Name Solution
(Print Name)

Section: (circle one) 10 MWF (Sauer) 2 MWF (Sauer)

ECE430 Final Exam, Fall 2005
Friday, Dec 16, 2005, 1:30 – 4:30 PM

One sheet (2-sided) provided

Problem 1 _______

Problem 2 _______

Problem 3 _______

Problem 4 _______

Problem 5 _______

Problem 6 _______

TOTAL: _______

Receiving or giving aid in a final examination is a cause for dismissal from the University

Problem 1 (25 pts.)

Four single-phase loads are connected in parallel across a 60 Hz, 7.2kV source. The nature of the four loads at the rated voltage are described below:

Load #1: Line current is 8 Amps at 0.9 power factor lag
Load #2: Line current is 12 Amps at 0.8 power factor lag
Load #3: Line current is 10 Amps at 0.85 power factor lead
Load #4: Line current is 7 Amps at unity power factor

Calculate the following:

a) The magnitude of the total single-phase source ~~current~~
b) The total complex power consumed by the three loads four
c) The minimum total single-phase source current magnitude you could achieve using power factor correction capacitors

Now assume that the total complex power you computed in part b) above could be divided up and arranged in a 3-phase balanced configuration and served from a 3-phase source of 12.5kV (line-line).

Calculate the following:

d) The magnitude of the line current of the 3-phase source
e) The minimum line current magnitude you could achieve using power factor correction capacitors

b) $\bar{S}_T = 7{,}200 \times 8 \angle 26^\circ + 7200 \times 12 \angle 37^\circ + 7200 \times 10 \angle -32^\circ$

$+ 7200 \times 7 = 51{,}770 + j25{,}250$
$69{,}000 + j52{,}000$
$61{,}059 - j38{,}154$
$50{,}400$
$\boxed{232{,}229 + j39{,}096} = 235{,}496 \angle 9.6^\circ$

a) $|\bar{S}_T| = 235{,}496 = 7200\, I$ $\quad \boxed{I = 32.7\text{ A}}$

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c) $232{,}229 = 7200\, I_{min}$ $\quad \boxed{I_{min} = 32.3\text{A}}$

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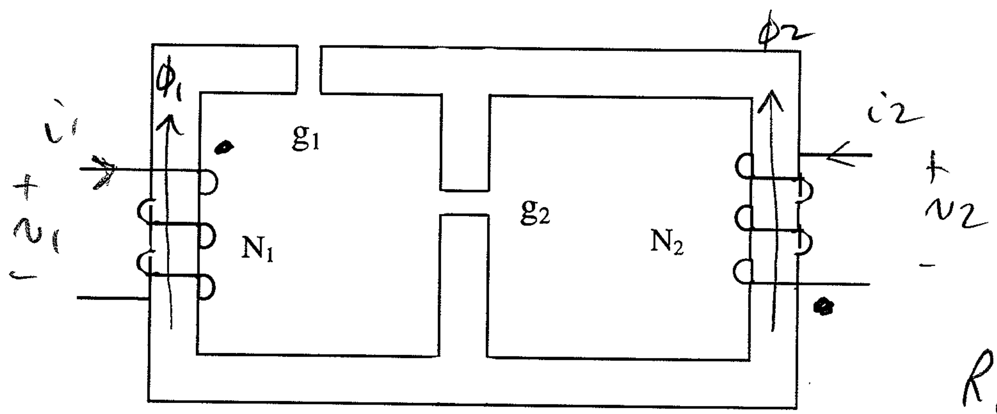
d) $235{,}496 = \sqrt{3} \times 12{,}500\, I_\ell$ $\boxed{I_\ell = 10.9\,A}$

e) $232{,}229 = \sqrt{3} \times 12{,}500\, I_{\ell_{min}}$ $\boxed{I_{\ell_{min}} = 10.7\,A}$

Problem 2 (25 pts.)

