
ECE 333 – GREEN ELECTRIC ENERGY

6. Limits on Conversion of Wind Into Electricity

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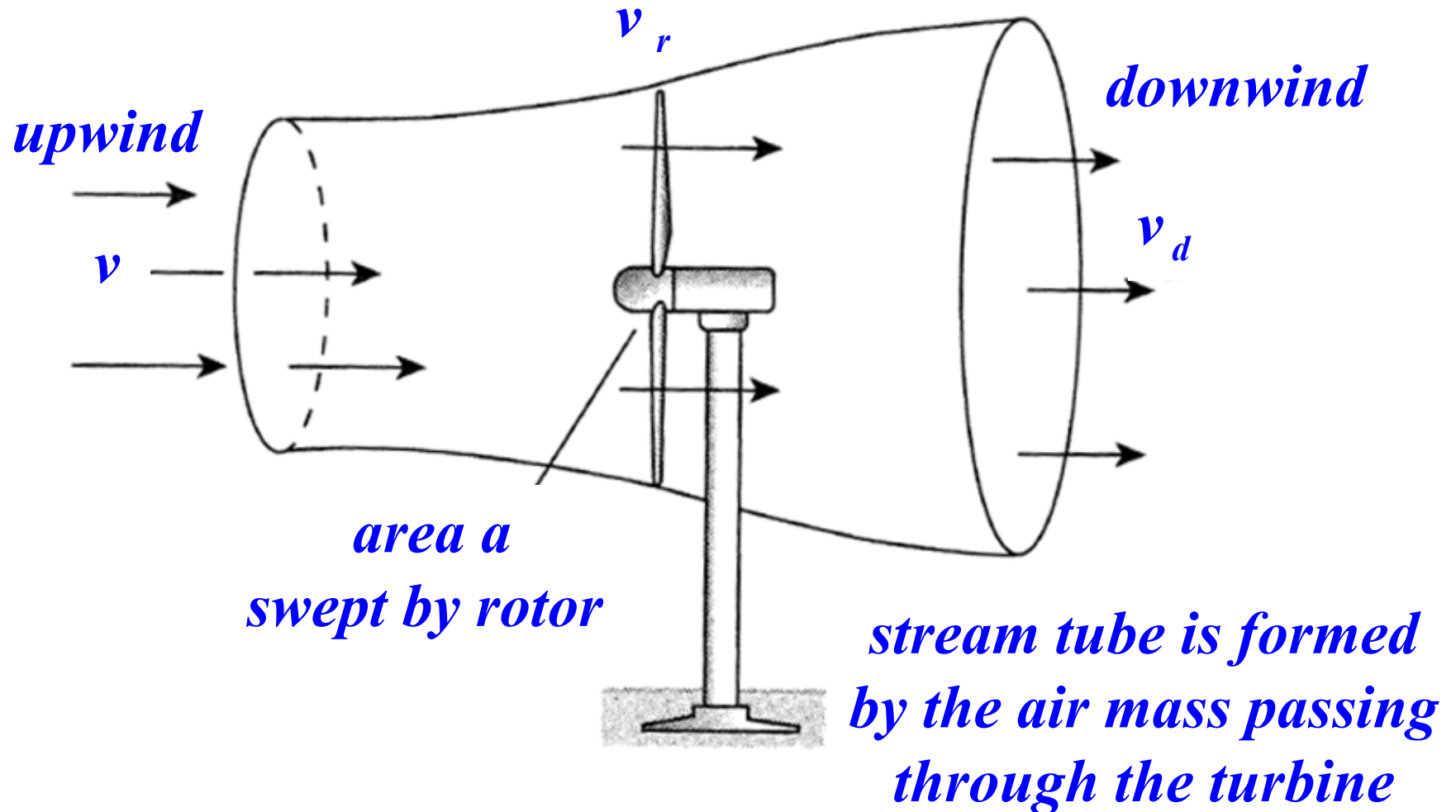
CONVERSION OF WIND INTO ELECTRICITY

