ECE 333 – GREEN ELECTRIC ENERGY

12. The Solar Energy Resource

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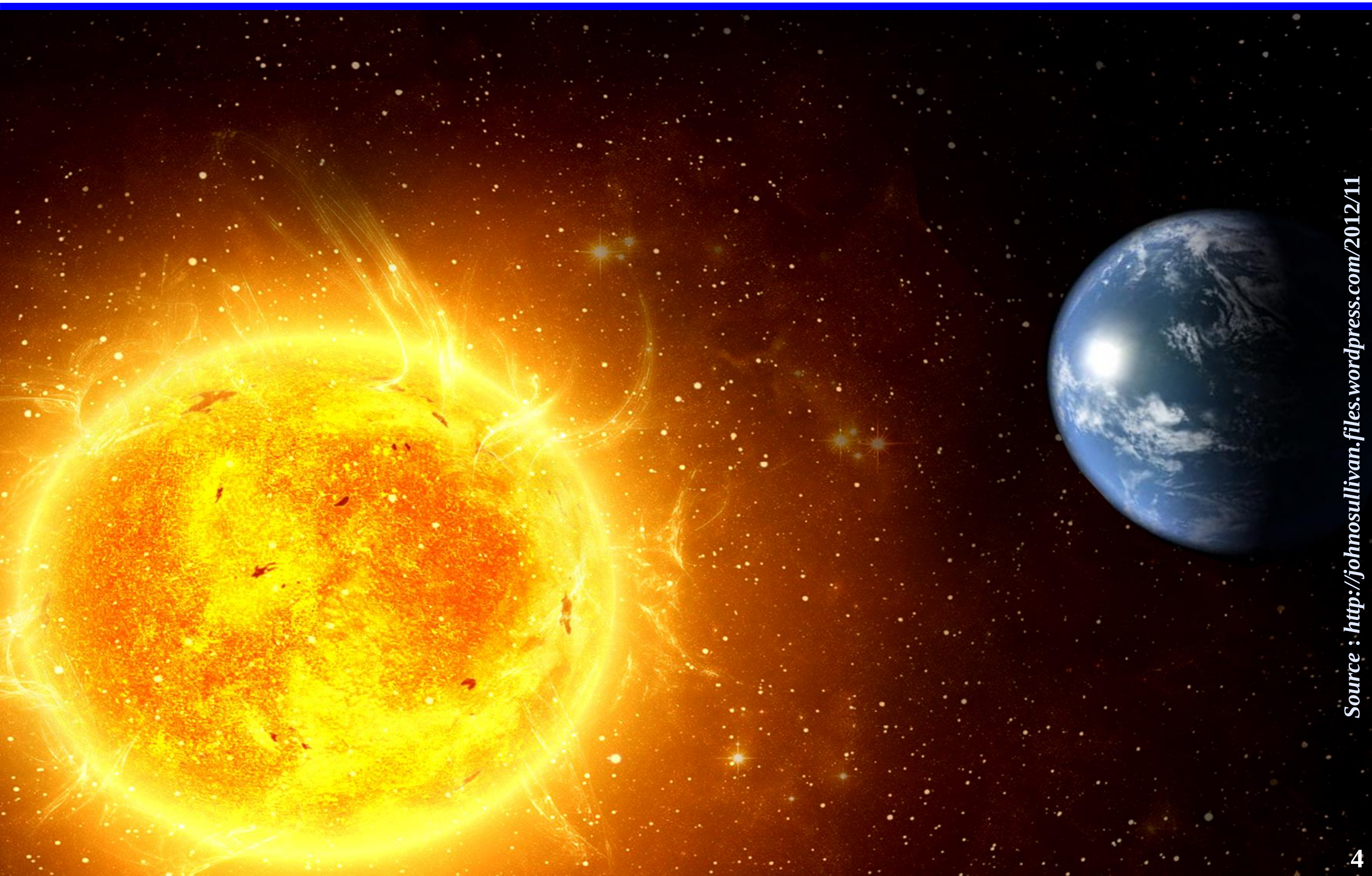
SOLAR ENERGY

- **Solar energy – a very clean energy source – is the most abundant renewable energy source**
- **Solar energy is harnessed for many applications, including electricity generation, lighting and steam and hot water production**

SOLAR RESOURCE LECTURE OUTLINE

- The solar energy source
- Extraterrestrial solar irradiation
- Analysis of solar position in the sky and its application to the determination of
  - the optimal tilt angle design for a solar panel
  - the sun path diagram for shading analysis
  - the solar time and civil time relationship

UNDERLYING BASIS: THE SUN IS A LIMITLESS ENERGY SOURCE



SOLAR ENERGY

- The *thermonuclear reactions*, as the hydrogen atoms fuse together to form helium in the sun, are the source of solar energy
- In every *second*, about 4 *billion kg* of mass are converted into energy, according to Einstein's well-known *mass–energy equation* $E = mc^2$
- This immense energy generated is huge and keeps the sun at *very high temperatures* at all times

SOLAR ENERGY

- **The plentiful solar energy during the past 4 or 5 *billion years* is expected to continue in the future**
- **An object emits radiant energy in an amount that is a function of its temperature; the sun emits *solar energy* into space via *radiation***
- ***Insolation* or *solar irradiation*, stated in units of $\frac{W}{m^2}$, measures the power density of the solar energy**

PLANCK'S LAW

- **Physicists use the *theoretical concept of a blackbody* – defined to be both a perfect emitter and a perfect absorber – to discuss radiation**
- **The emissive power intensity of a *blackbody* is a function of its wavelength λ and temperature τ as expressed by *Planck's law*:**

PLANCK'S LAW

emissive power intensity

$W / m^2\text{-}\mu m$

$$\rho_{\lambda}(\tau) = \frac{3.74 \times 10^{8}}{\lambda^{5}\left[exp\left(\frac{14,400}{\lambda \tau} \right) - 1 \right]}$$

μm

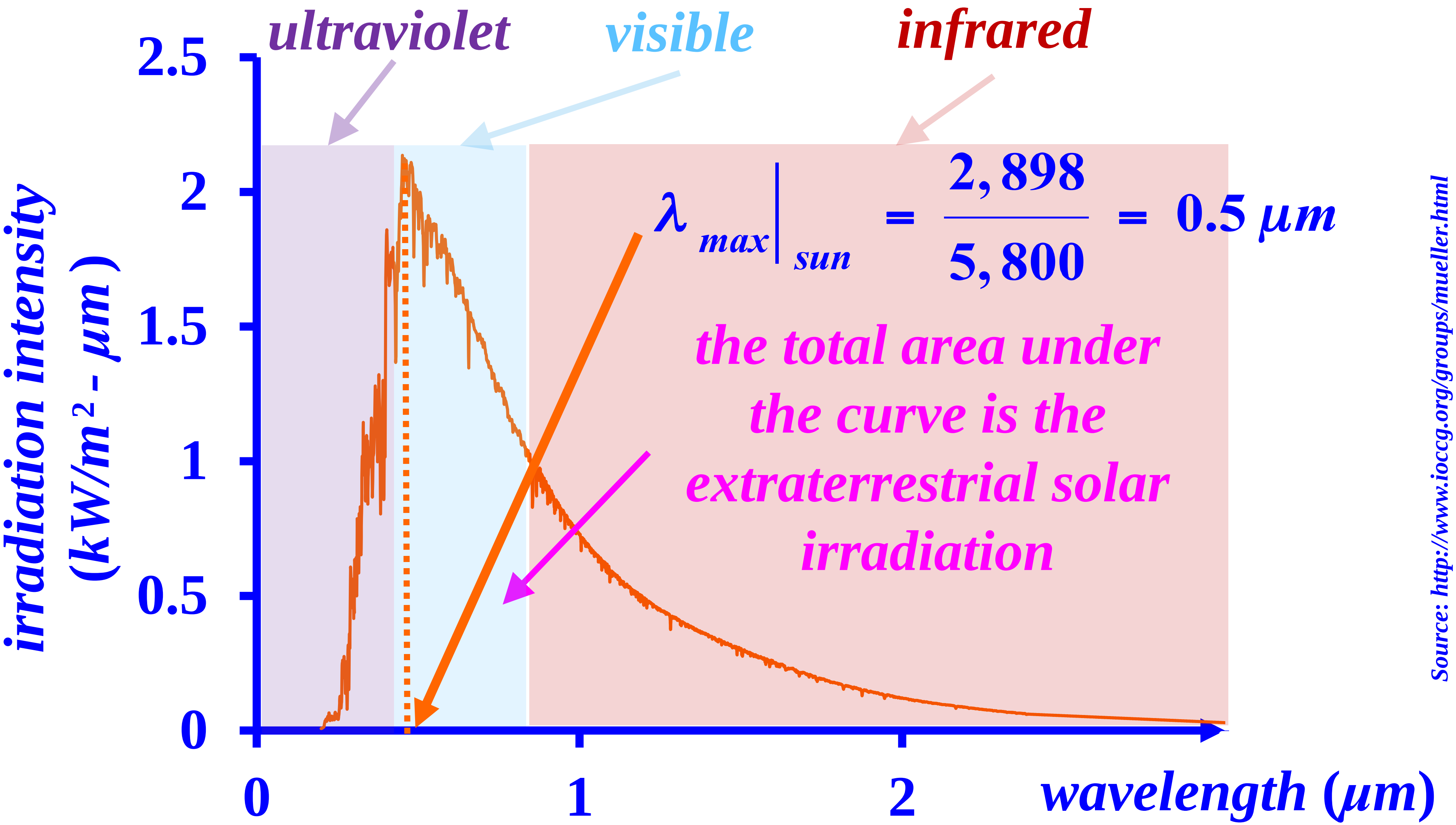
K

WIEN'S DISPLACEMENT RULE

An important feature of *blackbody* radiation is given by *Wien's displacement rule*, which determines the *wavelength* λ_{max} at which the emissive power intensity reaches its peak value

$$\lambda_{max} = \frac{2,898}{\tau} \quad \mu m$$

EXTRATERRESTRIAL SOLAR SPECTRUM



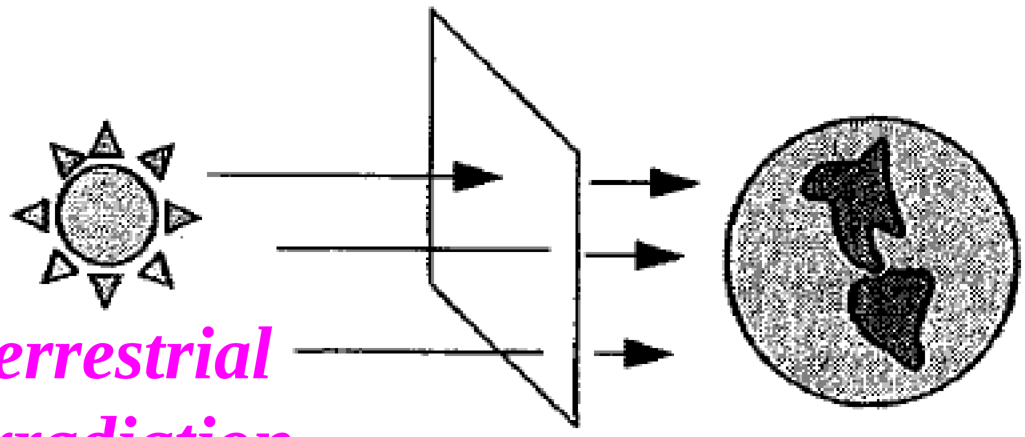
THE SOLAR IRRADIATION

- The sun's surface temperature is estimated to be 5,800 K and its power density value is assumed as 1.37 kW/m^2 – the value of insolation or solar irradiation just outside the earth's atmosphere
- The sun emits maximum energy at the *wavelength*

$$\lambda_{max}\Big|_{sun} = \frac{2,898}{5,800}\,\mu m = 0.5\,\mu m$$

EXTRATERRESTRIAL SOLAR IRRADIATION

Extraterrestrial solar irradiation **is defined as the solar irradiation that strikes an imaginary surface at the top of the earth's atmosphere; the surface lies perpendicular to the line from the earth's center to the sun's center**



extraterrestrial solar irradiation

STEFAN–BOLTZMANN LAW OF RADIATION

- **The total area under the power intensity curve is the *blackbody* radiant power density emitted over all the wavelengths**
- **The *Stefan–Boltzmann law of radiation* states that**

the total radiant power in W → $p_{blackbody}$

the surface area of the blackbody in m^2 → A

$$p_{blackbody} = \sigma A \tau^4$$

Stefan-Boltzmann constant: $5.67 \times 10^{-8} W / m^2 - K^4$ → σ

THE EARTH'S RADIATION

- **We consider the earth to be a blackbody with average surface temperature $15\,^{o}C$ and area equal to $5.1 \times 10^{14}\ m^{2}$**

