Problem #1

a) (20 pts)A two electrical and one mechanical port system has the equations

$$\lambda_1 = \frac{L_o}{\left(1+\frac{x}{a}\right)}\left(1-e^{-i_1/I_o}\right) + M\left(1-\frac{x}{b}\right)i_2$$

$$\lambda_2 = M\left(1-\frac{x}{b}\right)i_1 + \frac{L_2 i_2}{\left(1-x/c\right)^2}$$

Find the force of electric origin $f^e(i_1, i_2, x)$.

$$W_m' = \cancel{\int_0^x f^e(0,0,x')\,dx'} + \int_0^{i_1} \lambda_1(i_1', 0, x)\,di_1' + \int_0^{i_2} \lambda_2(i_1, i_2', x)\,di_2'$$

$$= \frac{L_o}{(1+\frac{x}{a})}\int_0^{i_1}\left(1-e^{-i_1'/I_o}\right)di_1' + \int_0^{i_2}\left[M\left(1-\frac{x}{b}\right)i_1 + \frac{L_2 i_2'}{(1-x/c)^2}\right]di_2'$$

$$= \frac{L_o}{(1+\frac{x}{a})}\,i_1 + \frac{L_o I_o}{(1+\frac{x}{a})}\left(e^{-i_1/I_o}-1\right) + M i_1 i_2\left(1-\frac{x}{b}\right) + \frac{L_2 i_2^2}{2\left(1-\frac{x}{c}\right)^2}$$

$$f^e(i_1, i_2, x) = \left.\frac{\partial W_m'}{\partial x}\right|_{i_1, i_2} = \frac{-L_o i_1}{a\left(1+\frac{x}{a}\right)^2} - \frac{L_o I_o}{a\left(1+\frac{x}{a}\right)^2}\left(e^{-i_1/I_o}-1\right)$$

$$-\frac{M i_1 i_2}{b} + \frac{L_2 i_2^2}{c\left(1-\frac{x}{c}\right)^3}$$

b) (15 pts) If $\lambda = \dfrac{\propto i^3}{(x + a)}$ where $\propto$ and a are constants, find both $W_m'(i,x)$ and $W_m(\lambda,x)$

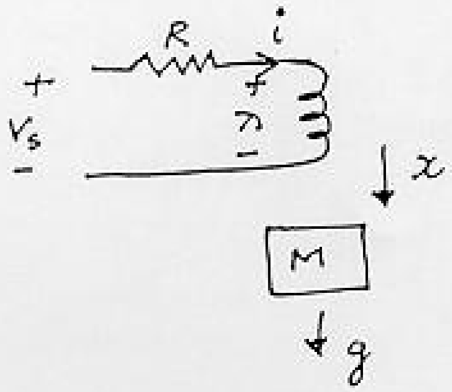
$$W_m' = \int_0^i \lambda(i',x)\,di' = \frac{\alpha i^4}{(x+a)\,4}$$

$$W_m(\lambda,x) = \int_0^\lambda i(\lambda',x)\,d\lambda' = \int_0^\lambda \left[\frac{\lambda'(x+a)}{\alpha}\right]^{\frac{1}{3}} d\lambda'$$

$$= \frac{3}{4}\,\lambda'^{\,4/3} \left[\frac{x+a}{\alpha}\right]^{\frac{1}{3}}$$

Question #2

a) (15 pts) $\lambda = \frac{L_o i}{x}$ and the force of electric origin f^e acting in positive x direction is $\frac{-L_o i^2}{2x^2}$ in the system below. Write the state space equations in terms of x, v and i where $v = \frac{dx}{dt}$.



Elec

$$V_s = iR + \frac{d\lambda}{dt}$$

$$= iR + \frac{L_o}{x}\frac{di}{dt} - \frac{L_o i}{x^2}\frac{dx}{dt}$$

Mech

$$M\frac{d^2x}{dt^2} = f^e + Mg$$

$$= -\frac{L_o i^2}{2x^2} + Mg$$

$$\dot{x} = v$$

$$\dot{v} = \frac{1}{M}\left(-\frac{L_o i^2}{2x^2} + Mg\right)$$

$$\dot{i} = \frac{x}{L_o}\left[-iR + \frac{L_o i}{x^2} v + V_s\right]$$

$$= \frac{-iRx}{L_o} + \frac{iv}{x} + \frac{xV_s}{L_o}.$$

b) (15 pts) Integrate the following system from $t = 0$ to $t = 0.02$ sec with a step size of $\Delta t = 0.01$ sec and enter the values below.

$$\frac{dx}{dt} = v$$
$$\frac{dv}{dt} = v - 10x^3 + 5$$

The initial conditions are $x(0) = 0.2$, $v(0) = 0$

$x(.01)$ 0.2

$v(.01)$ 0.0492

$x(.02)$ 0.200492

$v(.02)$ 0.098892

$x(.01) = x(0) + \Delta t\, v(0)$
$= 0.2 + (0.01)0) = 0.2$

$v(.01) = v(0) + \Delta t\, (v(0) - 10\, x(\cdot)^3 + 5)$
$= 0 + .01\, (0 - 10(.2)^3 + 5)$
$= .0492$

$x(.02) = x(.01) + \Delta t\, v(.01)$
$= 0.2 + .01\, (.0492)$
$= 0.2 + .00492$
$= 0.200492$

$v(.02) = v(.01) + .01\, (v(.01) - 10\, x(.01)^3 + 5)$
$= .0492 + .01\, (.0492 - 10\, (0.2)^3 + 5)$

$= .098892$

Problem 3 (35 points total)

Assume the state space equations for an electromechanical system are

$$\dot{x}_1 = 2x_1 - x_1 x_2$$
$$\dot{x}_2 = 2x_1 x_2 - 4x_2 + x_2^2$$

This system of equations has three equilibrium points, including an equilibrium point at the origin ($x_1 = 0$, $x_2 = 0$).

(10 pts) a) Write down the linear state space equations found by linearizing about the equilibrium point at the origin.

(10 pts) b) Determine the eigenvalues associated with the linearized system from part a. Is the equilibrium point at the origin stable?

(8 pts) c) Determine another equilibrium point (note, there are two correct solutions; you only need to find one of them).

(7 pts) d) Linearize about this second equilibrium point. Determine the eigenvalues associated with the second equilibrium and tell if it is stable.

a) $$\begin{bmatrix} \Delta\dot{x}_1 \\ \Delta\dot{x}_2 \end{bmatrix} = \begin{bmatrix} 2-x_2 & -x_1 \\ 2x_2 & 2x_1-4+2x_2 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$$

evaluating at (0,0) $\Delta\dot{x} = \begin{bmatrix} 2 & 0 \\ 0 & -4 \end{bmatrix} \Delta x$

b) $\lambda_1, \lambda_2 = 2, -4$ unstable

c) Either (1, 2) or (0, 4)

d) For (1, 2) $\Delta\dot{x} = \begin{bmatrix} 0 & -1 \\ 4 & 2 \end{bmatrix} \Delta x$

$\lambda_1, \lambda_2 = 1 \pm j1.73$ unstable

For (0, 4) $\Delta\dot{x} = \begin{bmatrix} -2 & 0 \\ 8 & 4 \end{bmatrix} \Delta x$

$\lambda_1, \lambda_2 = 4, -2$ unstable