusoids, we say that the two sinusoids are *in phase* with each other

IDEAL RESISTOR IN *AC* NETWORKS

- The evaluation of the average power is

$$p_{avg} = V\,I\cos\underbrace{\left(\theta_v - \theta_i\right)}_{0} = V\,I = \frac{V^2}{R} = I^2 R$$

- In *AC* networks, power is always interpreted as *average power* and so we drop the *avg* subscript and write simply

$$P = V\,I = I^2 R = \frac{V^2}{R}$$

and *P* represents the average power

EXAMPLE: CUISINART TOASTER

- The two–slot Cuisinart toaster is a 1,500–W load when plugged into a 120–V socket at 60 Hz; we can model the appliance as a simple resistor
- We compute from

$$P = \frac{V^2}{R}$$

the value of the resistance

$$R = \frac{V^2}{P} = \frac{120 \cdot 120}{1{,}500} = \frac{14{,}400}{1{,}500} = 9.6\ \Omega$$

EXAMPLE: CUISINART TOASTER

- The current is

$$I = \frac{V}{R} = \frac{120}{9.6} = 12.5\ A$$

- Now, consider a voltage spike of $125\ V$ and so the dissipated power becomes

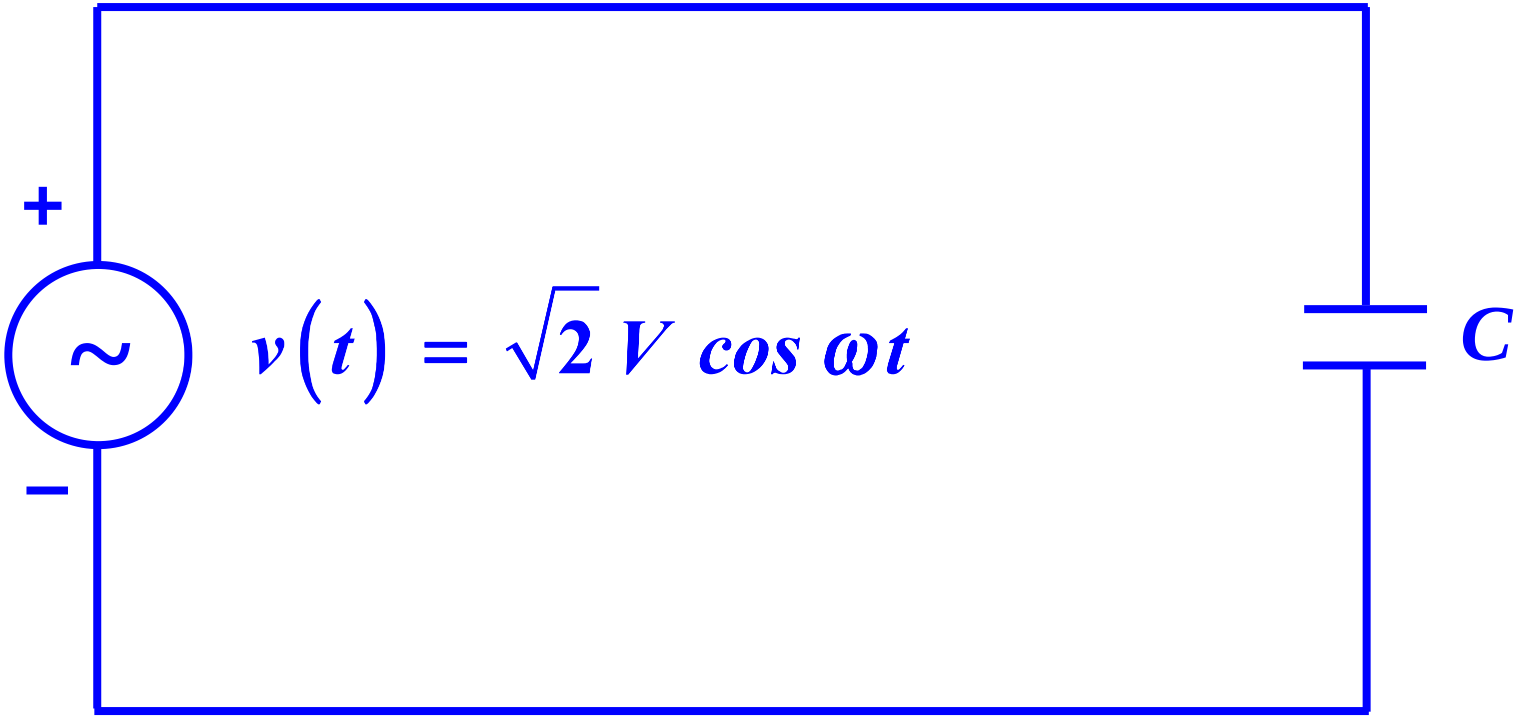
$$P = \frac{V^2}{R} = \frac{125 \cdot 125}{9.6} = 1{,}627.6\ W$$

representing an increase of $127.6\ W$ in the toaster consumption – a rather serious $8.5\ \%$ increase

IDEALIZED CAPACITOR IN *AC* NETWORKS

- **Recall the *equation of motion* for a capacitor**

$$i(t) = C\,\frac{dv}{dt}$$



IDEALIZED CAPACITOR IN *AC* NETWORKS

- For a sinusoidal voltage in an *AC* network

$$i(t) = C\frac{d}{dt}\left[\sqrt{2}V\cos\omega t\right] = -\omega C\sqrt{2}V\sin\omega t$$

- We use the identity

$$\sin\phi = \cos\left(\frac{\pi}{2}-\phi\right) = -\cos\left(\frac{\pi}{2}-\phi-\pi\right) = -\cos\left(\phi+\frac{\pi}{2}\right)$$

- Thus

$$i(t) = \omega C\sqrt{2}V\cos\left(\omega t+\frac{\pi}{2}\right)$$

IDEALIZED CAPACITOR IN *AC* NETWORKS

- Thus, the voltage across the capacitor and the current through it are
  - identical frequency sinusoids
  - there is a $\frac{\pi}{2}$ *radians* difference between the two waveforms
  - the current *leads* the voltage by $\frac{\pi}{2}$ *radians*

IDEALIZED CAPACITOR IN *AC* NETWORKS

- **Let**

$$I = \omega C V$$

and so

$$i(t) = \sqrt{2} I \cos\left(\omega t + \frac{\pi}{2}\right)$$

IDEALIZED CAPACITOR IN *AC* NETWORKS

- **We summarize**

$$V = \left(\frac{1}{\omega C} \right) I \longleftarrow$$

"*AC* version" of *Ohm's Law* for capacitors

- **The power dissipated by the capacitor is**

$$p(t) = v(t)\, i(t) = \sqrt{2} V \cos \omega t \, \sqrt{2}\, I \cos\left(\omega t + \frac{\pi}{2} \right)$$

and this simplifies to

IDEALIZED CAPACITOR IN *AC* NETWORKS

$$p(t) = 2VI \cdot \frac{1}{2}\left[\left[cos\left(2\omega t + \frac{\pi}{2}\right)\right] + \underbrace{cos\left(-\frac{\pi}{2}\right)}_{0}\right]$$

$$= VI\, cos\left(2\omega t + \frac{\pi}{2}\right)$$

- **Since the average power value of a sinusoid is 0,**

$$p_{avg} = 0$$

and so for a capacitor

$$P = 0$$

CAPACITOR EXAMPLE

- We consider the current through a $200-\mu F$ capacitor supplied by a $120-V$, $60-Hz$ source
- The voltage is given by