- **The *Stefan–Boltzmann law of radiation* states that the earth radiates**

THE EARTH'S RADIATION

$$p_{earth} = \sigma A \tau^{4}$$

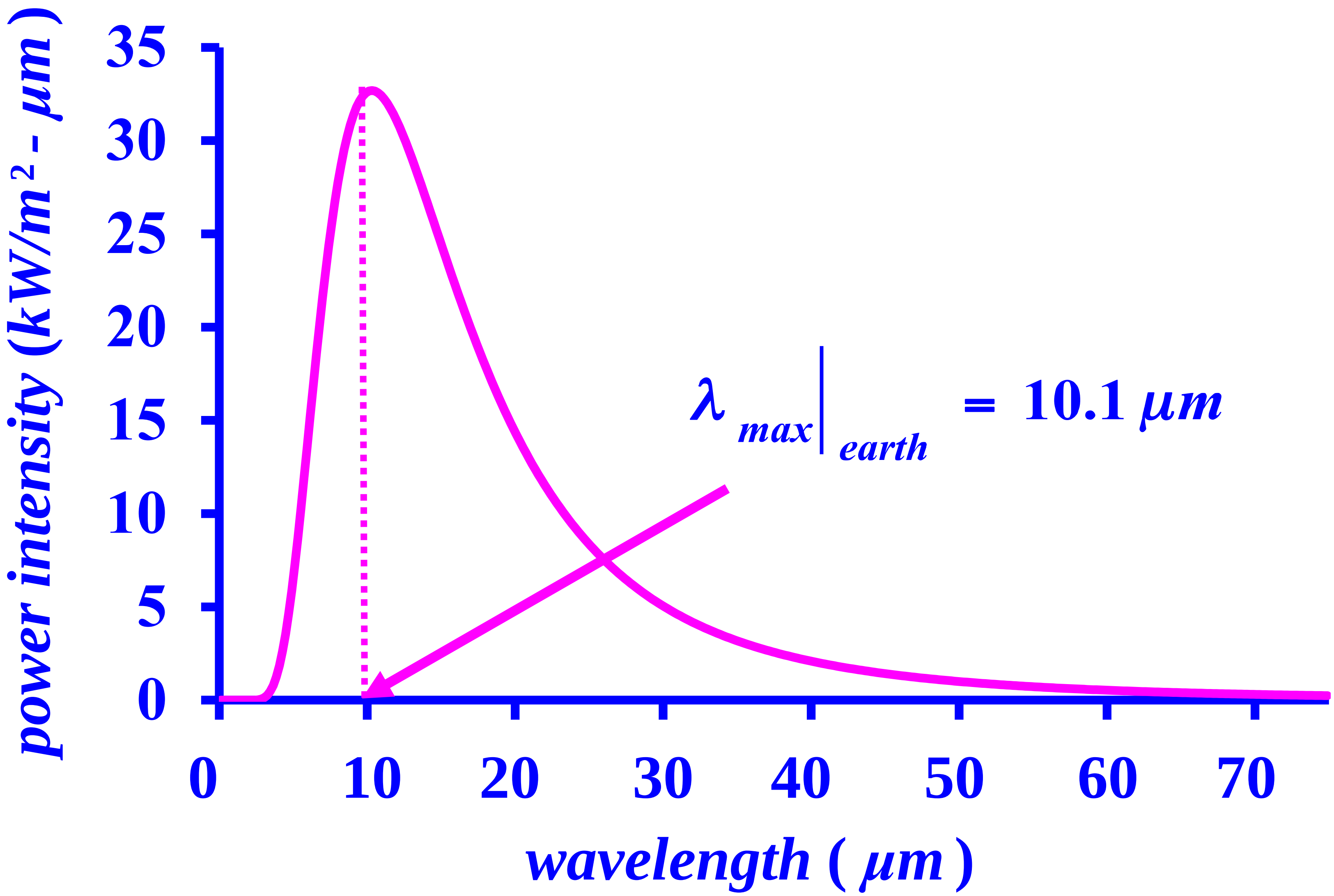
$$= (5.67 \times 10^{-8})(5.1 \times 10^{14})(15 + 273)^{4}$$

$$= 2 \times 10^{17}\ W$$

- The wavelength at which the maximum power is emitted by earth is given by *Wien's displacement rule*

$$\lambda_{max}\Big|_{earth} = \frac{2,898}{288}\ \mu m = 10.1\ \mu m$$

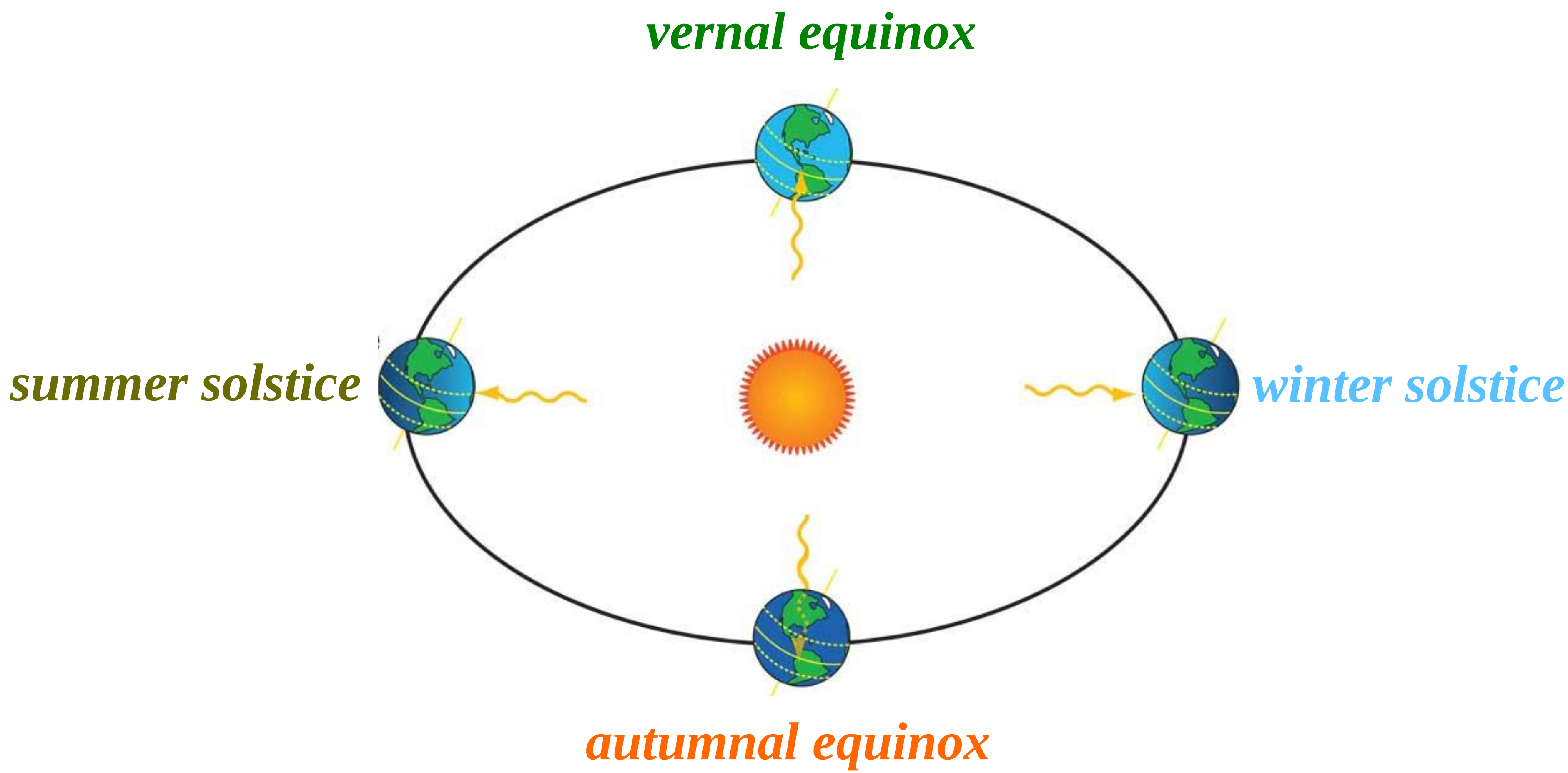
THE SPECTRAL EMISSIVE POWER INTENSITY OF A 288 – *K* BLACKBODY



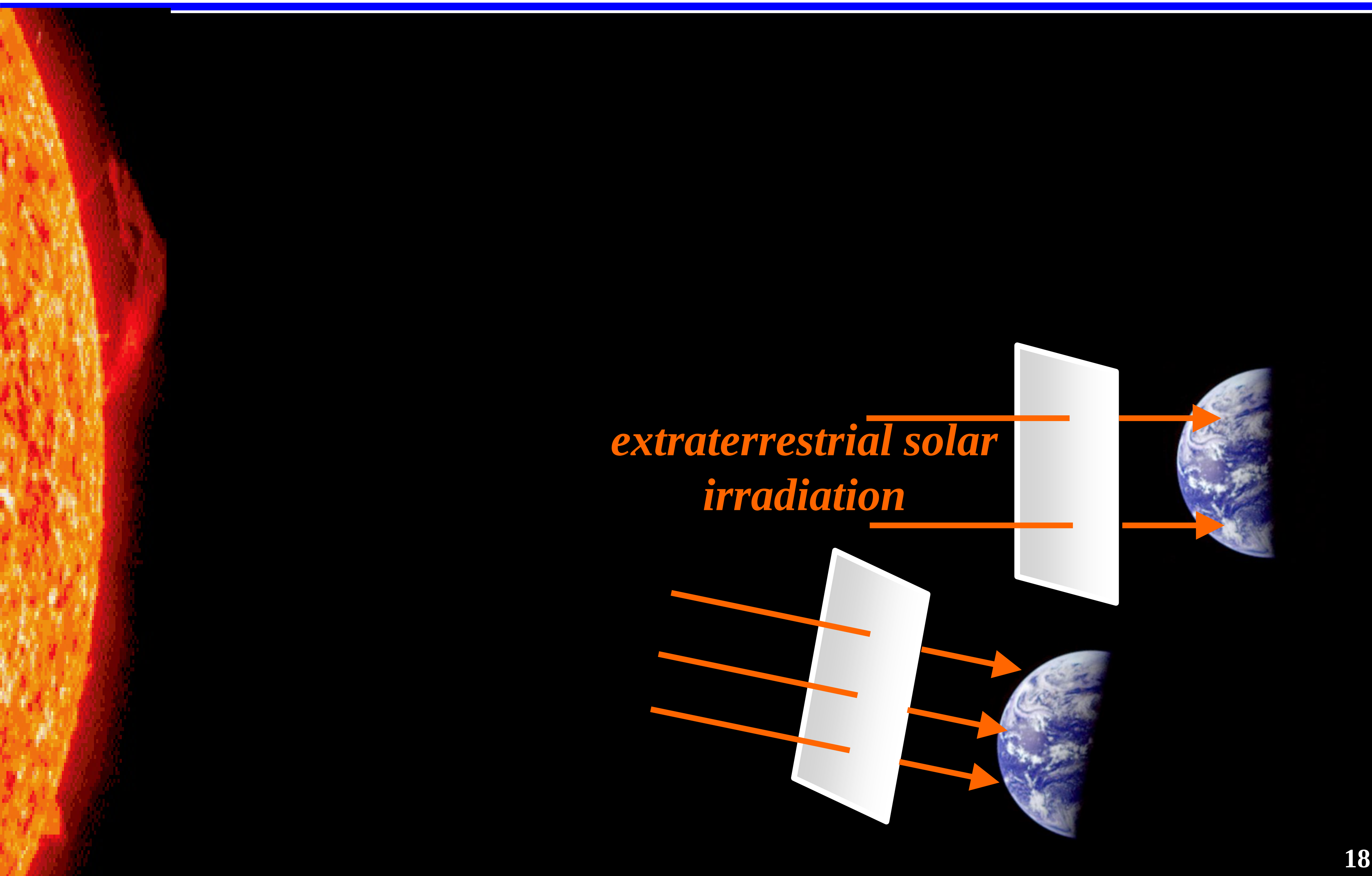
Source: http://www.ioccg.org/groups/mueller.html