- ❑ We analytically characterize the power in wind as a cubic function of the wind speed v
- ❑ The wind energy is in the form of kinetic energy, which is extracted from the wind and used to rotate the generator shaft mounted in the nacelle
- ❑ We examine the constraint – the so-called *Betz limit* – that restricts the ability of a wind turbine to convert kinetic energy into mechanical energy to spin the turbine generator shaft

THE BETZ LIMIT

- The limit was derived in 1919 by Albert Betz, a German physicist**
- We consider the wind as it passes through a wind turbine and we examine the wind stream**
- We explain the conceptual basis on the limit of the conversion of wind into electricity using the diagram below**

THE BETZ LIMIT



THE BETZ LIMIT

- ❑ Clearly, the turbine cannot extract all the kinetic energy in the wind because that implies that the air would have to stop completely after passing the turbine – an impossible situation **since it would prevent all the continuing wind to pass through the rotor**
- ❑ Furthermore, the downwind velocity v_d *cannot equal v* since that would imply that no energy is extracted by the turbine

THE BETZ LIMIT

- Betz formulated the relationship to determine the maximum mechanical power obtainable from wind
- We focus on what happens with the wind as it passes through the plane of the rotor blades at velocity v_r , with

$$v_d < v_r < v,$$

where, we explicitly take into account that as the wind mass of air goes through the stream tube, the downwind speed is lower than the upwind speed

THE BETZ LIMIT

- The conservation of energy implies that

$$\begin{array}{l} \textit{kinetic energy} \\ \textit{upwind} \end{array} = \begin{array}{l} \textit{kinetic energy} \\ \textit{downwind} \end{array} + \begin{array}{l} \textit{energy extracted} \\ \textit{by blade rotor} \end{array}$$

- Therefore, as the mass flow rate $\frac{dm}{dt}$ throughout the stream tube remains unchanged, the power extracted by the rotor blades is

$$\begin{aligned} P_r &= \frac{d}{dt} \left(\begin{array}{l} \textit{kinetic} \\ \textit{energy} \\ \textit{upwind} \end{array} - \begin{array}{l} \textit{kinetic} \\ \textit{energy} \\ \textit{downwind} \end{array} \right) \\ &= \frac{1}{2} \frac{dm}{dt} \left(v^2 - v_d^2 \right) \end{aligned}$$

THE BETZ LIMIT

- Now, we can determine $\frac{dm}{dt}$ anywhere in the stream tube and at the rotor blade plane since

$$\frac{dm}{dt} = \rho a v_r$$

and we assume that

$$v_r = \frac{v + v_d}{2}$$

and so

$$P_r = \frac{1}{2} \rho a \left(\frac{v + v_d}{2} \right) \left(v^2 - v_d^2 \right)$$

THE BETZ LIMIT

- We introduce the ratio λ defined by

$$v_d = \lambda v$$

so that the expression for p_r becomes

$$p_r = \frac{1}{2} \rho a v \left(\frac{1 + \lambda}{2} \right) v^2 (1 - \lambda^2)$$

$$= \underbrace{\frac{1}{2} \rho a v^3}_{\text{power in the wind}} \underbrace{\frac{1}{2} (1 + \lambda)(1 - \lambda^2)}_{\text{fraction extracted}}$$

power in the wind *fraction extracted*

THE BETZ LIMIT

- We can think of the fraction of power extracted by the rotor as the efficiency η_r of the rotor

$$\eta_r = \frac{1}{2}(1 + \lambda)(1 - \lambda^2)$$

so that

$$P_r = \frac{1}{2} \rho a v^3 \eta_r$$

THE BETZ LIMIT

- To determine the maximum rotor efficiency, we evaluate the derivative of p_r w.r.t. λ

$$\begin{aligned}\frac{dp_r}{d\lambda} &= \frac{1}{2} \rho a v^3 \frac{1}{2} \left[(1 + \lambda)(-2\lambda) + (1)(1 - \lambda^2) \right] \\ &= \frac{1}{4} \rho a v^3 \left[-2\lambda^2 - 2\lambda + 1 - \lambda^2 \right] \\ &= \frac{1}{4} \rho a v^3 \left[1 - 2\lambda - 3\lambda^2 \right] \\ &= \frac{1}{4} \rho a v^3 \left[(1 + \lambda)(1 - 3\lambda) \right]\end{aligned}$$