$$v(t) = \sqrt{2}\, 120 \cos \omega t$$

and the current is therefore

$$i(t) = \sqrt{2}\, I \cos\left(\omega t + \frac{\pi}{2}\right)$$

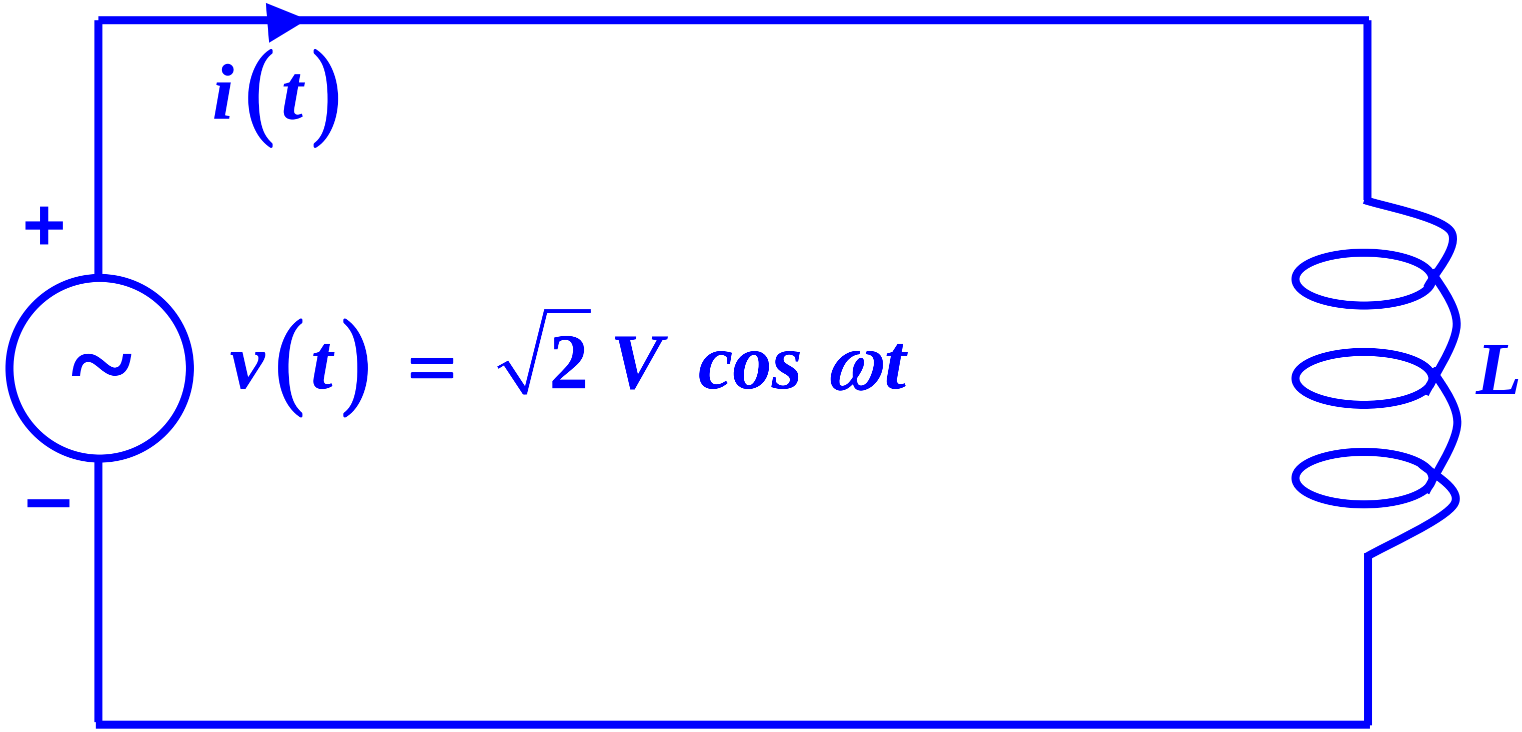
with

$$I = (2\pi 60)(120)(200 \cdot 10^{-6}) = 9.048\, A$$

IDEALIZED INDUCTOR IN *AC* NETWORKS

- **Recall the equation of motion for an inductor**

$$v(t) = L\frac{di}{dt}$$



IDEALIZED INDUCTOR IN *AC* NETWORKS

and so

$$i(t) = \frac{1}{L}\int_0^t v(\xi)\, d\xi$$

- For the sinusoidal voltage

$$v(t) = \sqrt{2}\, V \cos \omega t$$

we have

$$i(t) = \frac{1}{L}\int_0^t \sqrt{2}\, V \cos \omega \xi \, d\xi = \frac{\sqrt{2}\, V}{\omega L} \sin \omega t$$

- We use again the identity

$$\sin \phi = \cos\left(\phi - \frac{\pi}{2}\right)$$

IDEALIZED INDUCTOR IN *AC* NETWORKS

- **Thus**

$$i(t) = \sqrt{2}\,\frac{V}{\omega L}\cos\left(\omega t - \frac{\pi}{2}\right)$$

- **Therefore, the voltage across the inductor and the current through it are**
  - **identical frequency sinusoids**
  - **there is a $\frac{\pi}{2}$ *radians* difference between the two waveforms**

IDEALIZED INDUCTOR IN *AC* NETWORKS

- the current *lags* behind the voltage by $\frac{\pi}{2}$

- Let

$$I = \frac{1}{\omega L} V$$

and so

$$i(t) = \sqrt{2}\, I \cos\left(\omega t - \frac{\pi}{2}\right)$$

- We summarize:

$$V = \omega L I$$

"*AC* version" of *Ohm's Law* for inductors

IDEALIZED INDUCTOR IN *AC* NETWORKS

- The power dissipated by the inductor is

$$p(t) = v(t)\, i(t) = \sqrt{2} V \cos \omega t \, \sqrt{2}\, I \cos\left(\omega t - \frac{\pi}{2}\right)$$

and this simplifies to

$$p(t) = 2VI \cdot \frac{1}{2}\left[\cos\left(2\omega t - \frac{\pi}{2}\right) + \underbrace{\cos\left(\frac{\pi}{2}\right)}_{0}\right]$$

$$= VI \cos\left(2\omega t - \frac{\pi}{2}\right)$$

IDEALIZED INDUCTOR IN *AC* NETWORKS

- Clearly

$$p_{avg} = 0$$

and so

$$P = 0$$

- Neither capacitors nor inductors consume real power

POWER FACTOR

- We generalize the expressions for resistors, capacitors and inductors for a sinusoidal voltage

$$v(t) = \sqrt{2}\, V \cos(\omega t + \theta_v)$$

and a sinusoidal current

$$i(t) = \sqrt{2}\, I \cos(\omega t + \theta_i)$$

POWER FACTOR

- Now, we have shown that the angle θ takes on the specific values

$$\theta = \begin{cases} 0 & \textit{for a resistor} \\ \dfrac{\pi}{2} & \textit{for an inductor} \\ -\dfrac{\pi}{2} & \textit{for a capacitor} \end{cases}$$

POWER FACTOR

but for a network with an arbitrary combination of R, L and C components, θ is unknown

- We also showed earlier that the average value of power is

$$p_{avg} = V\,I\,cos(\theta) \qquad (*)$$

for

$$\theta = \theta_v - \theta_i$$

- Power engineers call the quantity $cos\,\theta$ the *power factor*

$$p.f. \triangleq cos\,\theta$$

POWER FACTOR

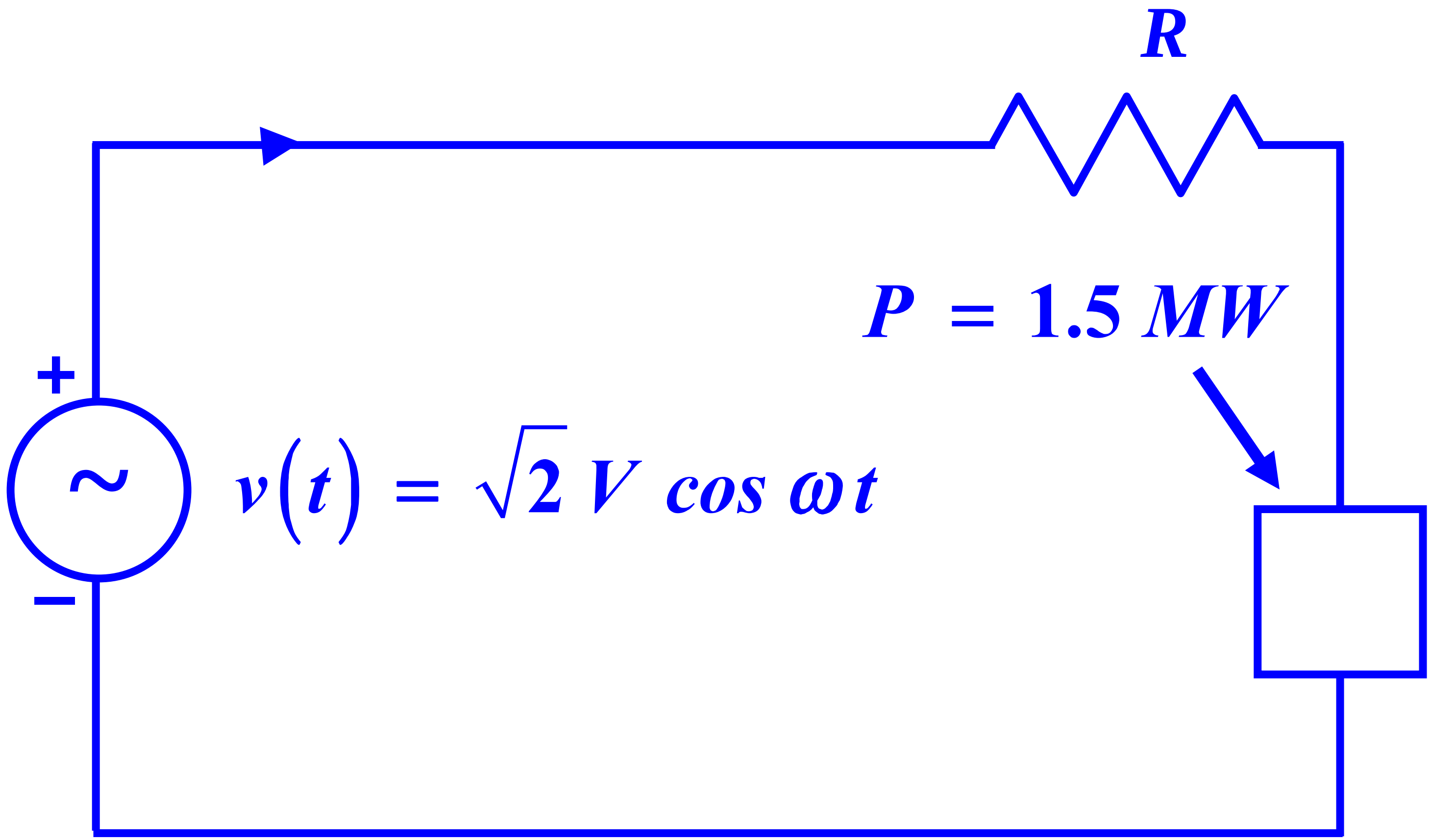
- The expression in (*) is general and may apply to any circuit or circuit element, any combination of *R*, *L* and *C* elements and, more importantly, any component with sinusoidal voltage and current
- We interpret *p.f.* to be the fraction that the real power represents of the total apparent power used by a particular component or system