A multi-coil magnetic circuit is illustrated in the diagram below. The cross-sectional area of the magnetic core is A everywhere. The permeability of the magnetic core is infinite. There are two air gaps with lengths g_1 and g_2 as shown and permeability μ_o. You may neglect fringing. You may define the two voltages and two currents any way that you like.

a) Put the polarity "dots" on the proper side of each coil
b) Find λ_1 and λ_2 in terms of your currents i_1 and i_2 where $v_1 = d\lambda_1/dt$ and $v_2 = d\lambda_2/dt$
c) Find an expression for the force that will try to close gap g1 when the circuits are energized.
d) Find an expression for the force that will try to close gap g2 when the circuits are energized.

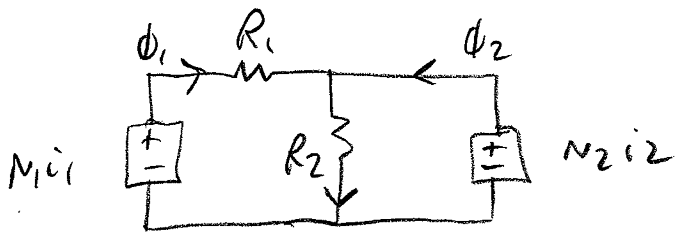


$$R_1 = \frac{g_1}{\mu_0 A}$$

$$R_2 = \frac{g_2}{\mu_0 A}$$

b) $\lambda_1 = N_1\phi_1$

$\lambda_2 = N_2\phi_2$



$$(\phi_1+\phi_2)R_2 = N_2 i_2$$

$$-N_1 i_1 + R_1\phi_1 + N_2 i_2 = 0 \qquad \phi_1 = \frac{N_1 i_1 - N_2 i_2}{R_1}$$

$$\phi_2 = \frac{N_2 i_2}{R_2} - \phi_1 = \frac{N_2 i_2}{R_2} + \frac{N_2 i_2}{R_1} - \frac{N_1 i_1}{R_1}$$

$$\lambda_1 = \frac{\mu_0 A N_1^2}{g_1} i_1 - \frac{\mu_0 A N_1 N_2}{g_1} i_2$$

$$\lambda_2 = -\frac{\mu_0 A N_1 N_2}{g_1} i_1 + \left(\frac{\mu_0 A N_2^2}{g_2} + \frac{\mu_0 A N_2^2}{g_1}\right) i_2$$

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c) $$w_m' = \frac{\mu_0 A N_1^2}{2 g_1} i_1^2 - \frac{\mu_0 A N_1 N_2 i_1 i_2}{g_1} + \left(\frac{\mu_0 A N_2^2}{2}\right)\left(\frac{1}{g_2} + \frac{1}{g_1}\right) i_2^2$$

$$f_1^e = \frac{\partial w_m'}{\partial g_1} = -\frac{\mu_0 A N_1^2 i_1^2}{2 g_1^2} + \frac{\mu_0 A N_1 N_2}{g_1^2} i_1 i_2 - \frac{\mu_0 A N_2^2}{2 g_1^2} i_2^2$$

d) $$f_2^e = \frac{\partial w_m'}{\partial g_2} = -\frac{\mu_0 A N_2^2 i_2^2}{2 g_2^2}$$

Problem 3 (25 pts):

a) If $\lambda = \frac{i^2}{x+2}$, find both $W_m'(i,x)$, $W_m(i,x)$, and then f^e at x =0.1m and i = 10A.

$$W_m' = \frac{1}{3}\frac{i^3}{(x+2)} \qquad W_m = \frac{i^3}{x+2} - \frac{1}{3}\frac{i^3}{(x+2)} = \frac{2}{3}\frac{i^3}{(x+2)}$$

$$f^e = -\frac{1}{3}\frac{i^3}{(x+2)^2} \qquad f^e\Big|_{\substack{x=.1\\ i=10}} = -\frac{1}{3}\frac{10^3}{2.1^2} = -76 \text{ N}$$

b) Use Euler's method with time step Δt = 0.1s to compute x(t) and v(t) at t =0.1 and 0.2s in the following system