THE BETZ LIMIT

□ We set $\frac{dp_r}{d\lambda}$ to be 0 and we solve for λ

□ The only physically meaningful solution is

$$\lambda = \frac{1}{3},$$

i.e., the efficiency is maximized when the ratio of

v_d to v is 1/3 so that

THE BETZ LIMIT

$$\eta_r = \frac{1}{2} \left(1 + \frac{1}{3} \right) \left(1 - \frac{1}{3^2} \right) = \frac{16}{27} = 59.3 \%$$

□ This optimal theoretical efficiency – better known

as the *Betz efficiency* – cannot be higher than

59.3 %; this value is the essence of the *Betz limit*

THE BETZ LIMIT

- The *Betz* limit implies that even under ideal conditions less than 60 % of the power in wind can be extracted; indeed, in actual systems, the best that is attainable is, typically, below 50 % – in other words, at most half of the energy in wind can be converted into mechanical energy to rotate the generator shaft

TIP SPEED RATIO

- The tip speed of the rotor is a function of the rate of rotation of the rotor specified by its *r.p.m.*: in each revolution of the rotor, the tip traverses a distance πd and so the tip speed is $(\pi d)(r.p.m.)$
- A convenient way to express rotor efficiency is in terms of the tip speed ratio τ , where

$$\tau = \frac{\text{rotor tip speed}}{v} = (r.p.m.) \cdot \frac{\text{min}}{60 \text{ sec}} \cdot \frac{\pi d}{v}$$

TIP SPEED RATIO

- ❑ Studies indicate that modern turbines attain maximum efficiency for $4 \leq \tau \leq 6$: the tip of the blade moves 4 – 6 times faster than the wind speed
- ❑ It follows that for maximum efficiency it is desirable that turbine blades change their speed as wind speed changes – as is the case in the so-called *variable speed generators*

INTERPRETATION OF η_r

- Betz's law states that the maximum power that we can extract from wind is

$$P_r = \left(\frac{1}{2} \rho a v^3 \right) 0.593$$

- Engineers define the efficiency as the ratio of the output to the input quantity

$$\eta_r = \frac{P_{out}}{P_{in}}$$

and so the natural question is what is P_{in}

INTERPRETATION OF η_r

- A convenient way to think about p_{in} is that p_{in} is the power in the wind prior to the installation of a turbine: absent the turbine, $v_r = v$ and so

$$p_{in} = \frac{1}{2} \rho a v^3$$

- The Betz efficiency determines the limit on the conversion of p_{in} into mechanical power to rotate the generator shaft