A *p.f.* EXAMPLE

- A small industrial customer is supplied by a $24\text{–}kV$, $60\text{–}Hz$ source to run a $1.5\text{–}MW$ real power load through a line whose resistance is R
- We compute the ratio of the real power line losses on the feeder line under two distinct *p.f.* values:

$$S\Big|_{p.f._1=\frac{1}{2}} \quad \text{and} \quad S\Big|_{p.f._2=\frac{\sqrt{3}}{2}}$$

EXAMPLE ON *p.f.*



- Basic assumption: the voltage drop through R is negligibly small

EXAMPLE ON *p.f.*

- Since

$$P = V\, I\, cos\theta = 1.5\, MW,$$

the *r.m.s.* value of the feeder current we compute

under $p.f._1$

$$I_1 = \frac{1.5\, MW}{\frac{1}{2}\left(24\, kV\right)}$$

and also under $p.f._2$

$$\frac{MW}{kV} = kA$$

$$I_2 = \frac{1.5\, MW}{\frac{\sqrt{3}}{2}\left(24\, kV\right)}$$

EXAMPLE ON *p.f.*

- The ratio of the losses is therefore

$$\frac{I_1^2 R}{I_2^2 R} = \left(\frac{I_1}{I_2}\right)^2 = \left(\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right)^2 = \left(\sqrt{3}\right)^2 = 3$$

- The losses are 3 times higher under the poor value $p.f._1$ than under the higher value $p.f._2$

APPARENT, REAL AND REACTIVE POWER

$$\vec{S} = \vec{V}\,\vec{I}^{\,*}$$

$$\vec{S} = Ve^{j\theta_v}(I\,e^{j\theta_i})^{*} = Ve^{j\theta_v}(I\,e^{-j\theta_i})$$

$$\vec{S} = V\,I\cos(\theta_v - \theta_i) + j\,V\,I\sin(\theta_v - \theta_i)$$

$$\vec{S} = V\,I\cos\left(\underbrace{\theta_v - \theta_i}_{\theta}\right) + j\,V\,I\sin\left(\underbrace{\theta_v - \theta_i}_{\theta}\right)$$

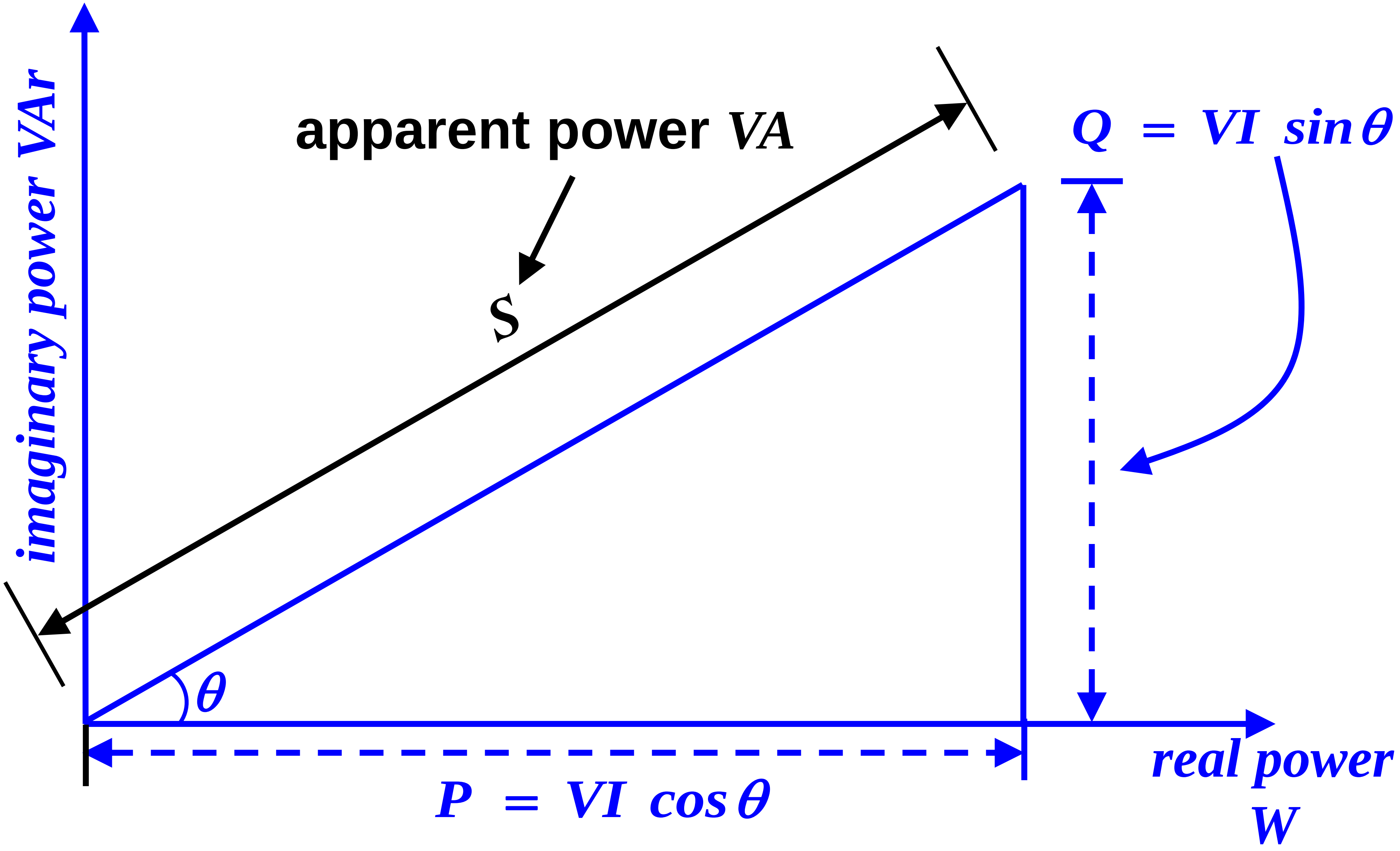
$$\vec{S} = \underbrace{V\,I\cos(\theta)}_{P} + j\underbrace{V\,I\sin(\theta)}_{Q}$$

$$\vec{S} = P + jQ$$

THE POWER TRIANGLE

- There is an important relationship between the apparent power S, the real power P and the reactive power Q ; we represent this relationship by the so-called *power triangle* in the complex plane
- The power triangle is drawn as follows

THE POWER TRIANGLE



THE POWER TRIANGLE

$\theta > 0$ current *lags* voltage

$\theta < 0$ current *leads* voltage

$$S = VI$$

$$P = S\cos\theta$$ ⟵ real power

$$Q = S\sin\theta$$ ⟵ reactive power

$$S^2 = P^2 + Q^2$$ ⟵ apparent power

THE POWER TRIANGLE

- For any arbitrary load

$$P > 0$$

but,

$$Q > 0 \qquad \text{for an inductive load}$$

$$Q < 0 \qquad \text{for a capacitive load}$$

- The real power consumed by a load is the rate at which work is done and is measured in W
- The reactive power is incapable to do work and its average is 0 for a capacitive/inductive element

THE POWER TRIANGLE

- Power suppliers, typically, charge for the P consumption but are also impacted by the Q, as the larger the Q, the higher the line losses; in certain cases, charges are imposed on the basis of S or explicitly take into account the *p.f.*
- The presence of electric motors, which are highly inductive loads, leads to real power losses on transmission lines that may be significant

EXAMPLE: MOTOR POWER TRIANGLE

- We consider a $250\text{–}V$ induction motor that draws $20\,A$ of current to generate $4.33\,kW$ of real power delivered to its shaft
- We draw the power triangle using