$$\frac{dx}{dt} = v \qquad x(0) = 0.5$$

$$\frac{dv}{dt} = 1 - xv + x^2 \qquad v(0) = 0$$

$$x(.1) = 0.5 + v(0) \times .1 = 0.5 \text{ m}$$

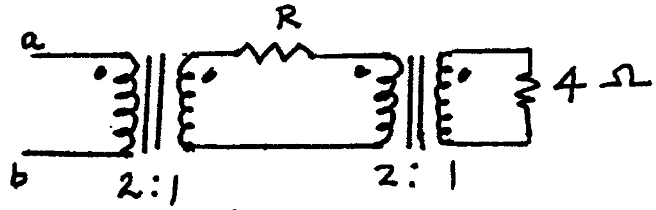
$$v(.1) = 0 + (1 - x(0)v(0) + x(0)^2) \times .1 = 0.125 \text{ m/s}$$

$$x(.2) = 0.5 + v(.1) \times .1 = 0.5 + .0125 = 0.5125 \text{ m}$$

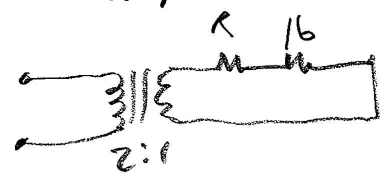
$$v(.2) = 0.125 + (1 - x(.1)v(.1) + x(.1)^2) \times .1$$

$$= 0.125 + (1 - .5 \times .125 + 0.5^2) \times .1 = 0.244 \text{ m/s}$$

c) Input resistance at “ab” is $100\Omega\,(R>0)$. The value of R is 9.



Ideal transformers

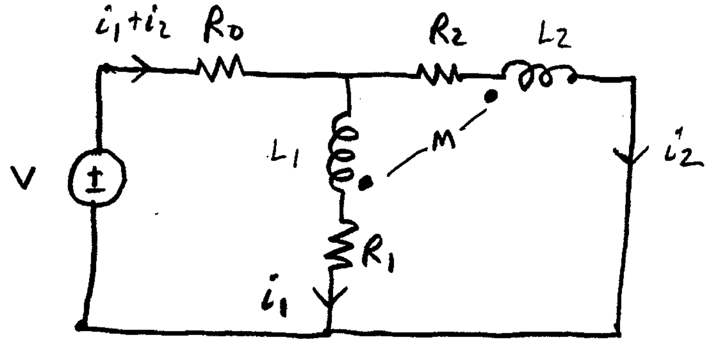


$$100 = 4R + 64$$
$$4R = 36 \qquad R = 9$$

d) Write the two loop equations for the circuit shown

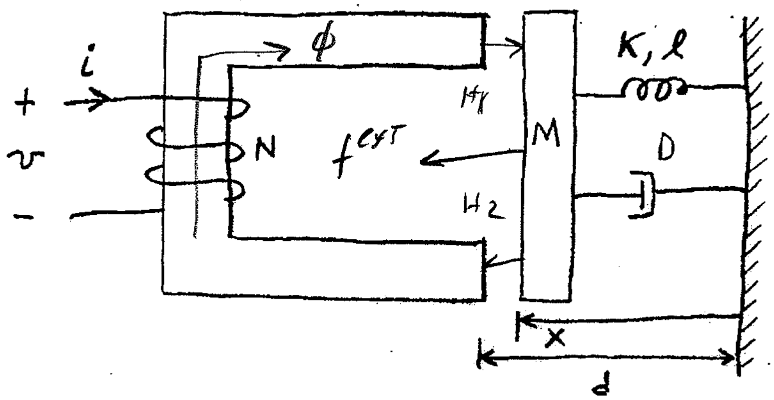
Left $-V + R_0(i_1+i_2) + L_1\frac{di_1}{dt} - M\frac{di_2}{dt} + R_1 i_1 = 0$

Right $0 + R_2 i_2 + L_2\frac{di_2}{dt} - M\frac{di_1}{dt} - R_1 i_1 - L_1\frac{di_1}{dt} + M\frac{di_2}{dt} = 0$



Problem 4 (25 pts.)