WIND TURBINE GENERATORS

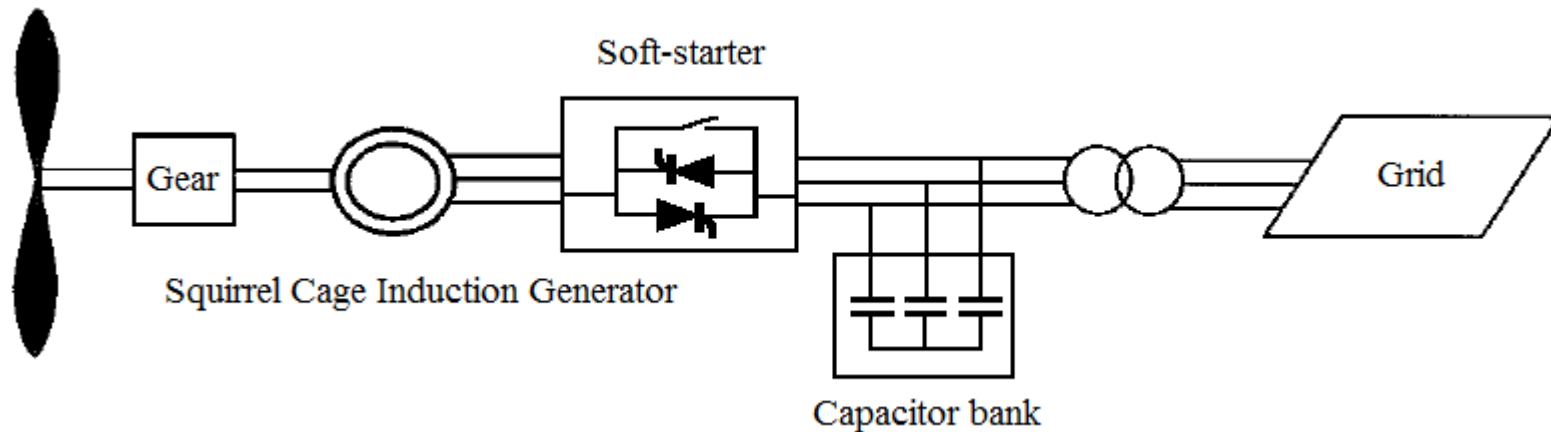
- ❑ The wind turbine generators or wind energy conversion systems may be classified into two principal categories
 - variable–speed rotors
 - fixed–speed rotors
- ❑ The variable–speed turbines have the ability to take advantage of the fact that wind speed varies

WIND TURBINE GENERATORS

and adjust the rotor speed so as to *optimally match* wind speed

- ❑ The fixed-speed rotor generators are simpler but cannot operate at optimal efficiency; moreover, the stresses from the rapidly varying wind speeds, typically, require sturdier design of such turbines