$$S = V\,I = 250 \bullet 20 = 5{,}000\;VA = 5\,kVA$$

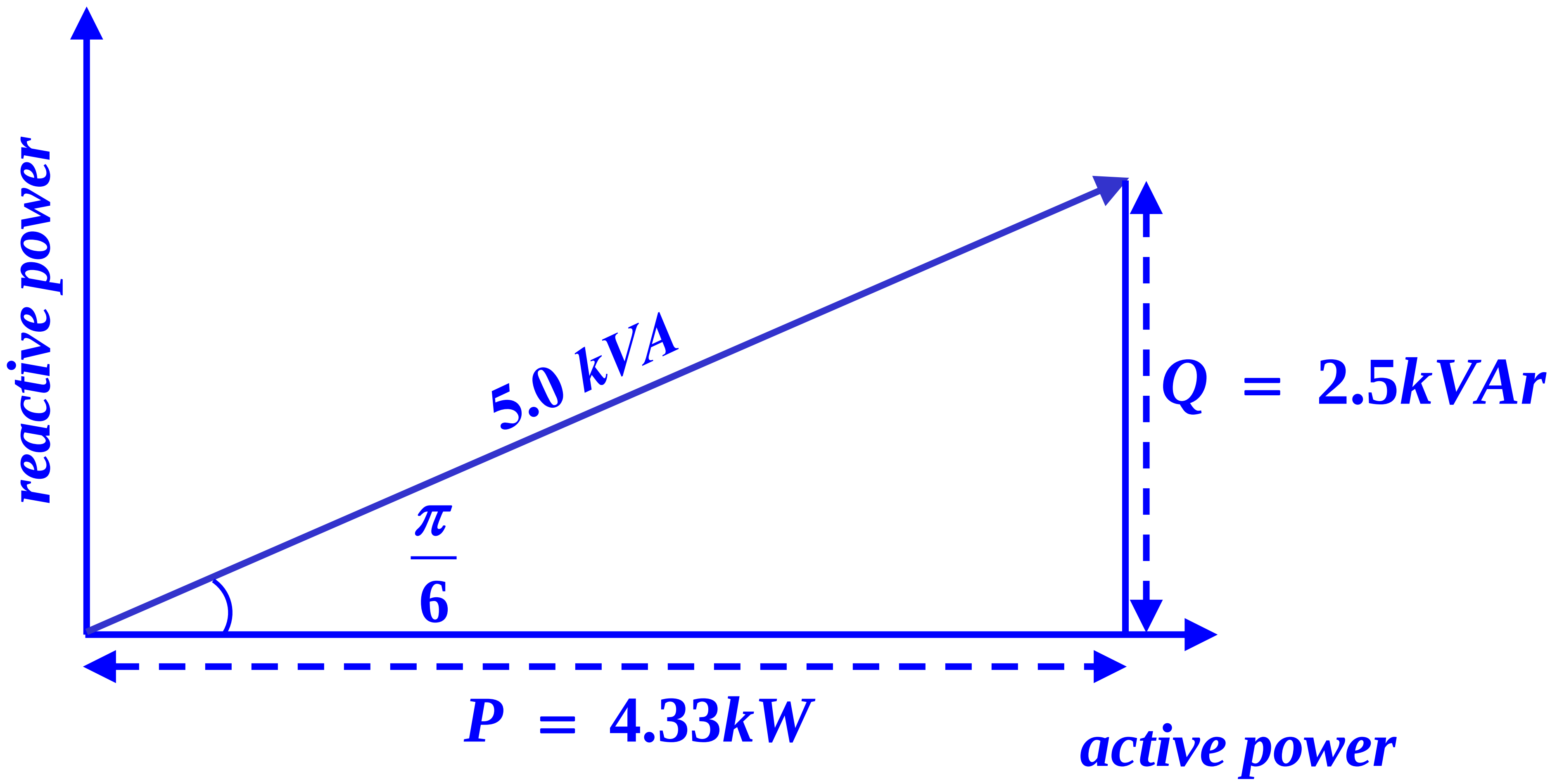
$$P = 4.33\,kW$$

$$\cos\theta = \frac{P}{S} = \frac{4.33}{5} = 0.866$$

$$\theta = \cos^{-1}\left(0.866\right) = \frac{\pi}{6}$$

$$Q = S\sin\theta = 2.5\;kVAr$$

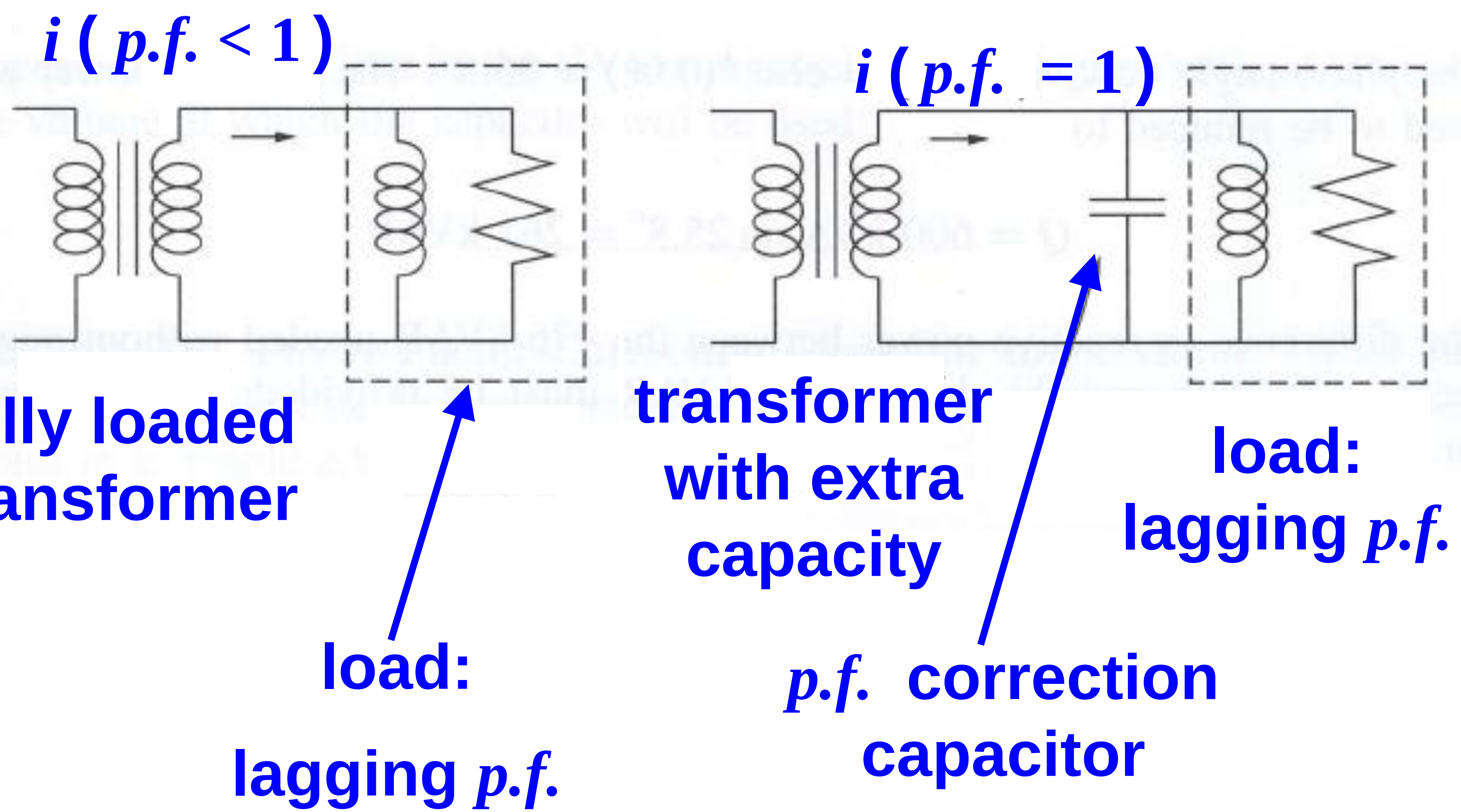
EXAMPLE: MOTOR POWER TRIANGLE



POWER FACTOR CORRECTION

- The smaller the *p.f.*, the worse the utilization of power is; the ideal is to get as near as possible to the *perfect* *p.f.* *of* 1.0
- Sometimes, it is either desirable or necessary to use capacitors to correct the *p.f.* to offset the *VArs* of the inductive elements
- A *p.f.* corrective action can result in increased real power delivery to the loads

EXAMPLE: POWER FACTOR CORRECTION



EXAMPLE: POWER FACTOR CORRECTION

- A transformer is operating close to its kVA rating and is used to deliver 600 kVA at a 0.75 $p.f.$
- There is a 20 % forecasted growth in the real power demand for next year
- This growth needs to be accommodated without any investment in a new transformer with the installation of capacitors for $p.f.$ correction

EXAMPLE: POWER FACTOR CORRECTION

- **The current situation is characterized by**

$$p.f. = 0.75 \qquad = \cos\theta$$

$$\theta = \cos^{-1}(0.75) = 0.72 \; radians$$

$$P = 600 \cdot 0.75 \qquad = 450 \; kW$$

$$Q = 600 \cdot 0.66 \qquad = 397 \; kVAr$$

- **The forecasted situation is**

$$P_{new} = 450(1.2) = 540 \; kW$$

$$p.f._{new} = \frac{540}{600} \qquad = 0.9$$

EXAMPLE: POWER FACTOR CORRECTION

$$\theta = cos^{-1}(0.9) = 0.45\, radians$$

$$Q_{new} = 600(0.435) = 261\, kVAr$$

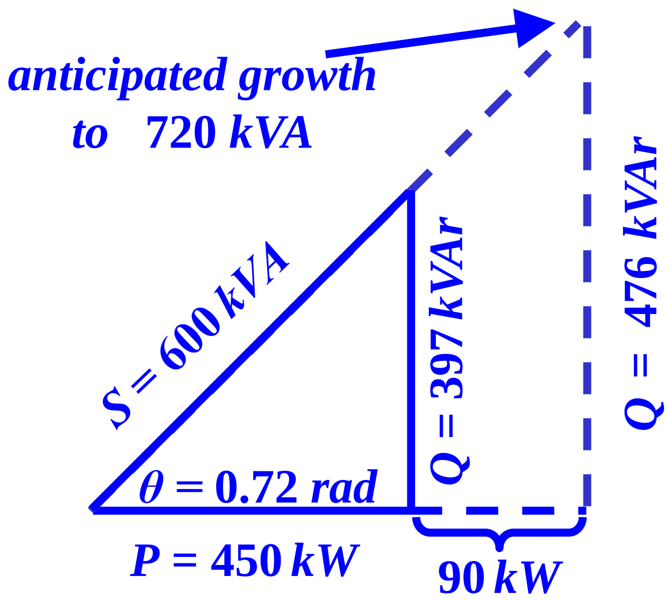
- The difference between $Q = 476\, kVAr$ and