EARTH'S ORBIT OVER ITS YEARLY REVOLUTION AROUND THE SUN

Source*: *http://scijinks.nasa.gov



EXTRATERRESTRIAL SOLAR IRRADIATION

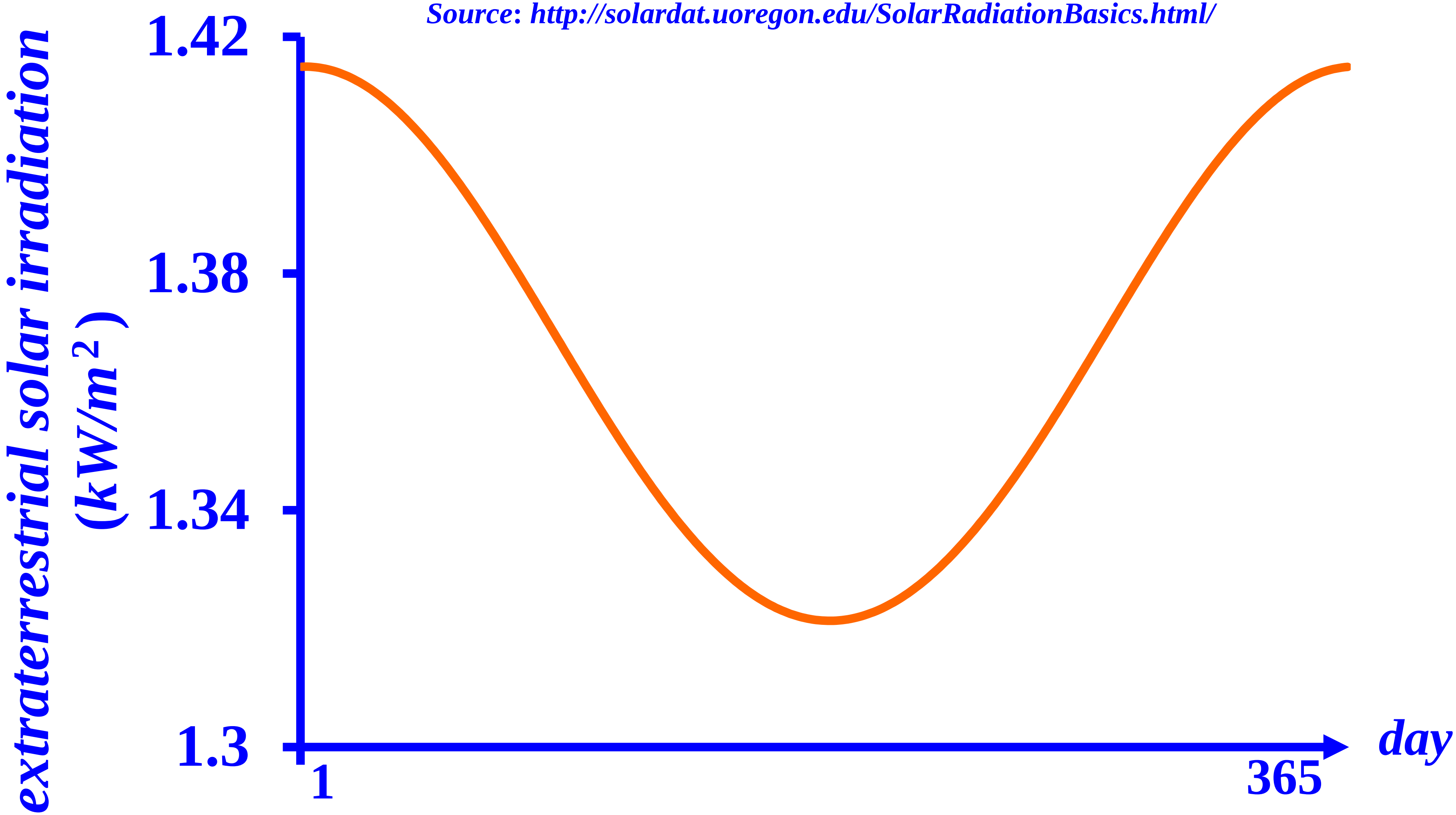


EXTRATERRESTRIAL SOLAR IRRADIATION OVER A YEAR

- **In the analysis of all solar issues, we use *solar time* based on the sun's position with respect to the earth, instead of *clock* or *civil time***
- **Extraterrestrial solar irradiation depends on the distance between the earth and the sun and therefore is a function of the day of the year**

THE ANNUAL EXTRATERRESTRIAL SOLAR IRRADIATION

Source*: *http://solardat.uoregon.edu/SolarRadiationBasics.html/



EXTRATERRESTRIAL SOLAR IRRADIATION OVER A YEAR

- **The extraterrestrial solar irradiation variation in a day is *negligibly small* and we assume that its value $i_0\big|_d$ is constant as the earth rotates each day**

- **We use the approximation for $i_0\big|_d$ given by:**

$$i_0\big|_d = 1{,}367\left[1 + 0.034\cos\left(2\pi\frac{d}{365}\right)\right] \qquad d = 1,2,\ldots,365/366$$

W/m^2

EXTRATERRESTRIAL SOLAR IRRADIATION

- We consider the approximation of extraterrestrial solar irradiation on January 1, with $d = 1$, is

$$i_0\Big|_1 = 1{,}367\left[1 + 0.034\cos\left(2\pi\frac{1}{365}\right)\right] = 1{,}413\,\frac{W}{m^2}$$

- Now, for August 1, $d = 213$ and the extraterrestrial solar irradiation is approximately

$$i_0\Big|_{213} = 1{,}367\left[1 + 0.034\cos\left(2\pi\frac{213}{365}\right)\right] = 1{,}326\,\frac{W}{m^2}$$

EXTRATERRESTRIAL SOLAR IRRADIATION

- We observe that in the Northern hemisphere, the extraterrestrial solar irradiation is higher on a cold winter day than on a hot summer day
- This phenomenon results from the fact that the sunlight enters into the atmosphere with different incident angles; these angles impact greatly the

EXTRATERRESTRIAL SOLAR IRRADIATION

fraction of extraterrestrial solar irradiation *received* on the earth's surface at different times of the year

- **As such, at a specified geographic location, we need to determine the *solar position in the sky* to evaluate the *effective amount* of solar irradiation at that location**

THE SOLAR POSITION IN THE SKY

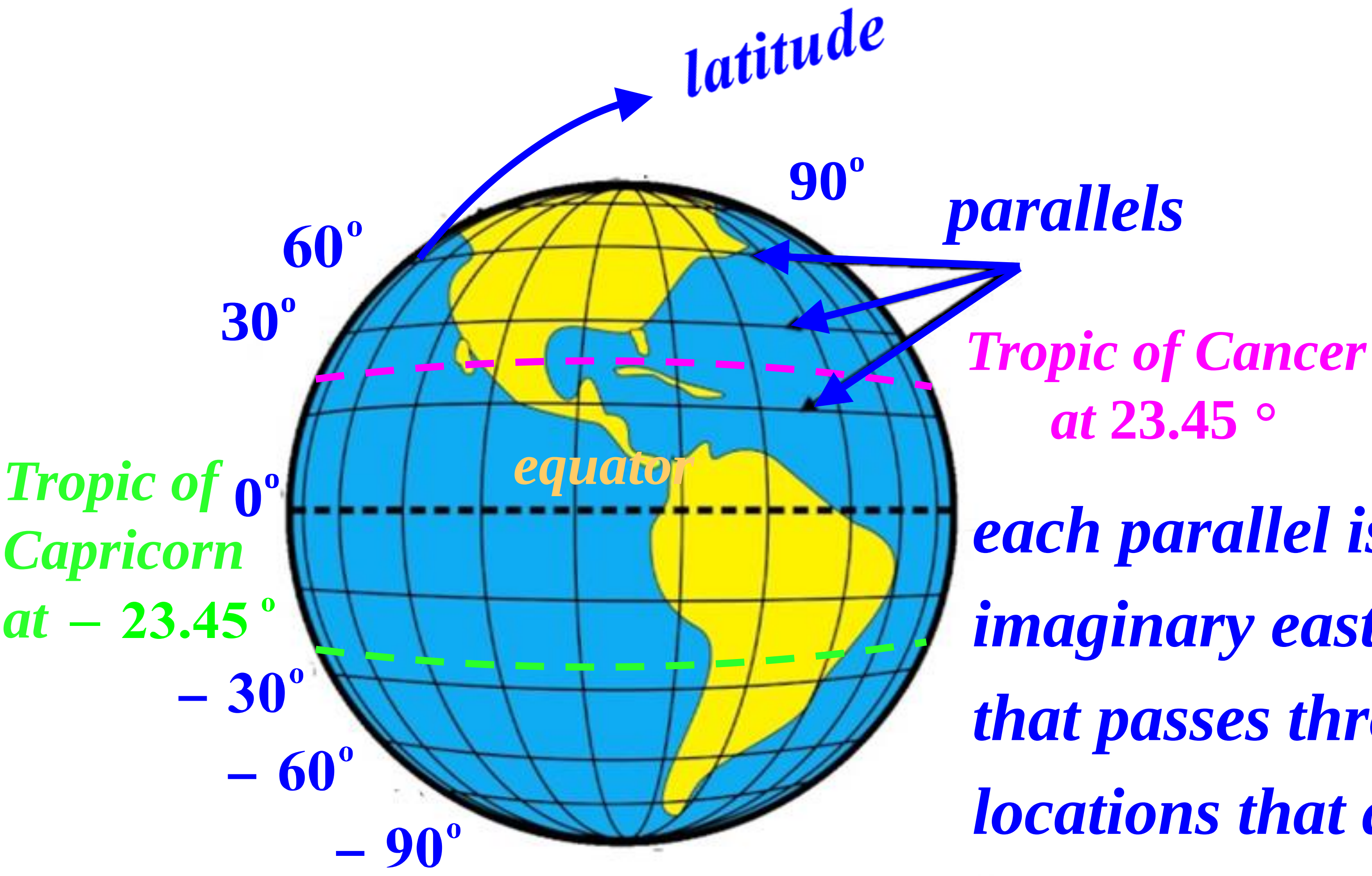
The *solar position in the sky* varies as a function of:

- **the *specific* geographic location of interest;**
- **the time of day due to the earth's rotation around its tilted axis; and,**
- **the day of the year that the earth is on its orbital revolution around the sun**

LATITUDE AND LONGITUDE

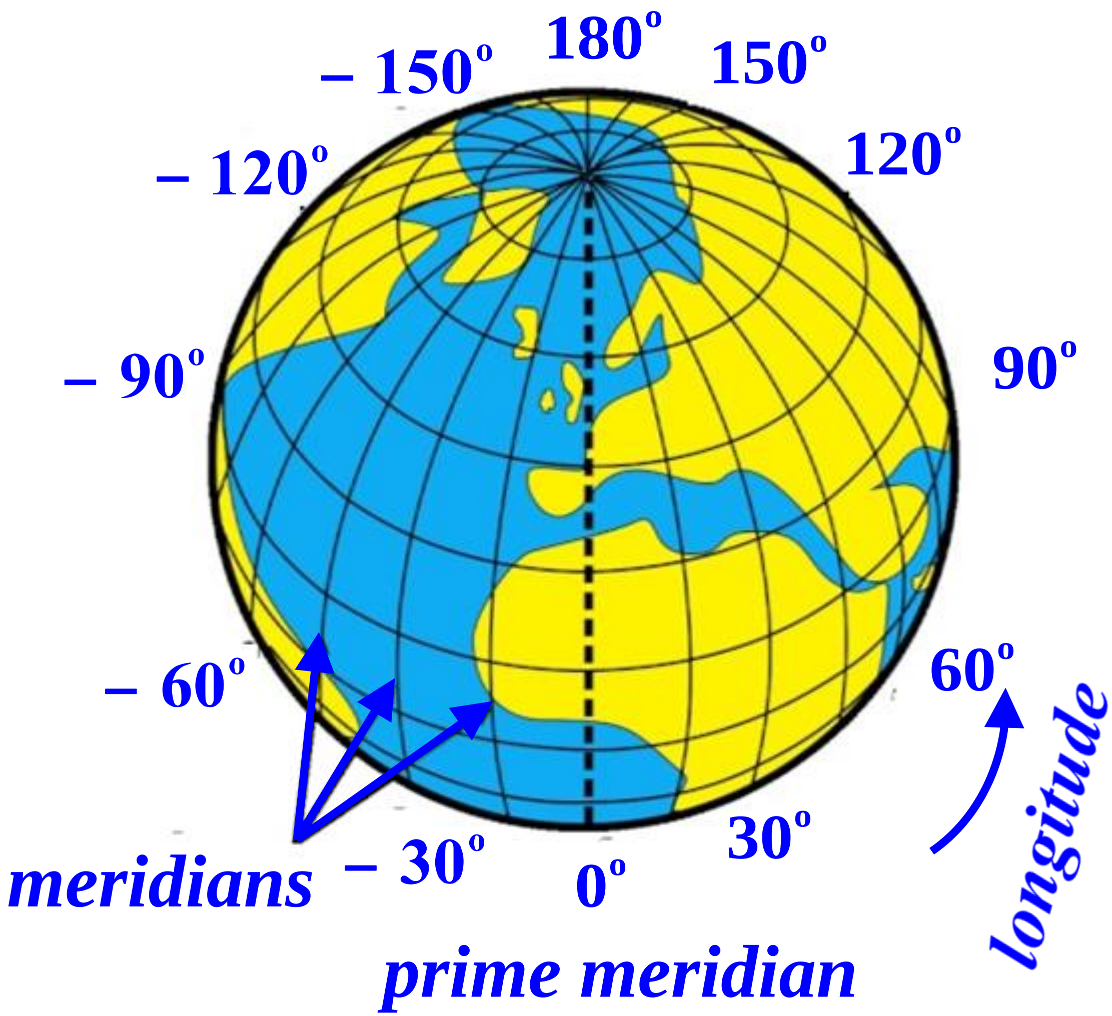
- **A geographic location on earth is specified in full by the local *latitude* and *longitude***
- **The *latitude* and *longitude pair* of geographic coordi–nates specify the North–South and the East–West positions of a location on the earth's surface; the coordinates are expressed in *degrees* or *radians***

LATITUDE AND LONGITUDE



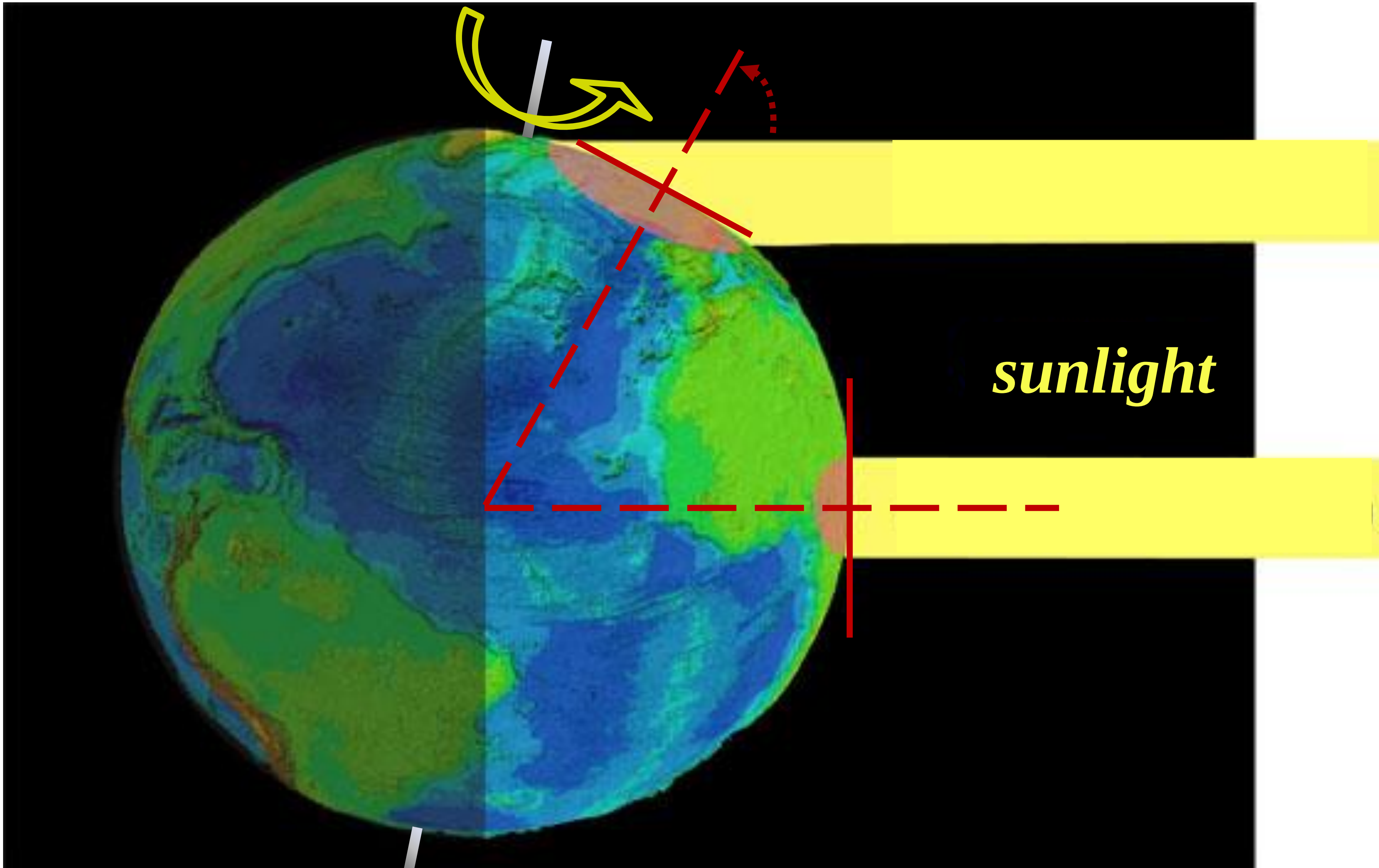
each parallel is an imaginary east-west circle that passes through all the locations that are at the corresponding latitude

LATITUDE AND LONGITUDE



a meridian is an ***imaginary arc*** ***on the earth's surface that connects the North and the South poles***

THE SOLAR IRRADIATION VARIES WITH THE GEOGRAPHIC LOCATION



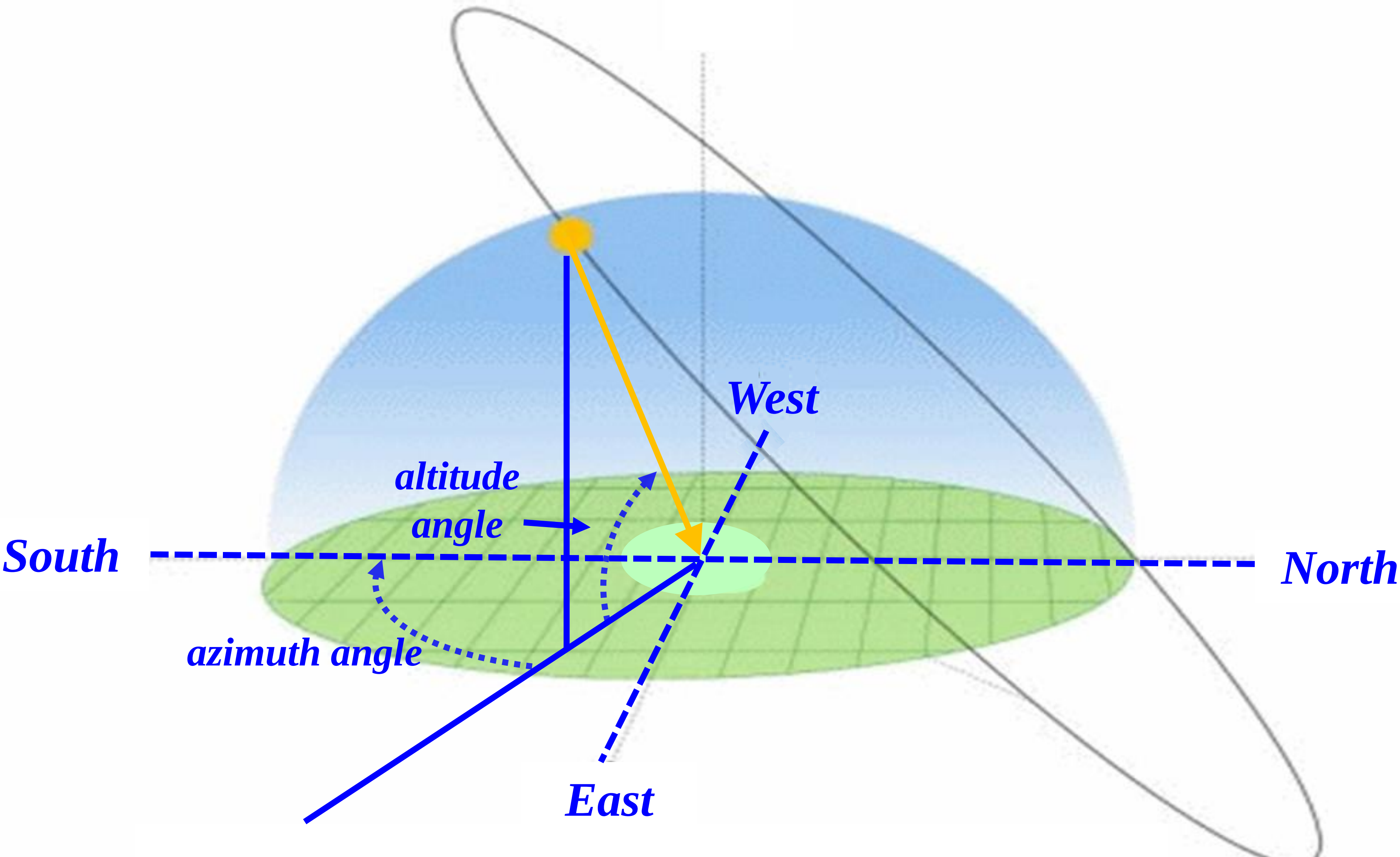
EARTH’S ROTATION

http://upload.wikimedia.org/wikipedia/commons/d/d7/Earth%27s_Axis.gif

EARTH'S ROTATION

- Although the sun's position is fixed in space, the earth's rotation around its tilted axis results in the "movement" of the sun from east to west during each day's sunrise–to–sunset period
- The "movement" of the sun's position in the sky causes variations in the solar irradiation received at a specified location on the earth's surface