Given the electromechanical relay shown below with the typical parameters as indicated. The rectangular pieces are iron with infinite permeability and cross sectional area A. The spring zero-force distance (script ℓ) is calibrated to the distance x.



a) Find an expression for the force of electrical origin acting on the mass M in terms of the various parameters in the figure and the coil current.
b) Write the complete dynamic model for this device in state space form (3 ordinary differential equations). Assume inputs are voltage and external force. Add a resistor to the coil.
c) If the mass is held fixed at an initial position x_0 while the coil is energized to a current value of i_0 and a flux linkage value of λ_0, find an expression for the energy transferred from the electrical system into the coupling field.
d) If the flux linkage is then held constant at λ_0 while the mass is moved from position x_0 to x_1, find an expression for the energy transferred from the mechanical system into the coupling field.

a) $H_1(d-x)+H_2(d-x)=Ni$ $\qquad \mu_0 H_1 A-\mu_0 H_2 A=0$

$H_1=H_2=\dfrac{Ni}{2(d-x)}$ $\qquad \lambda=N\phi=N\mu_0 A\dfrac{Ni}{2(d-x)}=\dfrac{\mu_0 A N^2}{2(d-x)}i$

$W_m'=\dfrac{\mu_0 A N^2}{4(d-x)}i^2$ $\qquad f^e=\dfrac{\mu_0 A N^2 i^2}{4(d-x)^2}=\dfrac{\lambda_0^2}{\mu_0 A N^2}$

b) $v=iR+\dfrac{d\lambda}{dt}=iR+\dfrac{\mu_0 A N^2}{2(d-x)}\dfrac{di}{dt}+\dfrac{\mu_0 A N^2 i v}{2(d-x)^2}$

$\dfrac{dx}{dt}=v$ $\qquad m\dfrac{dv}{dt}=-k(x-\ell)-Dv+f^{ext}+\dfrac{\mu_0 A N^2 i^2}{4(d-x)^2}$

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c) $$\underset{a-b}{EFE} = \int_0^{\lambda_0} i\Big|_{x=x_0} d\lambda = \int_0^{\lambda_0} \frac{2(d-x_0)\lambda}{\mu_0 A N^2} d\lambda$$

$$= \frac{(d-x_0)\lambda_0^2}{\mu_0 A N^2} = \frac{(d-x_0)}{\mu_0 A N^2}\left(\frac{\mu_0 A N^2 i_0}{2(d-x_0)}\right)^2$$

$$= \frac{\mu_0 A N^2 i_0^2}{4(d-x)}$$ (This is also $W_{m_0} = W'_{m_0}$)

d) $$\underset{b-c}{EFM} = -\int_{x_0}^{x_1} f^e\Big|_{\lambda_0 = \text{CONST}} dx = -\int_{x_0}^{x_1} \frac{\lambda_0^2}{\mu_0 A N^2} dx$$

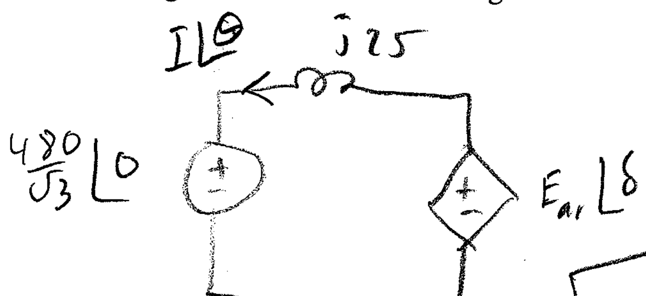
$$= \frac{\lambda_0^2}{\mu_0 A N^2}(x_0 - x_1)$$

Problem 5 (25 pts.)

A 3-phase, 4-pole, 60Hz, round rotor, Y-connected synchronous machine is delivering 6,000 Watts (3-phase) to a network at 480 Volts (line to line). This 6,000 Watt output will be constant throughout this problem. The machine has a synchronous reactance of 25 Ohms and negligible stator (armature) resistance.

When the rotor (field) current is 4 Amps, the generator line current is 8 Amps while serving this 6,000 Watt load. When the rotor (field) current is reduced slowly, the generator line current decreases to a minimum of 7.22 Amps (unity power factor operation) and then starts to increase as the rotor (field) current is reduced further. Eventually the synchronous machine becomes unstable (torque angle equal to 90 degrees) when the rotor (field) current reaches its minimum allowable value for this level of real power load.

a) What is the rotor (field) current when the machine is operating at unity power factor?
b) What is the torque angle when the machine is operating at unity power factor?
c) What is the rotor (field) current when the machine becomes unstable?
d) What is the generator line current magnitude when the machine becomes unstable?