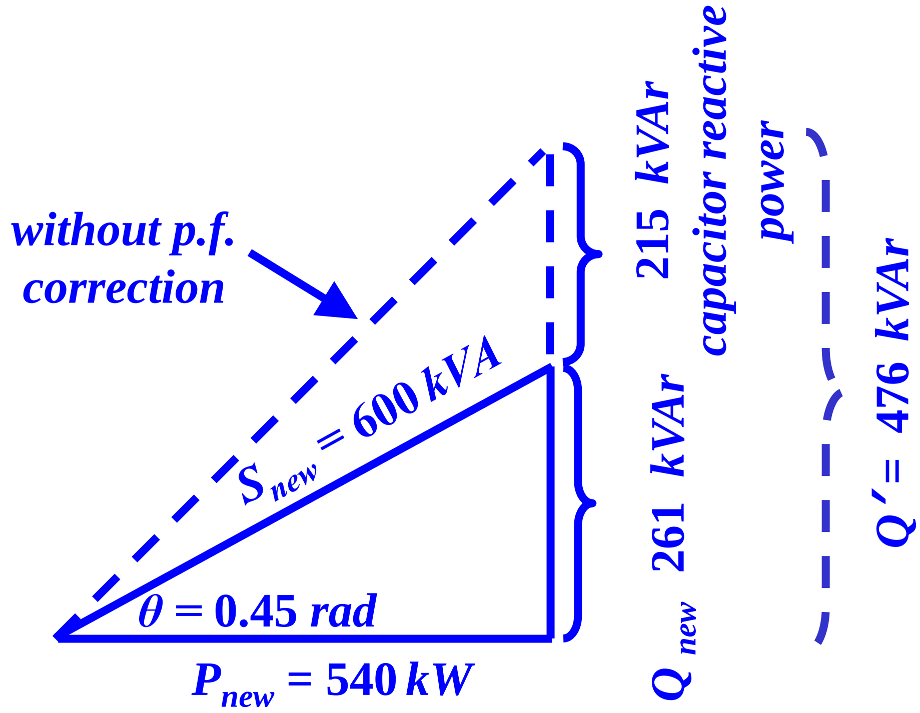
  $Q_{new} = 261\, kVAr$ is compensated by the installation

  of capacitors with

$$Q_c = 476 - 261 = 215\, kVAr$$

EXAMPLE: POWER FACTOR CORRECTION





before correction **with 215-*kVAr* correction**

EXAMPLE: POWER FACTOR CORRECTION

- We can determine the capacitance of the $p.f.$ correction capacitors

$$Q_c = V_c I_c = V_c \left(\omega C V_c \right)$$

$$C = \frac{Q_c}{\omega V_c^2}$$

- We assume that the input voltage to the capacitors is at $12\,kV$, and so

$$C = \frac{215\,kVAr}{(377)(12)^2 (kV)^2} = (3.96)\,10^{-3}\,F$$

DISTRIBUTION SYSTEM CAPACITORS FOR *p.f.* CORRECTION

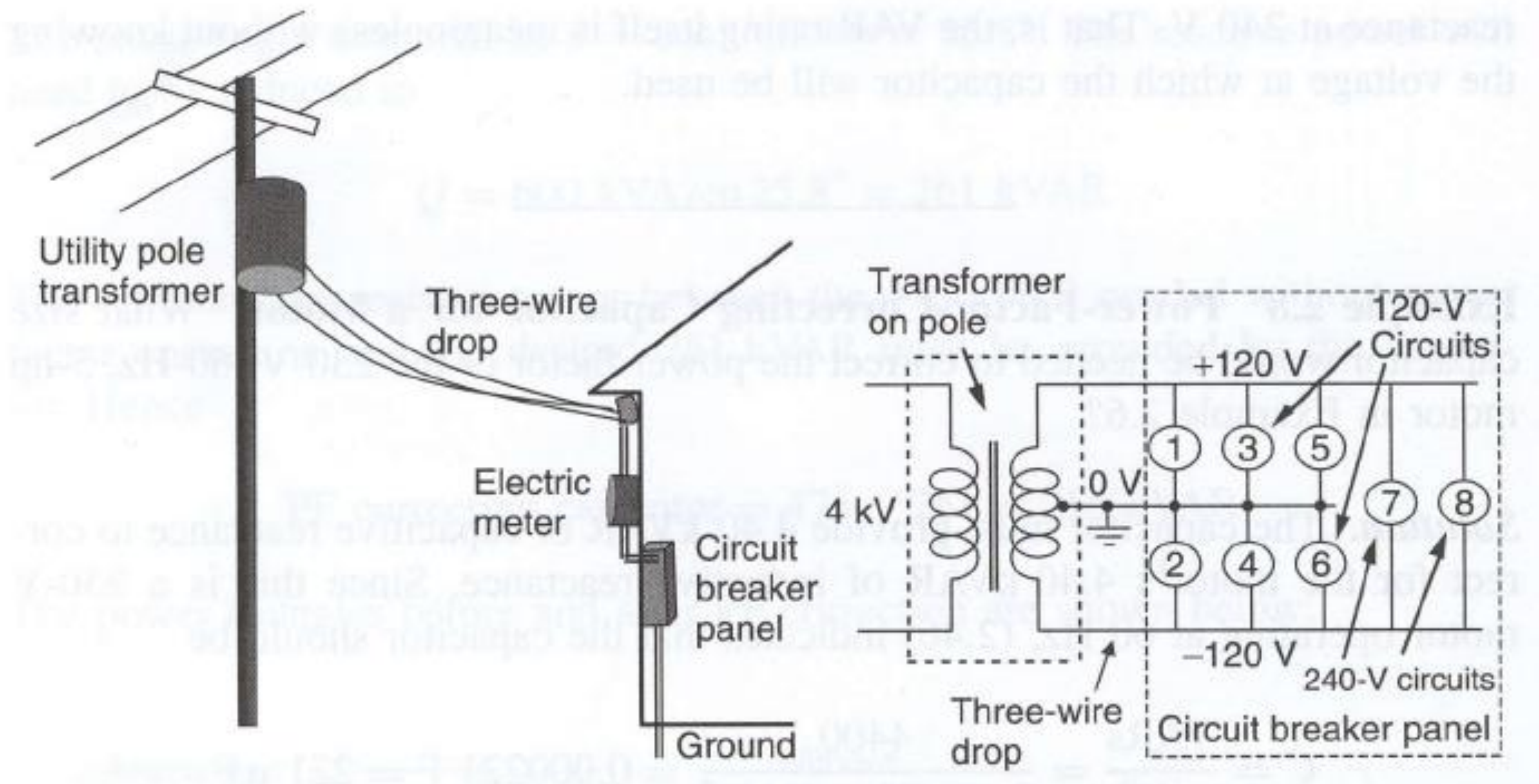
THE RESIDENTIAL ELECTRICITY SUPPLY

- **In the *US*, residential service is typically provided from a 4.16–*kV* feeder line through a step–down transformer to the 120/240 *V* household voltage**
  - **all outlets provide 120 *V***
  - **some outlets provide 240 *V* electricity (air conditioning, heavier duty appliances)**

THE RESIDENTIAL ELECTRICITY SUPPLY

- The provision of $240 - V$ service is done by
  - grounding the center tap of the secondary side of the transformer
  - using the other two ends of the windings at the $\pm\ 120\, V$ supply to obtain the $240 - V$ potential