THE SOLAR IRRADIATION VARIES AS A FUNCTION OF THE TIME OF DAY



THE SOLAR POSITION IN THE SKY AT ANY TIME OF THE DAY

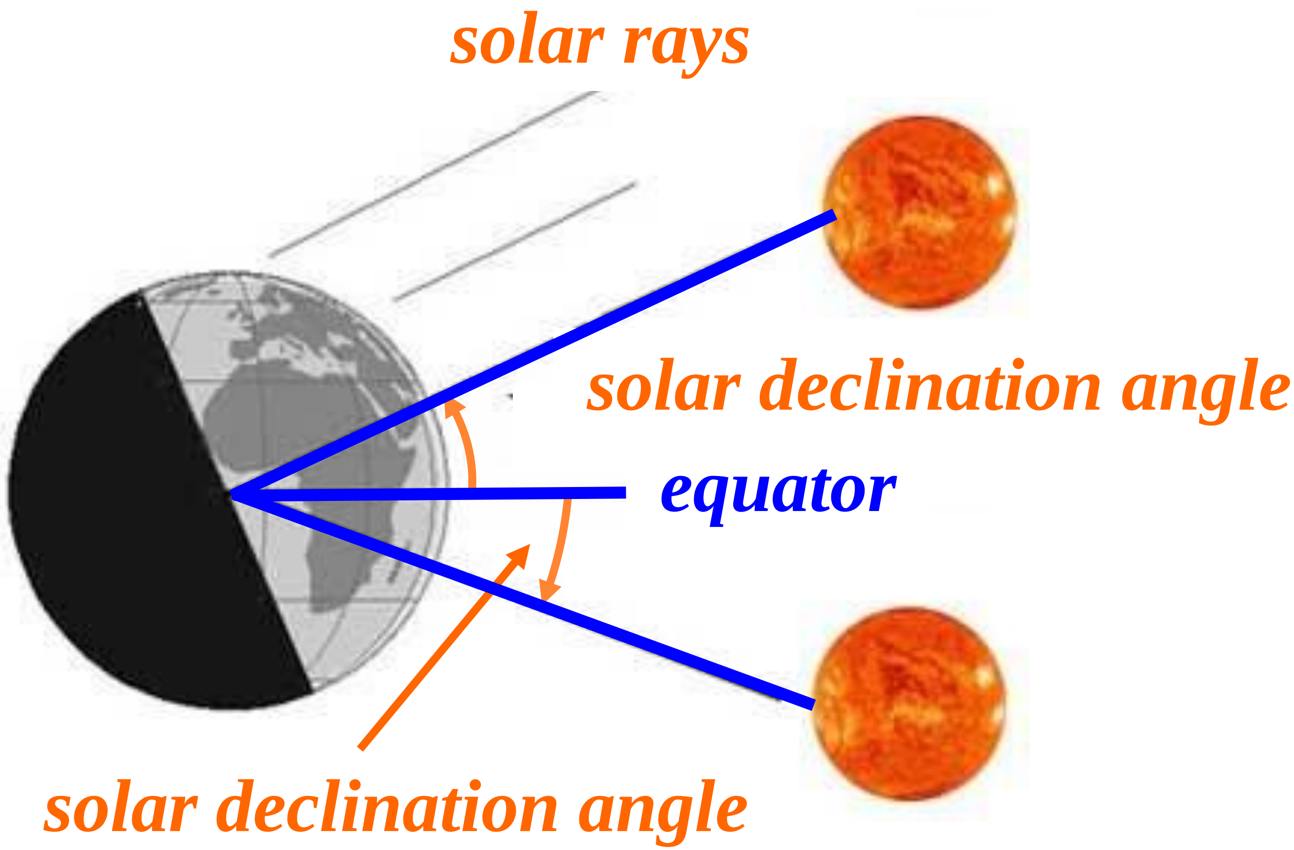
- **The solar position in the sky at any time of the day – sunrise–to–sunset period – is expressed in terms of the *altitude angle* and the *solar azimuth angle***
- **The *altitude angle* is defined as the angle between the sun and the local horizon, which depends on the location's *latitude*, *solar declination angle* and *solar hour angle***

SOLAR DECLINATION ANGLE

- The *solar declination angle* refers to the angle between the plane of the equator and an imaginary line from the center of the sun to the center of the earth
- The *solar declination angle* variation during a day is sufficiently small to allow us to assume it to be constant and represent it as a function of d by $\delta\big|_{d}$

SOLAR DECLINATION ANGLE

$$\delta\Big|_{d} = 0.41\ sin\left[\frac{2\pi}{365}(d-81)\right]\ radians$$



Source: https://encrypted-tbn3.gstatic.com/images?q=tbn:ANd9/

SOLAR HOUR ANGLE

- ***Solar noon*** **is the time at which the solar position in the sky is** ***vertically above the local meridian*** **– the line of longitude; the sun is due South (North) of the location in the Northern (Southern) Hemisphere at latitudes above the** ***Tropic of Cancer*** **(at latitudes below the** ***Tropic of Capricorn*****)**
- ***Solar hour angle*** $\theta(h)$ **refers to the angular rotation in** ***radians*** **the earth must go through to reach the** ***solar noon*****;** h **is** **+** ***before solar noon*** **–** ***ante meridiem*** **– and** **–** ***after solar noon*** **–** ***post meridiem***

SOLAR HOUR ANGLE

- We consider the earth to rotate $\frac{2\pi}{24}$ each *hour*; so

$$\theta(h) = \frac{\pi}{12} h \ \ radians$$

- At 11 *a.m.* in solar time

$$\theta(1) = \frac{\pi}{12}$$

and at 2 *p.m.* in solar time

$$\theta(-2) = -\frac{\pi}{6}$$

ALTITUDE ANGLE

Then, the relationship of *altitude angle* $\beta(h)\big|_d$ and the location's *latitude*, *solar declination angle* and *solar hour angle* is expressed as

$$\sin\left(\beta(h)\big|_d\right)$$

$$= \cos(\ell)\cos\left(\delta\big|_d\right)\cos(\theta(h)) + \sin(\ell)\sin\left(\delta\big|_d\right)$$

where ℓ is the local latitude

EXAMPLE: ALTITUDE ANGLE AT CHAMPAIGN

- Champaign's latitude is 0.7 *radians*

- October 22 corresponds to $d = 295$; the solar declination angle is given by

$$\delta\Big|_{295} = 0.41 \, sin\left[\frac{2\pi}{365}\left(295 - 81\right)\right] = -\,0.21 \; radians$$

- At 1 *p.m.* solar time, the hour angle is

$$\theta(-1) = \frac{\pi}{12}\cdot(-1) = -\frac{\pi}{12} \; radians$$

EXAMPLE: ALTITUDE ANGLE AT CHAMPAIGN

- We compute the *altitude angle* at Champaign using

$$sin\left(\beta(-1)\Big|_{295} \right)$$

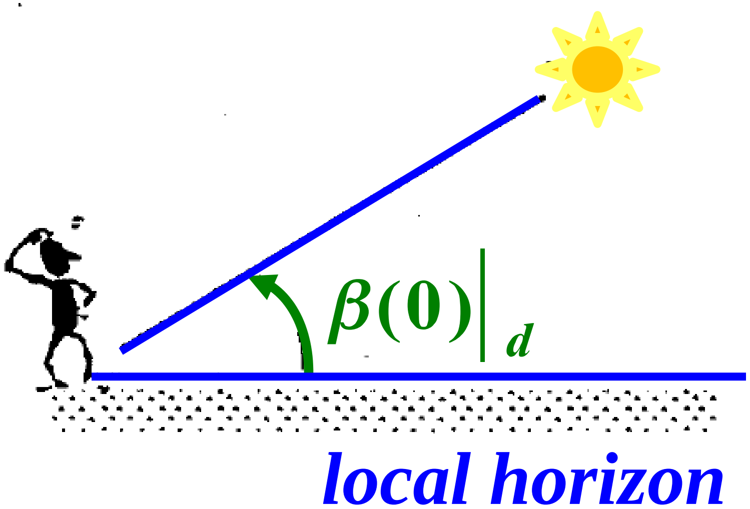
$$= \; cos(0.7)\, cos(-0.21)\, cos\left(-\frac{\pi}{12} \right) + sin(0.7)\, sin(-0.21)$$

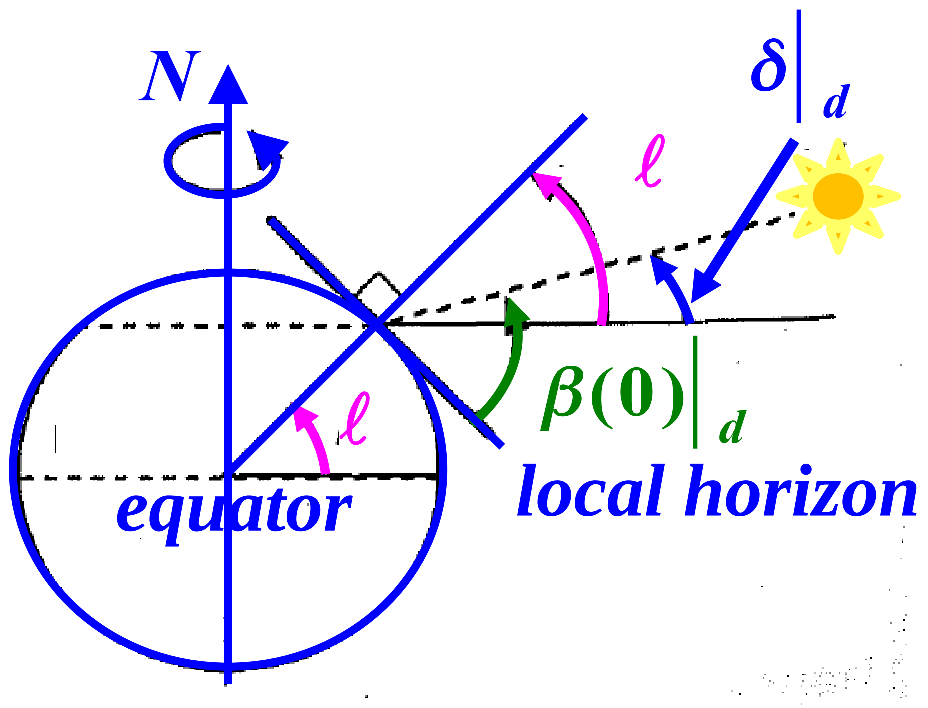
$$= \; 0.59$$

and so

$$\beta(-1)\Big|_{295} = \; sin^{-1}(0.59) = \; 0.623 \; radians$$

SPECIAL CASE: THE ALTITUDE ANGLE AT SOLAR NOON





SPECIAL CASE: ALTITUDE ANGLE AT SOLAR NOON

- The *altitude angle at solar noon* of day d satisfies

$$\sin\left(\beta(0)\Big|_{d}\right)$$

$$= \cos(\ell)\cos\left(\delta\Big|_{d}\right)\cos\left(\theta(0)\right) + \sin(\ell)\sin\left(\delta\Big|_{d}\right)$$