WECS TYPE A

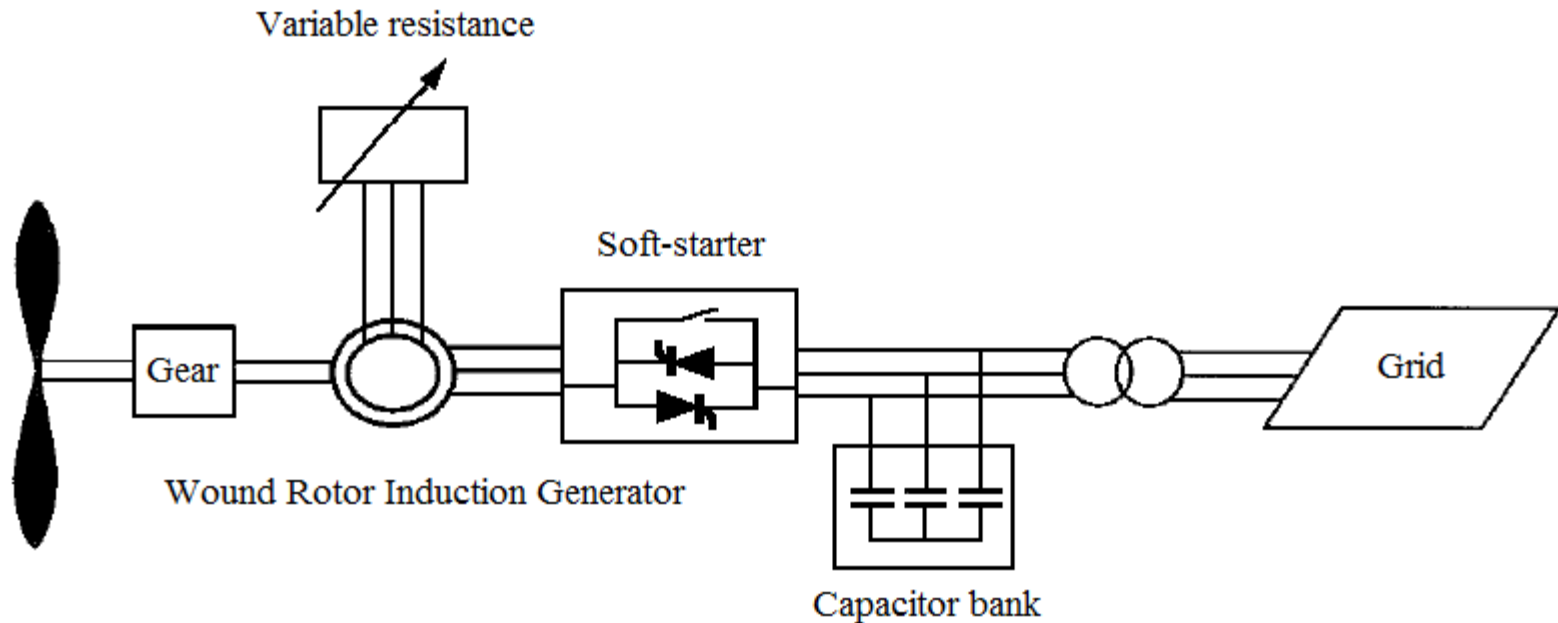


- ❑ The turbine design uses an induction generator connected to a **fixed-speed wind turbine**
- ❑ This design needs two additional components for grid connection:

WECS TYPE A

- a **soft-starter** to decrease current transients during startup phase
- a **capacitor bank** to compensate for reactive power
- As a result of the capacitor bank, the generator can work close to a zero value generation or consumption of reactive power
- Unfortunately, such capacitive compensation **fails to provide** flexible reactive power control by the wind turbine

WECS TYPE B



□ The *Vestas*-developed *type B WECS* generator is

designed to work with a **limited variable speed**

WECS TYPE B

wind turbine

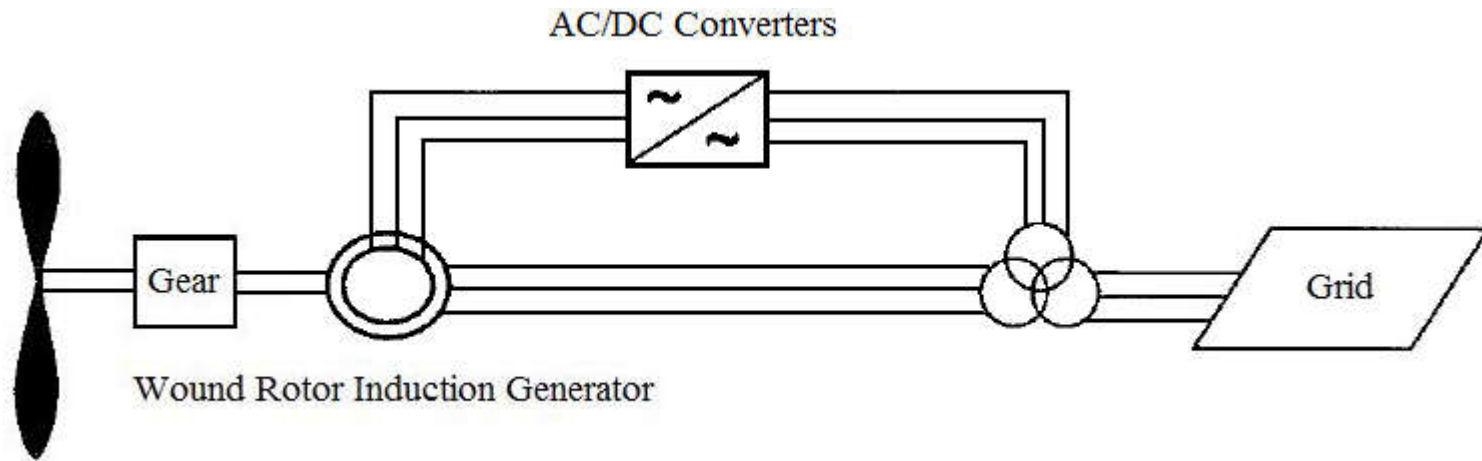
- The turbine uses the variable resistor in the rotor