THE RESIDENTIAL ELECTRICITY SUPPLY

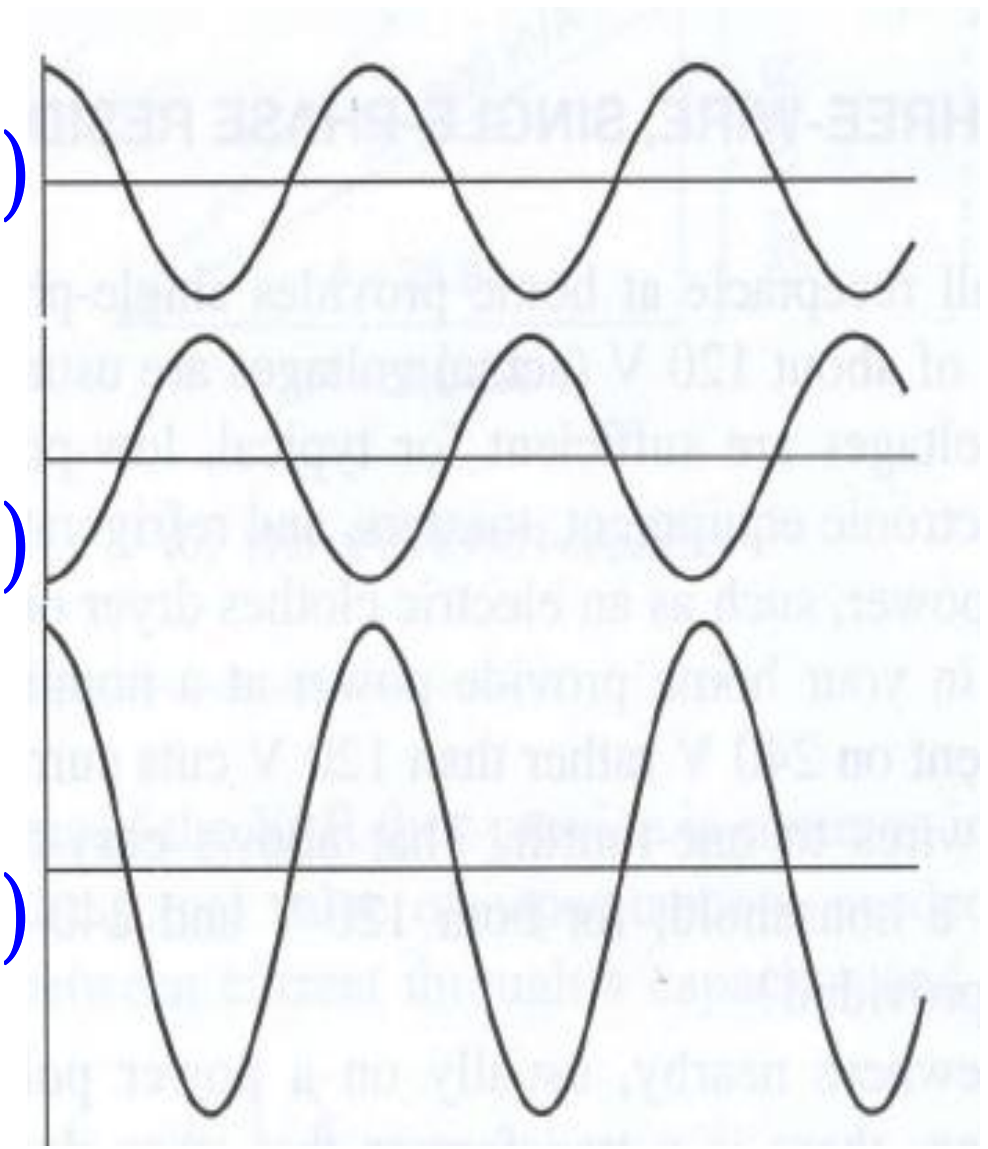


THE RESIDENTIAL ELECTRICITY SUPPLY

$$v_1 = 120\sqrt{2}\ \cos(377t)$$

$$v_2 = -120\sqrt{2}\ \cos(377t)$$

$$v_1 - v_2 = 240\sqrt{2}\ \cos(377t)$$



- **Analytically**

$$v_1(t) = 120\sqrt{2}\ \cos 377t$$

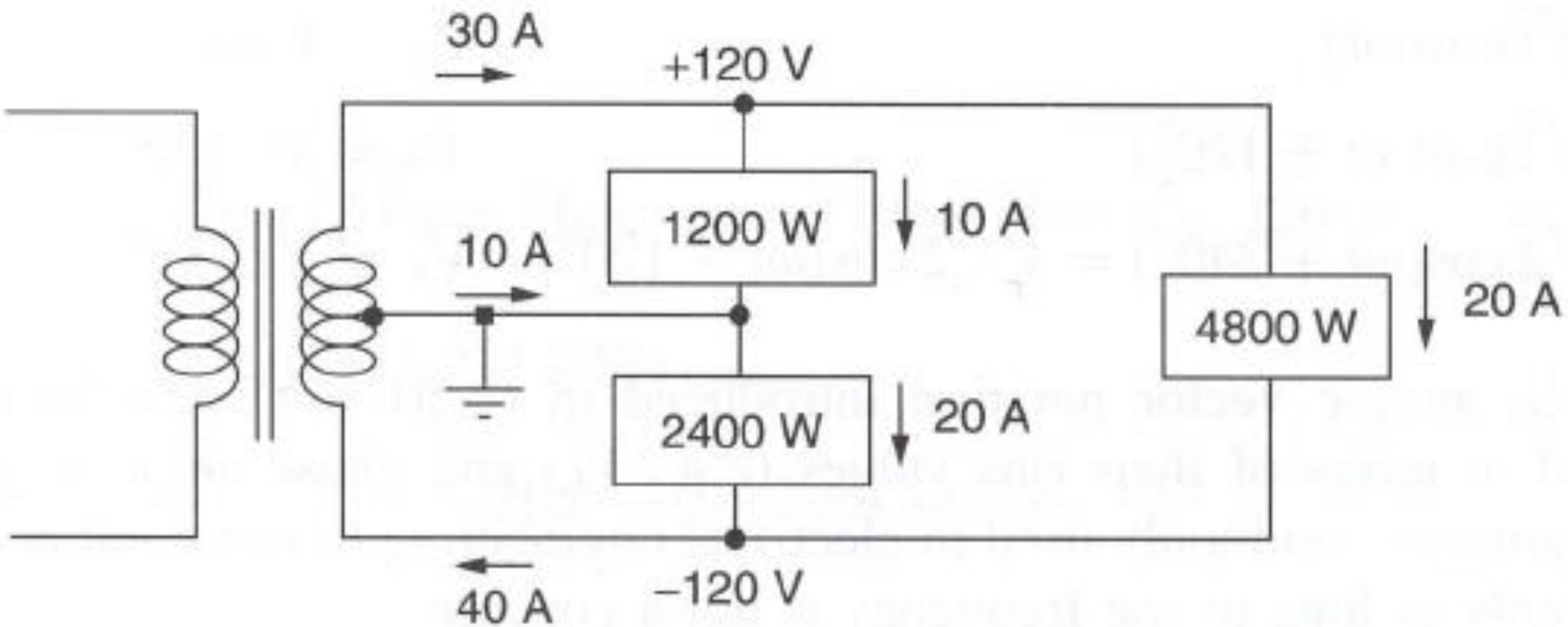
$$v_2(t) = 120\sqrt{2}\ \cos(377t + \pi)$$

$$= -120\sqrt{2}\ \cos 377t$$

and therefore

$$v_1(t) - v_2(t) = 240\sqrt{2}\ \cos 377t$$

RESIDENTIAL LOAD EXAMPLE



RESIDENTIAL LOAD EXAMPLE

- We consider the three loads served by a three–wire $120\,/\,240–V$ system with

  $$1,200\ W \text{ at } 120\ V \text{ on phase } a,\ \ p.f. = 1.0$$

  $$2,400\ W \text{ at } 120\ V \text{ on phase } b,\ \ p.f. = 1.0$$

  $$4,800\ W \text{ at } 240\ V,\ \ p.f. = 1.0$$

- We wish to compute the currents in the wires

- We start with the relationship

$$P = V\,I\,\cos\theta = V\,I$$

RESIDENTIAL LOAD EXAMPLE

- For the 4,800–W load

$$I_{4,800} = \frac{4,800}{240} = 20\ A$$

- For the 2,400–W load

$$I_{2,400} = \frac{2,400}{120} = 20\ A$$

- For the 1,200–W load

$$I_{1,200} = \frac{1,200}{120} = 10\ A$$

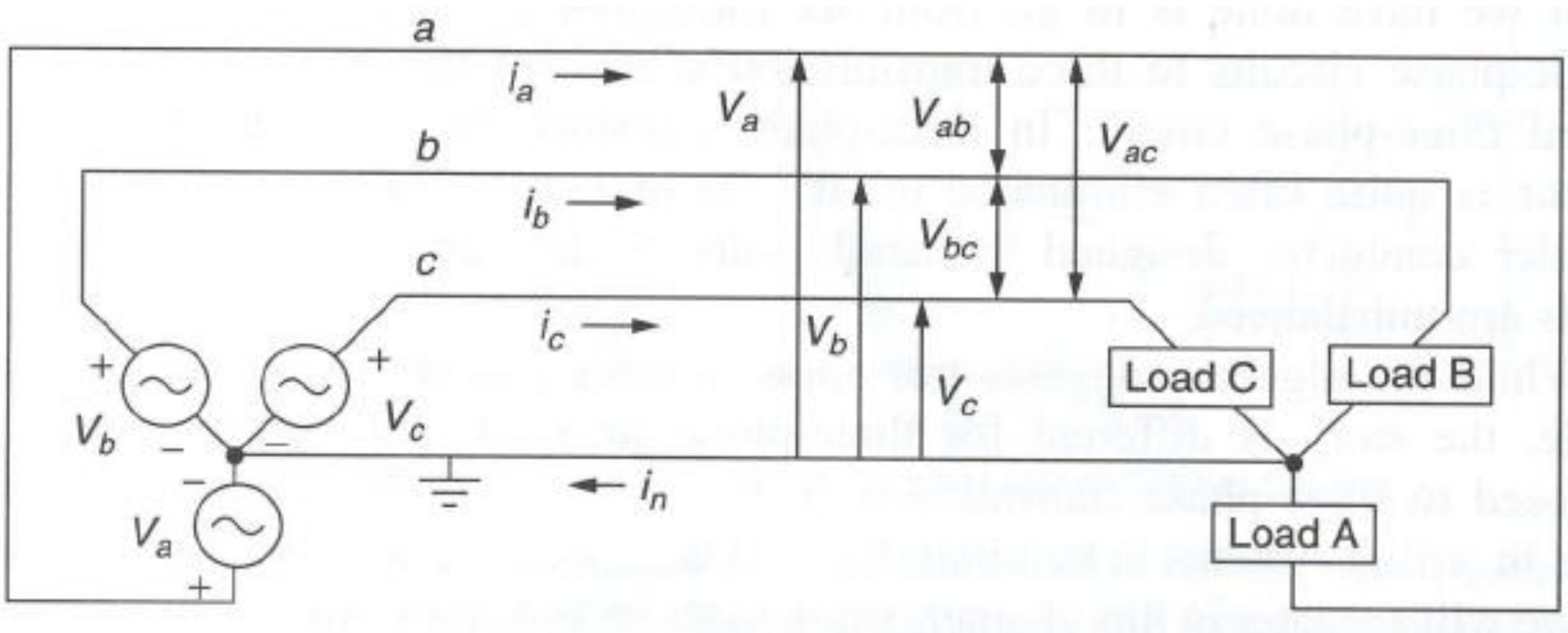
RESIDENTIAL LOAD EXAMPLE

- Note that *KCL* induces a current of $10\,A$ in the neutral leg and as such, the unbalanced load creates a nonzero current in the neutral
- This case differs from the typical, balanced conditions we encounter in which each *hot* leg has the same magnitude current and the neutral current vanishes

THREE – PHASE AC NETWORKS

- Today's power systems use the three–phase (3ϕ) generators to produce electricity and 3ϕ transmission lines to deliver it to various parts of the network
- The interconnection of network elements into a 3ϕ network is done typically using either the *delta* (Δ) or the *wye* (Y) configuration
- We examine a Y–connected 3ϕ *generator* to a 3ϕ *load*