- However, we obtain a direct expression for $\beta(0)\Big|_{d}$

  from the geometric relationship

$$\beta(0)\Big|_{d} = \frac{\pi}{2} - \ell + \delta\Big|_{d} \quad \textit{radians}$$

EXAMPLE: ALTITUDE ANGLE AT SOLAR NOON

- We determine the altitude angle for Champaign at

  $\ell = 0.7$ *radians*, at *solar noon* on March 1 ($d = 60$)

- The solar declination angle is

$$\delta\Big|_{60} = 0.41\ sin\left[\frac{2\pi}{365}\left(60 - 81\right)\right] = -\ 0.15\ radians$$

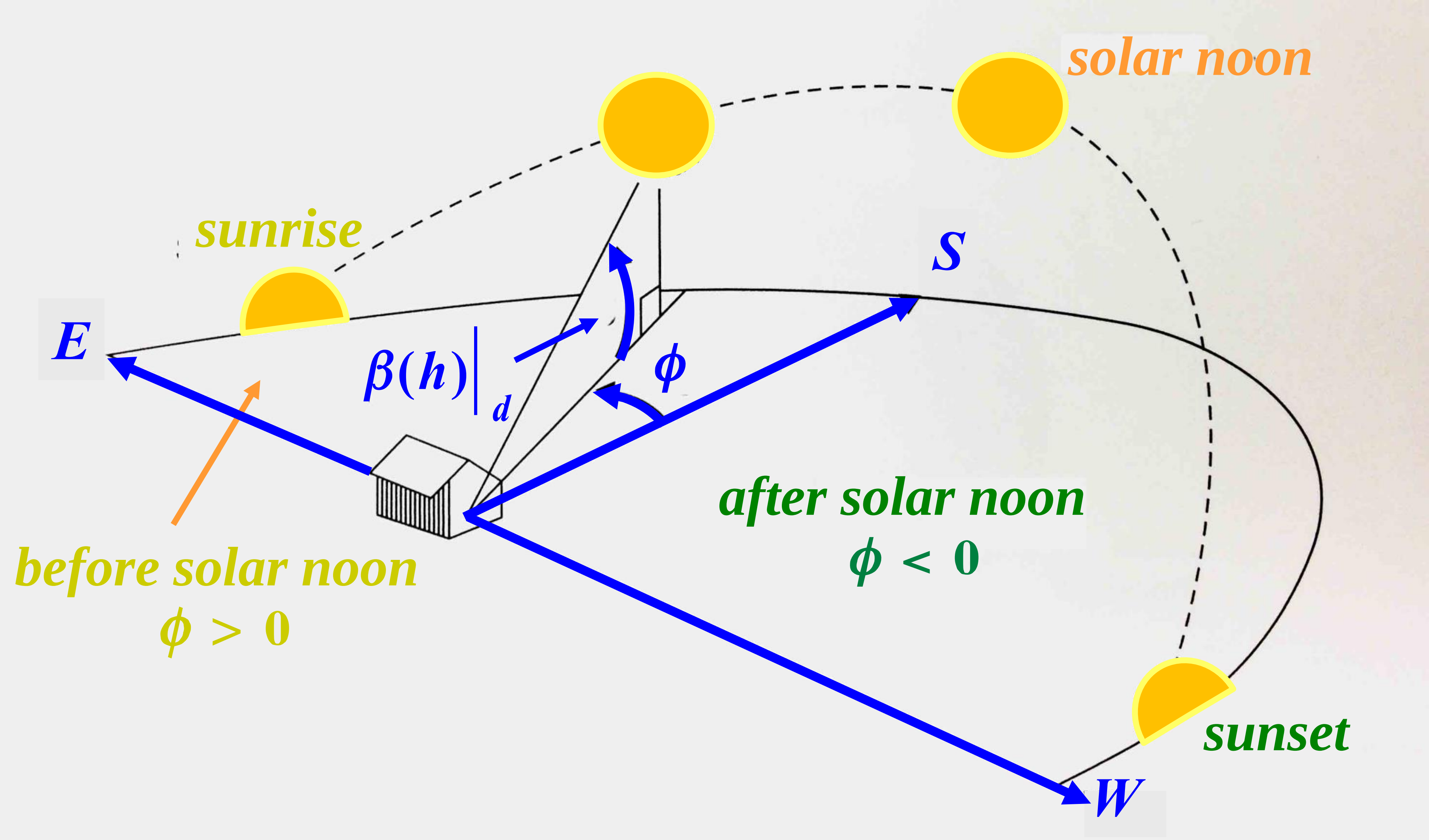
- The altitude angle at solar noon is

$$\beta(0)\Big|_{60} = \frac{\pi}{2} - \ell + \delta\Big|_{60} = 0.72\ radians$$

THE SOLAR AZIMUTH ANGLE

- *The solar azimuth angle* ϕ **in the** *Northern Hemisphere* **is defined as the angle between a due South line and the projection of the line of sight to the sun on the earth surface**
- **We use the convention that** ϕ **is** *positive* **when the sun is in the** *East* **– before solar noon – and** *negative* **when the sun is in the** *West* **– after solar noon**

THE SOLAR AZIMUTH ANGLE



THE SOLAR AZIMUTH ANGLE

- The equation for the *solar azimuth angle* $\phi(h)\big|_d$ is determined from the relationship

$$sin\left(\phi(h)\big|_d\right) = \frac{cos\left(\delta\big|_d\right)\, sin\left(\theta(h)\right)}{cos\left(\beta(h)\big|_d\right)}$$

- Since the sinusoidal function is subject to ambiguity because $sin\,x = sin\,(\pi - x)$, we need to

THE SOLAR AZIMUTH ANGLE

determine whether the azimuth angle is greater or

less than $\frac{\pi}{2}$:

if $\cos\left(\theta(h)\right) \geq \frac{\tan\left(\delta\big|_d\right)}{\tan(\ell)}$, then $\left|\phi(h)\big|_d\right| \leq \frac{\pi}{2}$;

else, $\left|\phi(h)\big|_d\right| > \frac{\pi}{2}$

EXAMPLE: WHERE IS THE SUN IN THE SKY

- **Determine the *altitude* and the *solar azimuth* angles at $3\ p.m.$ solar time in Champaign at the latitude $\ell = 0.7\ radians$ at the summer solstice on $d = 172$**

- **The solar declination is**

$$\delta\Big|_{172} = 0.41\ radians$$

- **The hour angle at $3\ p.m.$ solar time is**

$$\theta(-3) = -\frac{\pi}{4}$$

EXAMPLE: WHERE IS THE SUN IN THE SKY

- Then, we compute the altitude angle to be:

$$sin\left(\beta(-3)\Big|_{172}\right)$$

$$= cos(0.7)\, cos\left(0.41\right) cos\left(-\frac{\pi}{4}\right) + sin(0.7)\, sin\left(0.41\right)$$

$$= 0.75$$

- Then

$$\beta(-3)\Big|_{172} = 0.85 \;\; radians$$

EXAMPLE: WHERE IS THE SUN IN THE SKY

- The sine of the azimuth angle is obtained from

$$sin\left(\phi(-3)\Big|_{172} \right) = \frac{cos\left(0.41\right)\ sin\left(-\frac{\pi}{4}\right)}{cos\left(0.85\right)} = -0.9848$$

- Two possible values for the azimuth angle are

$$\phi(-3)\Big|_{172} = sin^{-1}(-\ 0.9848) = -\ 1.4\ radians$$

or

$$\phi(-3)\Big|_{172} = \pi - \ sin^{-1}(-\ 0.9848) = 4.54\ radians$$

EXAMPLE: WHERE IS THE SUN IN THE SKY

- Since

$$\cos\left(\theta(-3)\right) = 0.707 \quad \text{and} \quad \frac{\tan\left(\delta\big|_{172}\right)}{\tan(\ell)} = 0.515$$

- Then, we can see that

$$\cos\left(\theta(-3)\right) > \frac{\tan\left(\delta\big|_{172}\right)}{\tan(\ell)}$$

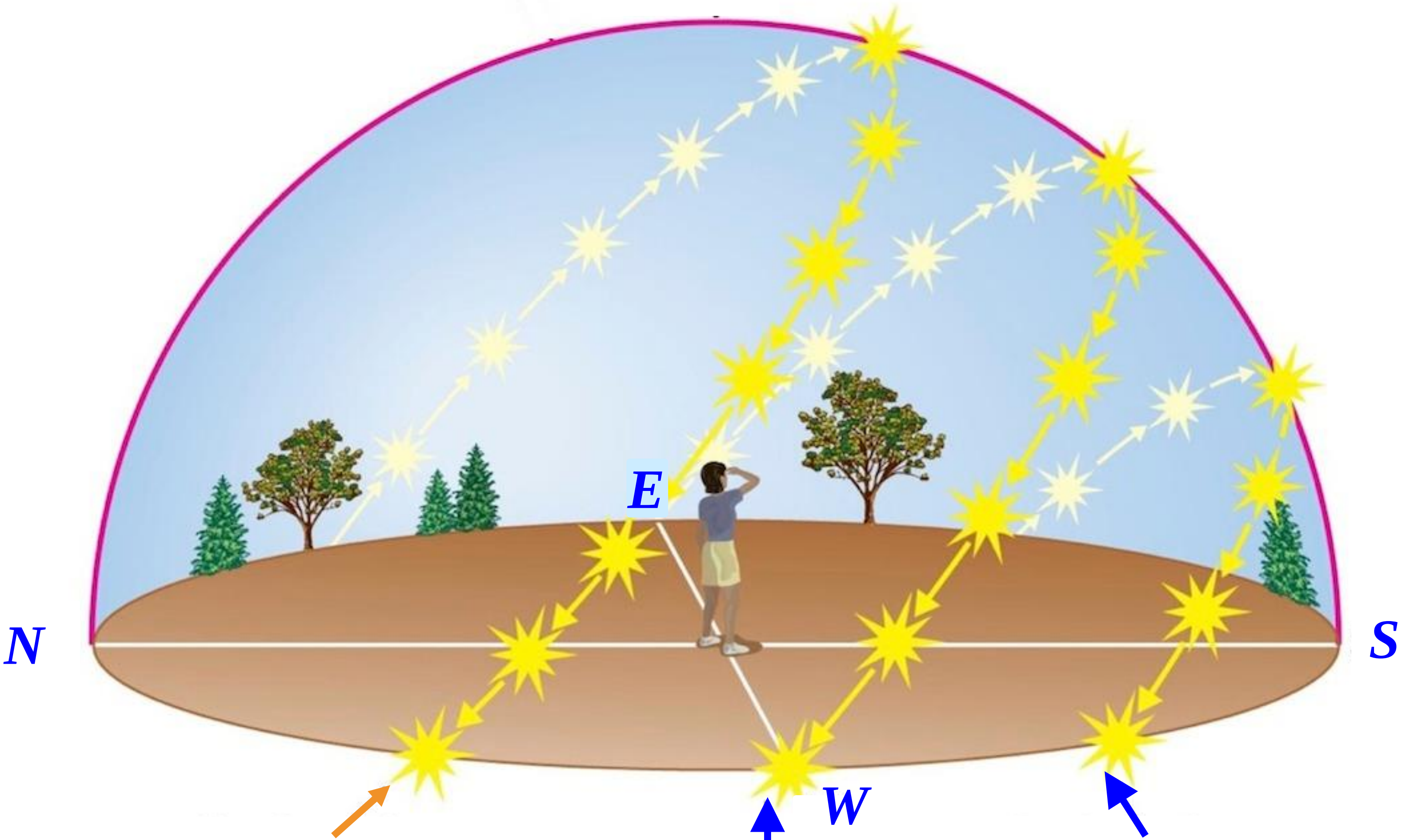
- Therefore,

$$\phi(-3)\big|_{172} = -\,1.4\ radians$$

IMPORTANCE OF THE ANALYSIS ON SUN'S POSITION IN THE SKY

- We are now equipped to determine the sun's position in the sky at any time and at any location
- To effectively design and analyze solar plants, the sun's position in the sky analysis has some highly impactful applications, including
  - to build ***sun path diagrams*** for shading analysis;
  - to determine sunrise and sunset times; and
  - to evaluate a solar panel's optimal ***position***