$$6000 = 3 \times \frac{480}{\sqrt{3}} \times 8\cos(-\theta)$$

$$\theta = \pm 25.6^\circ$$

Because reducing the field current lowers the armature current, the machine is overexcited, or producing VARS, so $\theta = -25.6^\circ$

$$K4\angle\delta = j25 \times 8\angle{-25.6^\circ} + \frac{480}{\sqrt{3}}\angle 0$$

$$= 200\angle 64.4 + 277$$

$$= 363 + j180 = 405\angle\delta$$

$$K = 101\ \Omega$$

a) $6000 = 3 \times \frac{480}{\sqrt{3}} \times I_{upf}$ $\quad I_{upf} = 7.2A$

$$101\, I_f \angle\delta = j25 \times 7.2\angle 0 + 277\angle 0$$

$$= 277 + j180 = 330\angle\delta$$

$$\boxed{I_f = 3.27A}$$

b) $\boxed{\delta = 33^\circ}$

c) $6000 = \frac{3 \times 277 \times 101\, I_f \sin 90^\circ}{25}$

$$\boxed{I_{f_{min}} = 1.8A}$$

d) $101 \times 1.8\angle 90 = j25\,\bar{I} + 277\angle 0$

$$\bar{I} = \frac{-277 + j182}{j25} = 7.3 + j11$$

$$\boxed{|\bar{I}| = 13.2\ A}$$

Problem 6 (25 pts.)

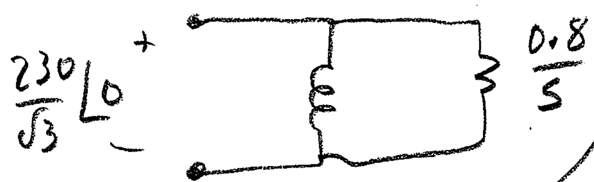
A 5 HP, 230 Volt (line to line), 60 Hz, 3-phase, Y-connected, 6 pole Induction motor is running at rated voltage and is loaded to rated HP. The stator resistance and leakage reactance may be neglected. The rotor resistance referred to the stator (R_r') is 0.8 Ohms. The rotor leakage reactance referred to the stator (X_{lr}') is negligible. Note, 1 HP = 746 Watts.

a) Plot the torque to the shaft versus speed for this machine – show numbers (for both torque and speed) on the plot.

b) Assuming that the HP rating is the power out the shaft (P_m), what is the rated speed in RPM? (you may assume that the slip at rated speed is much less than 1.0)

c) What is the frequency of the rotor currents at rated speed?

a) $P_{AG} = 3\times\left(\frac{230}{\sqrt{3}}\right)^2 \frac{s}{.8} = 66,129\, s$



$P_m = (1-s)P_{AG} = (s-s^2)\,66,129$

b) $= 746\times 5 = 3730$

$T = \frac{P_{AG}}{\omega_s \frac{2}{P}} = \frac{66,129}{2\pi 60 \frac{2}{6}} s$

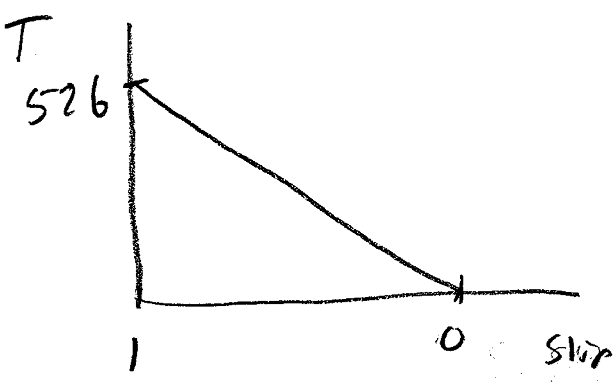
$= 526\, s$

$s^2 - s + 0.0564 = 0$

$s = +\frac{1}{2} \pm \frac{1}{2}\sqrt{1 - 4\times .0564}$

$= 0.5 \pm 0.44$

$= 0.94$ or 0.06



$\boxed{RPM = (1-s)\times 1200 = 1128\ RPM}$

c) $\boxed{f_r = s f_s = .06\times 60 = 3.6\ Hz}$