THREE – PHASE *AC* NETWORKS



THREE – PHASE *AC* NETWORKS

- The phase voltages are measured with respect to the neutral

$$v_a(t) = V\sqrt{2}\cos\omega t \quad \leftrightarrow \quad \vec{V}_a = Ve^{j0}$$

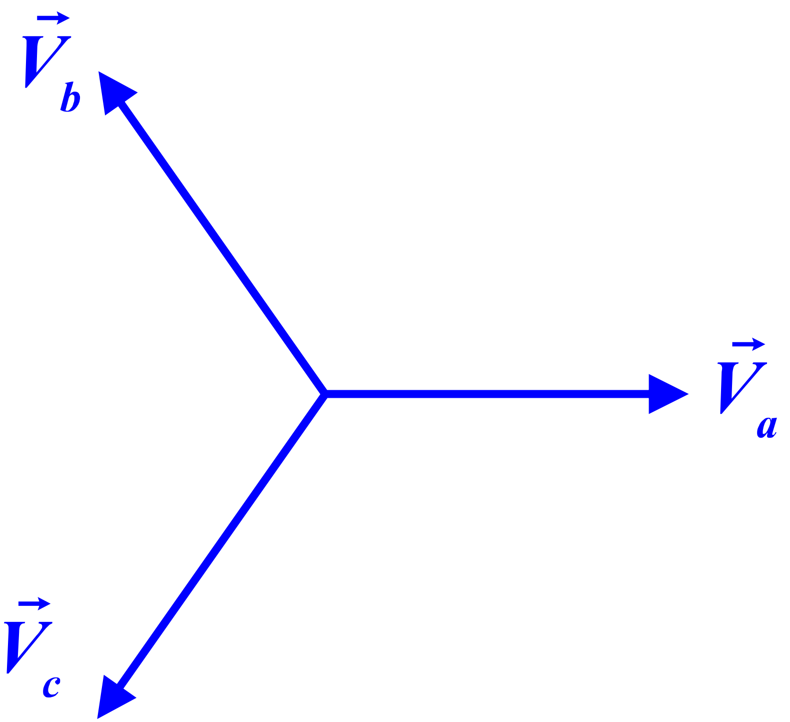
$$v_b(t) = V\sqrt{2}\cos\left(\omega t + \frac{2\pi}{3}\right) \quad \leftrightarrow \quad \vec{V}_b = Ve^{j\frac{2\pi}{3}}$$

$$v_c(t) = V\sqrt{2}\cos\left(\omega t - \frac{2\pi}{3}\right) \quad \leftrightarrow \quad \vec{V}_c = Ve^{-j\frac{2\pi}{3}},$$

where, the entities on the right represent the complex voltages in phasor notation

THREE – PHASE AC NETWORKS

- Note that the voltages are equal in magnitude and are exactly $\pm \frac{2\pi}{3}$ *radians* from one another (***balanced voltages***)



THREE – PHASE *AC* NETWORKS

- Consequently,

$$\vec{V}_a + \vec{V}_b + \vec{V}_c = 0$$

- The voltage between two–phases are typically called line voltages; for example the line *a* to the line *b* voltage is

$$v_{ab}(t) = v_{a0}(t) + v_{0b}(t) = v_{a0}(t) - v_{b0}(t)$$

and so

$$v_{ab}(t) = V\sqrt{2}\cos\omega t - V\sqrt{2}\cos\left(\omega t + \frac{2\pi}{3}\right)$$

THREE – PHASE *AC* NETWORKS

- Now, for a balanced network, the phase voltage *r.m.s.* values are equal

$$V_a = V_b = V_c = V_p \longleftarrow \text{ r.m.s. phase voltage}$$

- Therefore

$$v_{ab}(t) = V_p \sqrt{2} \cos \omega t - V_p \sqrt{2} \cos\left(\omega t + \frac{2\pi}{3} \right)$$

- We make use of the identity

$$\cos \phi - \cos \xi = -2 \sin\left[\frac{1}{2}(\phi + \xi) \right] \sin\left[\frac{1}{2}(\phi - \xi) \right]$$

THREE – PHASE *AC* NETWORKS

- So we obtain

$$v_{ab}(t) = V_p\sqrt{2}\cdot(-2)\,\sin\left(\omega t + \frac{\pi}{3}\right)\cdot\sin\left(-\frac{\pi}{3}\right)$$

$$= V_p\sqrt{2}\cdot 2\;\sin\frac{\pi}{3}\cdot\sin\left(\omega t + \frac{\pi}{3}\right)$$

$$= \underbrace{\sqrt{3}\,V_p}_{V_\ell}\sqrt{2}\;\sin\left(\omega t + \frac{\pi}{3}\right)$$

$$= V_\ell\sqrt{2}\;\sin\left(\omega t + \frac{\pi}{3}\right)$$

THREE – PHASE *AC* NETWORKS

- The relationship of the *r.m.s.* value of line-to-line voltage V_ℓ relative to that of the phase voltage V_p is given by

$$V_\ell = \sqrt{3}\, V_p$$

- Examples of typical values

service type	V_ℓ	V_p
buildings	202 *V*	120 *V*
commercial	480 *V*	277 *V*
residential	416 *V*	240 *V*

THREE – PHASE AC NETWORKS

- Each phase has apparent power

$$S_{\phi} = I_p V_p$$

and so the 3ϕ system has apparent power

$$S_{3\phi} = 3 I_p V_p$$

$$= \sqrt{3}\, I_p \sqrt{3}\, V_p$$

$$= \sqrt{3}\, I_p V_{\ell}$$

- Therefore,

$$P_{3\phi} = S_{3\phi} \cos\theta$$

$$Q_{3\phi} = S_{3\phi} \sin\theta$$

THREE – PHASE *AC* NETWORKS

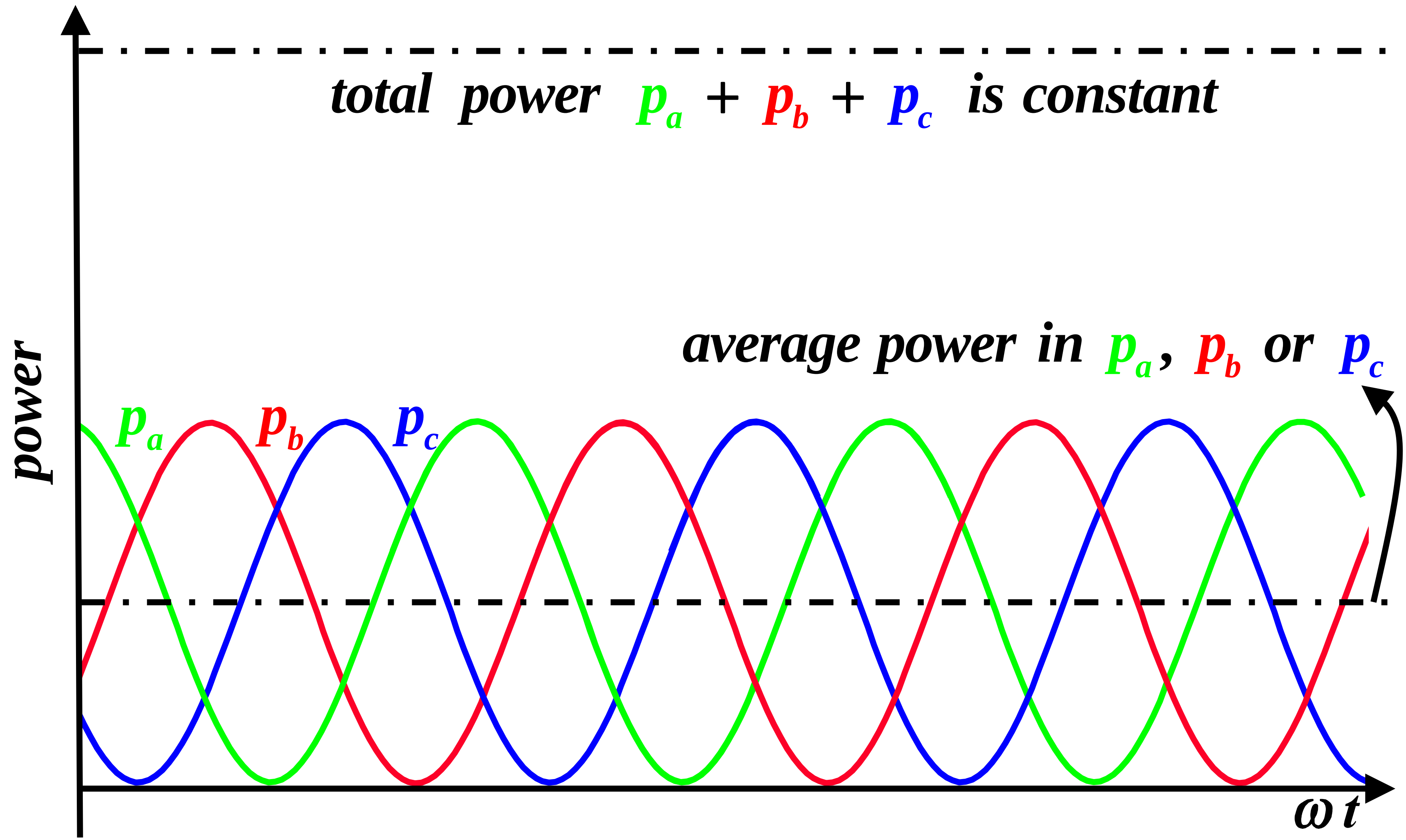
where, θ is the difference of the phase voltage and current angles and, *under balanced conditions*, the power is identical for each phase