SUN PATH


E
N
S
W
sun's path at the summer solstice
sun's path at an equinox
sun's path on winter solstice

SUN PATH DIAGRAM

- The *sun path diagram* is a chart used to illustrate the continuous changes of sun's location in the sky at a given location over each day's hours
- The sun's position in the sky is found for any *hour* of the specified day *d* of the year from the *azimuth* and the *altitude angles* in the *sun path diagram* that correspond to that *hour*

SUNRISE AND SUNSET

- An important issue is the determination of the sunrise/sunset times since solar energy can be collected only during the *sunrise–to–sunset* hours
- We estimate the sunrise/sunset times from the equation used to compute the solar altitude angle, whose value is *zero at sunrise and sunset, i.e.,*

SUNRISE AND SUNSET

$$\sin\left(\beta(h)\Big|_d\right) = 0$$

- The relationship for the solar angle results in:

$$\cos(\theta(h)) = -\frac{\sin(\ell)\sin\left(\delta\Big|_d\right)}{\cos(\ell)\cos\left(\delta\Big|_d\right)} = -\tan(\ell)\tan\left(\delta\Big|_d\right)$$

- Now we can determine the sunrise solar hour angle $\kappa_+\Big|_d$ and the sunset hour angle $\kappa_-\Big|_d$ to be:

SUNRISE AND SUNSET

- The corresponding sunrise and sunset angles are

$$\kappa_{+}\Big|_{d} = \cos^{-1}\left(-\tan(\ell)\tan\left(\delta\Big|_{d}\right)\right)$$

$$\kappa_{-}\Big|_{d} = -\cos^{-1}\left(-\tan(\ell)\tan\left(\delta\Big|_{d}\right)\right)$$

so that the solar times for sunrise/sunset are at

$$12:00 - \frac{\kappa_{+}\Big|_{d}}{\pi/12} \quad \text{and} \quad 12:00 - \frac{\kappa_{-}\Big|_{d}}{\pi/12}$$

SUNRISE TIME IN CHAMPAIGN

- Champaign is located at $\ell = 0.7$ *radians*

- On October 22, the solar declination angle is – 0.21 *radians* and the sunrise solar hour angle is :

$$\kappa_{+}\Big|_{295} = \cos^{-1}\big(-\tan(0.7)\tan(-0.21)\big) = 1.39 \; radians$$

- The sunrise expressed in solar time is at

$$12:00 - \frac{1.39}{\pi / 12} = 6:27 \; a.m.$$

SOLAR TIME AND CIVIL TIME

- **So far, we used exclusively *solar time* measured with reference to solar noon in all our analysis of insolation and its impacts**
- **However, in our daily life we typically use *civil* or *clock time*, which measures the time to align with the earth's daily rotation over exactly 24 *hours***

SOLAR TIME AND CIVIL TIME

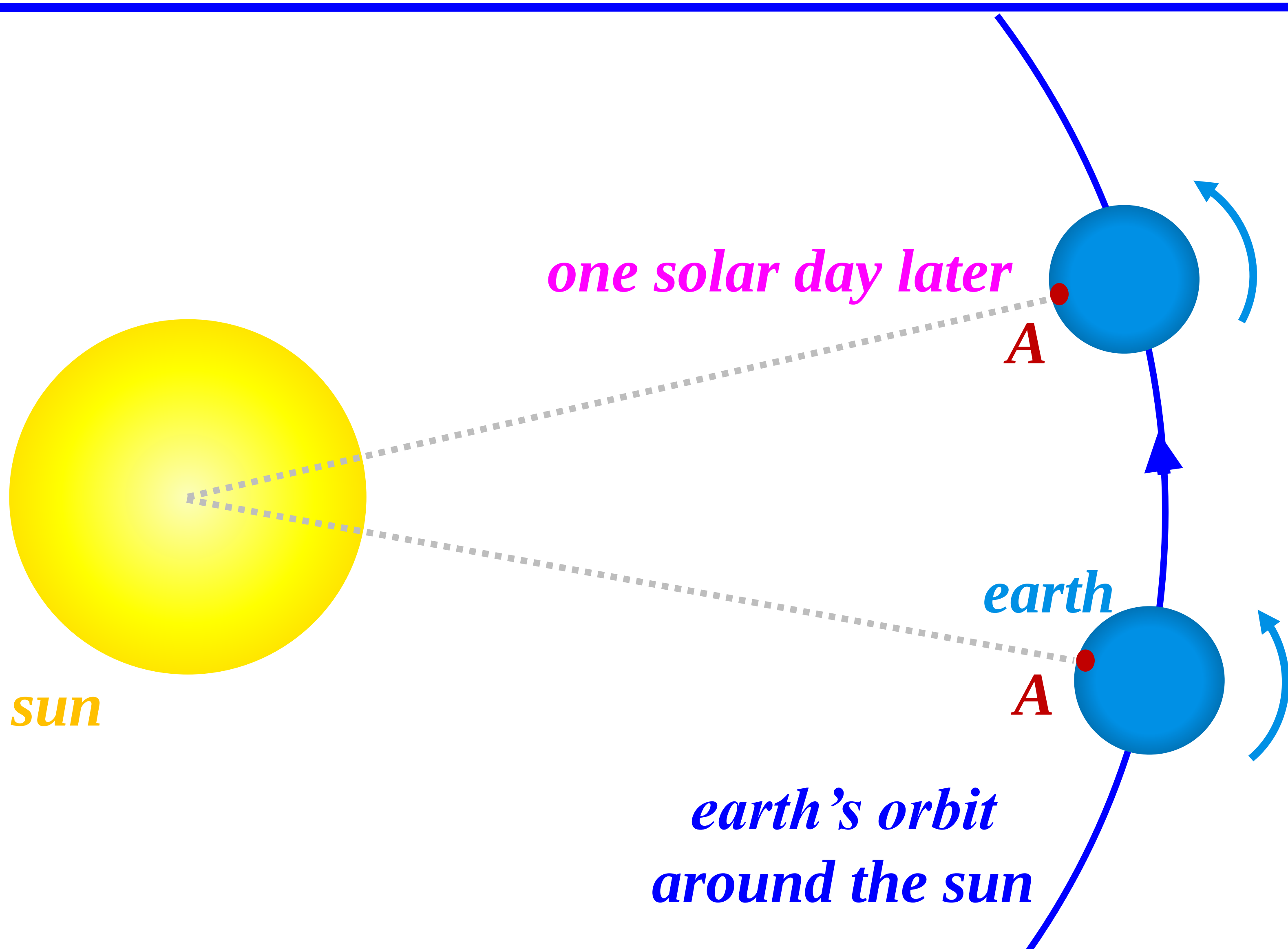
- The difference at a specified location on the earth surface between the *solar time* and the *civil time* arises from the earth's uneven movement along its elliptical orbit of the annual revolution around the sun and the deviation of the local time meridian from the actual locational longitude
- As such, two distinct adjustments are required in order to convert between *solar time* and *civil time*

SOLAR DAY AND 24–*HOUR* DAY

- **We examine the difference between a *solar day* and the corresponding *civil 24–hour* day**

- **We define a *solar day* as the time elapsed between two successive solar noons**

HOW LONG IS A SOLAR DAY



http://astronomy.nju.edu.cn/~lixd/GA/AT4/AT401/IMAGES/AACHCIR0.JPG

SOLAR DAY

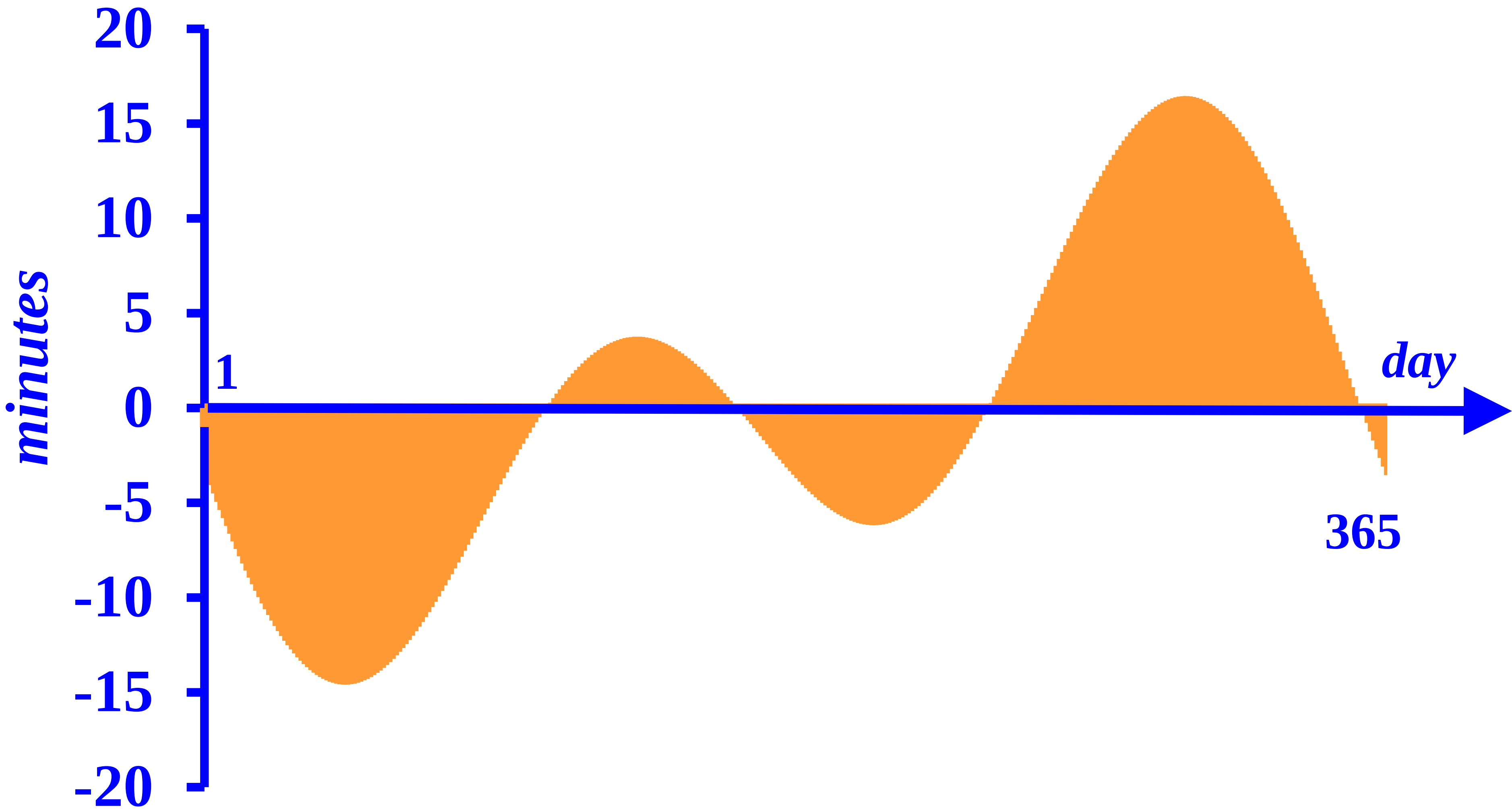
- The earth's elliptical orbit in its *revolution around the sun* results in a different duration of each solar day
- The difference between a solar day and a 24-*h* day is given by the deviation $e|_d$ in *minutes*