to control the real power output

- The capacitor bank and soft-starter device roles

are analogous to those in the the *type A* design

WECS TYPE C

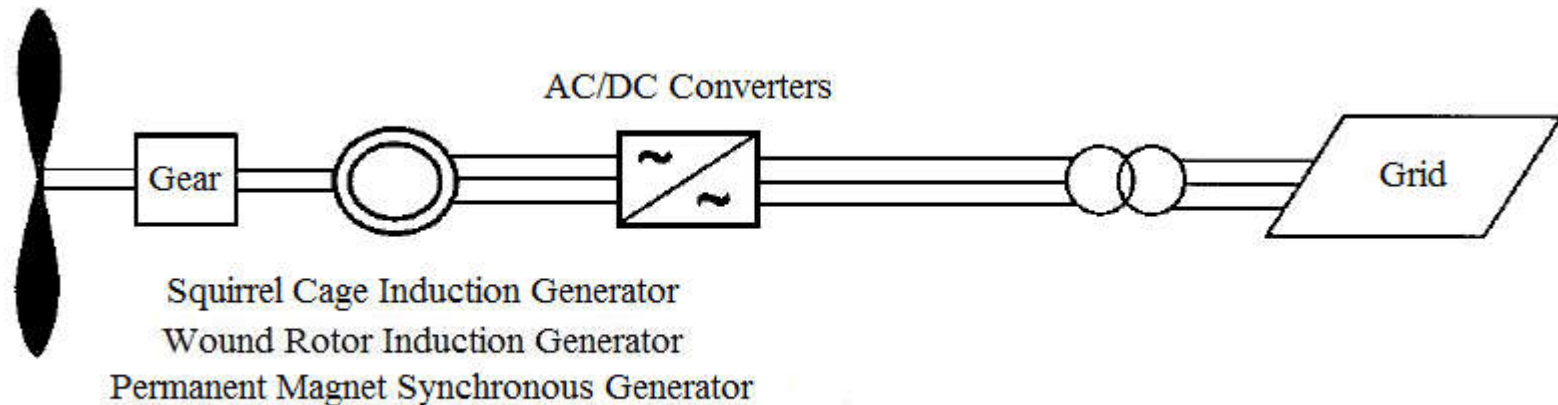


- ❑ This design uses two AC/DC converters rated at 25 % of total generator power with a capacitor between them to control the *WECS*
- ❑ The wound rotor induction generator topology is also known as a **doubly fed induction generator (DFIG)**

WECS TYPE C

- ❑ The term “doubly” comes from the fact that the rotor winding is not short-circuited – as in the classical “singly-fed” induction machine – but a voltage is induced from the rotor – side converter
- ❑ Depending on the operating scheme, they can keep a constant value of reactive power or keep the terminal voltage constant
- ❑ The *WECS type C* is the **most widespread of all wind turbines on the market**

WECS TYPE D



- ❑ The *type D* design uses a **full-scale frequency converter** with different types of generators
- ❑ The most common implementation is the **permanent magnet synchronous generator (*PMSG*)**

WECS TYPE D

- ❑ This design allows **full control over active and reactive power production** and has a high wind energy extraction value
- ❑ Full power control improves power and frequency stability in the grid and reduces the short circuit power
- ❑ Most *type D* designs **do not need a gearbox** – a key advantage of *type D WECS*

WIND TURBINE CLASSIFICATION