- In fact, we can show that

$$p_a(t) + p_b(t) + p_c(t) = 3P_\phi$$

is constant and such a smooth, constant level of power is a major advantage of $3\,\phi$ systems, in contrast to $1\,\phi$, for sinusoidal $p(t)$

THREE – PHASE *AC* NETWORKS



EXAMPLE: 3ϕ NETWORK $p.f.$ CORRECTION

- The 1ϕ–motors in a small enterprise are supplied by a 3ϕ, $208\text{–}V$ transformer
- The real power demand is $80\,kW$ with a $p.f. = 0.5$ and incurs losses of $4\,kW$
- We compute $S_{3\phi}$ using

$$P_{3\phi} = \sqrt{3}\,V_{\ell}\,I_{p}\,cos\theta = S_{3\phi}\,0.5 = 80\,kW$$

so that

$$S_{3\phi} = 160\,kVA$$

EXAMPLE: 3ϕ NETWORK $p.f.$ CORRECTION

- **We also evaluate**

$$I_p = \frac{S_{3\phi}}{\sqrt{3}V_\ell} = \frac{160}{\sqrt{3}\ 208} = .444\ kA$$

- **Next consider a $p.f.$ correction to 0.9 and so**

$$S'_{3\phi} = \frac{80}{0.9} = 88.9\,kVA \ll 160\,kVA$$

EXAMPLE: 3ϕ NETWORK $p.f.$ CORRECTION

- Also, the corresponding phase current is

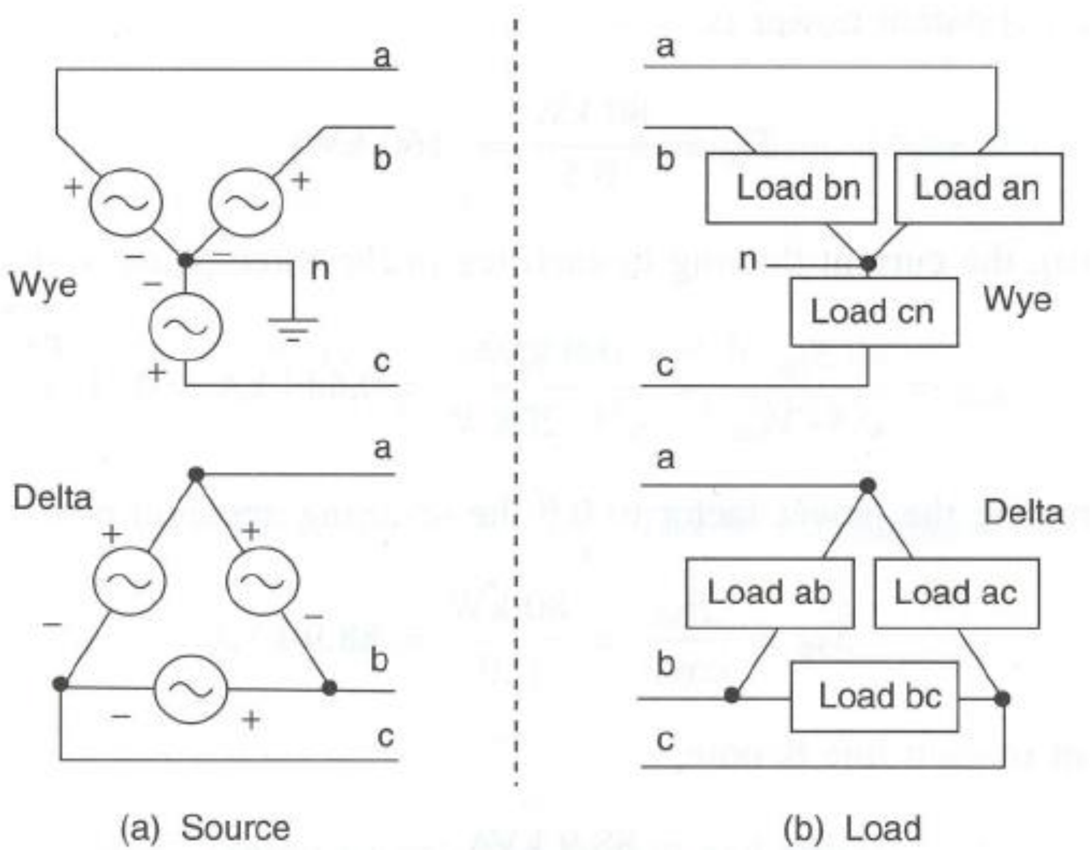
$$I'_p = \frac{88.9}{\sqrt{3}\,208} = .247\ kA$$

- We also evaluate the losses under corrected *p.f.*

$$R\left(I'_p\right)^2 = \frac{4}{\left(.444\right)^2}\left(.247\right)^2 = 1.24\ kW$$

THE 3ϕ DELTA CONNECTION

- The other way to connect 3ϕ elements is the Δ connection without a neutral line

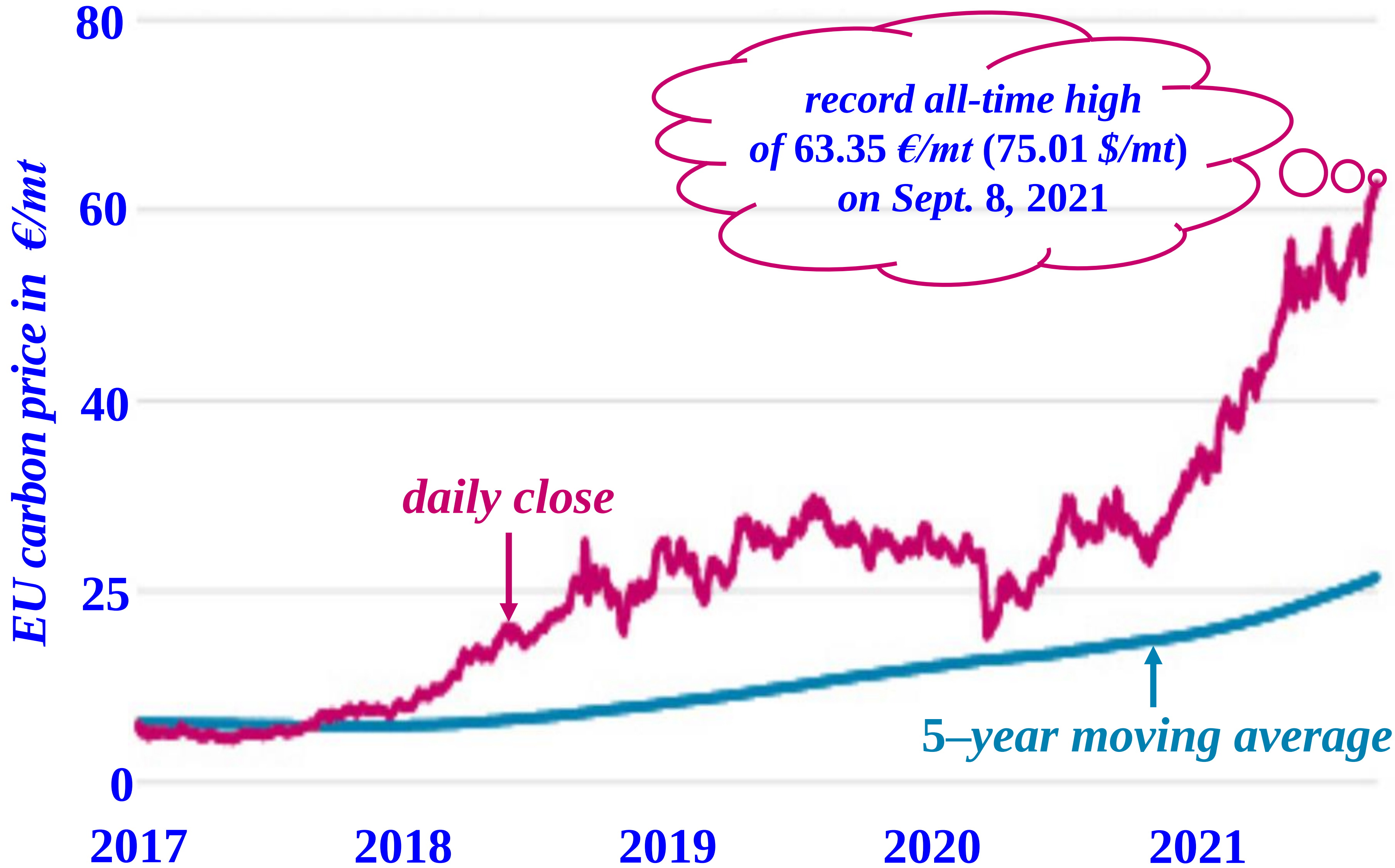


THE 3ϕ DELTA CONNECTION

- **The comparison of the key characteristics of the two connection schemes is summarized by the table**

variable	$Y - connection$	$\Delta - connection$
r.m.s. current	$I_{\ell} = I_{p}$	$I_{\ell} = \sqrt{3}\, I_{p}$
r.m.s. voltage	$V_{\ell} = \sqrt{3}\, V_{p}$	$V_{\ell} = V_{p}$
3ϕ *power*	$P_{3\phi} = 3\, V_{p}\, I_{p} \cos\theta$	$P_{3\phi} = \sqrt{3}\, V_{\ell} I_{\ell} \cos\theta$

EU CARBON PRICES CONTINUE TO SMASH RECORDS IN 2021



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