$$e|_d = 9.87\,sin\left(2(b|_d)\right) - 7.53\,cos(b|_d) - 1.5\,sin\,(b|_d),$$

where,

$$b|_d = \frac{2\pi}{364}\left(d - 81\right) \quad radians$$

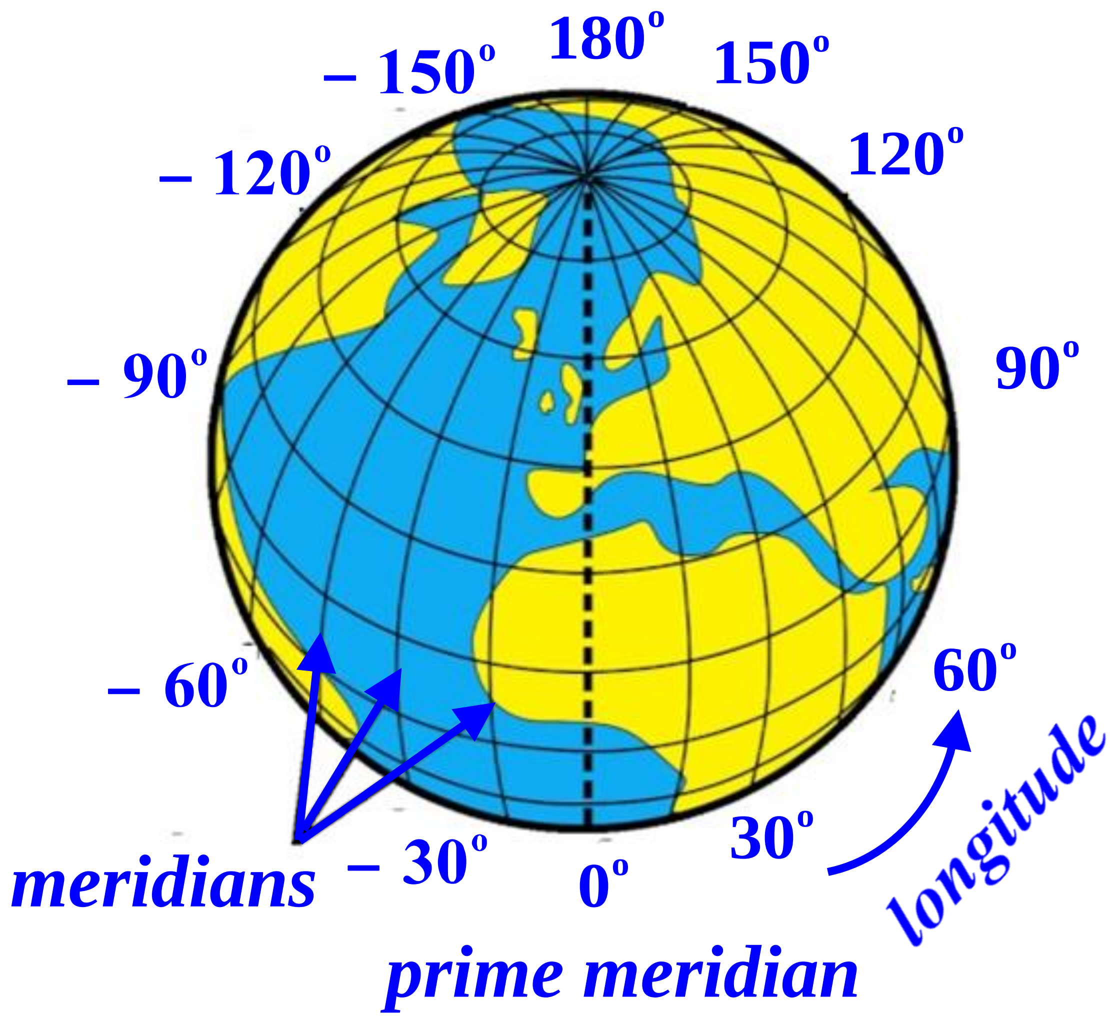
DIFFERENCE BETWEEN A SOLAR AND A 24–*HOUR* DAY OVER A YEAR



LOCAL TIME MERIDIAN AND LOCAL LONGITUDE

- There are *24 time zones* to cover the earth, each with its own time meridian with the 15° longitude gap between the time meridians of two adjacent time zones
- The second adjustment deals with the longitude correction for the fact that the clock time at any location within each time zone is defined by its local time meridian which differs from the time zone meridian for that location

LONGITUDES AND MERIDIANS



a meridian is an imaginary arc on the earth's surface that connects the North and the South poles

LOCAL TIME MERIDIAN AND LOCAL LONGITUDE

- For every degree of longitude difference, the solar time difference corresponds to

$$\frac{24\ hour \cdot 60\ m\ /\ hour}{360^{o}} = 4\ \frac{m}{degree\ longitude}$$

- The time adjustment in *minutes* due to the *degree longitude difference* between the specified location and the local time meridian is the product of 4 times the longitude difference in degrees
- The time adjustment is *positive* (*negative*) *if the local meridian is to the east* (*west*) *of the local time meridian*

LOCAL TIME MERIDIAN AND LOCAL LONGITUDE

- The sum of the adjustment $e|_d$ and the longitude correction results in:

$$solar\ time = clock\ time + e|_d + 4 \times \frac{180}{3.14} \times (local\ longitude - local\ time\ meridian)$$

- This relationship allows the conversion between *solar time* and *civil time* at any location on earth

EXAMPLE: SOLAR TIME AND CLOCK TIME

- **Find the clock time at *solar noon* in *Springfield*, *IL*, on July 1, the 182nd day of the year**

- **For d = 182, we have**

$$b\Big|_{182} = \frac{2\pi}{364}\left(182 - 81\right) = 1.72\ radians$$

$$e\Big|_{182} = 9.87\,sin\,(2\times 1.72) - 7.53\,cos\,(1.72) - 1.5\,sin\,(1.72)$$

$$= -\ 3.51\ minutes$$

EXAMPLE: SOLAR TIME TO CLOCK TIME

- **For *Springfield, IL*, with longitude – 1.55 *radians* and *local time meridian* – $\pi/2$, the clock time is given by:**

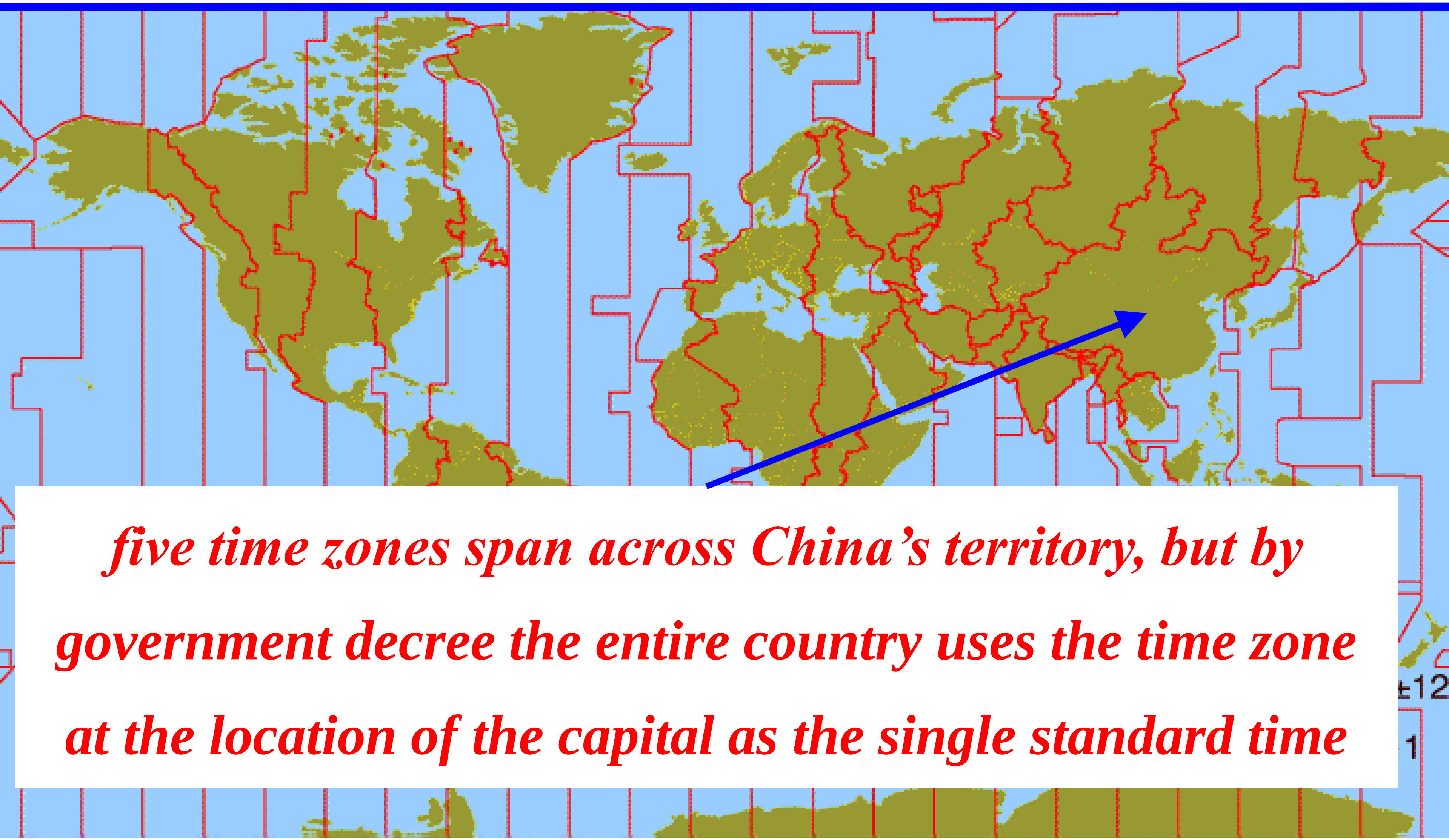
$$solar\ time\ -\ e|_d\ -\ 4 \times \frac{180}{\pi} \times$$

$$(local\ longitude\ -\ local\ time\ meridian)$$

$$=\ solar\ noon - (-3.51) - \frac{720}{\pi}\big((-1.5648) - (-\pi/2)\big)$$

$$=\ 12:02\ p.m.(CST) = 1:02\ p.m.(CDT)$$

WORLD TIME ZONE MAP



five time zones span across China's territory, but by government decree the entire country uses the time zone at the location of the capital as the single standard time

Source*: *http://www.physicalgeography.net/fundamentals/images/world_time2.gif

CONCLUSION

- **With the conversion scheme between the solar and clock times, the analysis of solar issues, *e.g.*, the expression of sunrise/sunset on civil time basis, makes the results far more meaningful for use in daily life**
- **Such a translation renders the analysis results to be much more concrete for all applications**