ECE 430 Exam #1, Spring 2008 Name: *Solution*

90 Minutes

Section (Check One) MWF 10 ______ TTH 1 ______ MWF 3 ______

1. ________ / 25 2. ________ / 25

3. ________ / 25 4. ________ / 25 Total ________ / 100

Useful information

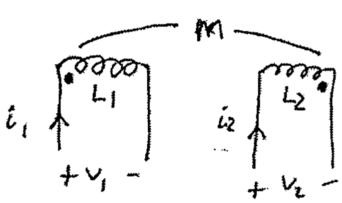
$\sin(x) = \cos(x - 90^\circ)$ $\quad \overline{V} = \overline{ZI}$ $\quad \overline{S} = \overline{VI}^*$ $\quad \overline{S}_{3\phi} = \sqrt{3} V_L I_L \angle\theta$

$0 < \theta < 180^\circ$ (lag) $\quad I_L = \sqrt{3} I_\phi$ (delta) $\quad \overline{Z}_Y = \overline{Z}_\Delta / 3$ $\quad \mu_0 = 4\pi \cdot 10^{-7}$ H/m

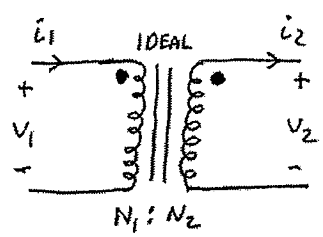
$-180^\circ < \theta < 0$ (lead) $\quad V_L = \sqrt{3} V_\phi$ (wye)

$\int_C \mathbf{H} \cdot \mathbf{dl} = \int_S \mathbf{J} \cdot \mathbf{n} da$ $\quad \int_C \mathbf{E} \cdot \mathbf{dl} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot \mathbf{n} da$ $\quad \Re = \frac{l}{\mu A}$ $\quad MMF = Ni = \phi \Re$

$\phi = BA$ $\quad \lambda = Li = N\phi$ $\quad k = \frac{M}{\sqrt{L_1 L_2}}$ $\quad$ 1 hp = 746 Watts



$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$



$a = \frac{N_1}{N_2}$ $\quad N_1 i_1 = N_2 i_2$

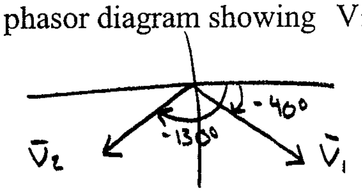
$\frac{V_1}{V_2} = \frac{N_1}{N_2}$

1. (25 points)

Give the r.m.s., cosine-referenced phasor for $v_1(t) = 1414\cos(377t-40^{\circ})$ $1000\angle -40^{\circ}$

Give the r.m.s., cosine-referenced phasor for $v_2(t) = 2000\sin(377t-40^{\circ})$ $1414\angle -130^{\circ}$

Draw the phasor diagram showing $\overline{V}_1$ and $\overline{V}_2$



How much does $\overline{V}_2$ lag $\overline{V}_1$ by? 90°

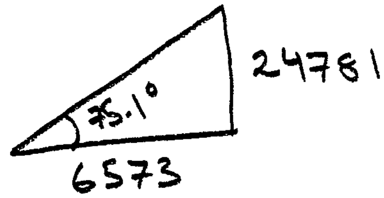
If $\overline{V}_1$ is connected to a load consisting of a series combination of a resistor of 10 Ω and an inductor of 100 mH, find the current and complex power "into" the load. (remember to use units and recall that the impedance of an inductor is $\overline{Z} = j\omega L$)

$$\bar{I} = \frac{100\angle -40^{\circ}}{10+j37.7} = 25.6\angle -115.4 \text{ A}$$
$$= -10.89 - j23.2 \text{ A}$$

$$\bar{S} = \bar{V}\bar{I}^* = \frac{|\bar{V}|^2}{\bar{Z}^*} = \frac{|1000|^2}{10-j37.7}$$

$$\bar{S} = 25.6\angle 75.1^{\circ} \text{ kVA}$$
$$= 6573 + j\,24781 \text{ VA}$$

Draw the power triangle.



Give the power factor angle **and** the power factor (specify lead or lag)

$\theta_{pf} = 75.1^{\circ}$

$pf = 0.257$, lagging

2. (25 points)

The following three-phase, balanced loads are connected across a three-phase, wye-connected source (60 Hz and 480 V – line to line). The nature of the three loads are described below:

Load #1: Wye-connected load with 100 kVA at 0.9 PF lag;
Load #2: Wye-connected load with 60 kW at 0.7 PF lead;
Load #3: Delta-connected load, with 75 A phase current and 0.9 PF lag.

Calculate the following:

a) The total complex power (three phase) consumed by the three loads;
b) The magnitude of the total source line current;
c) The reactive power per phase needed to be added to the delta connected load to make the overall PF 0.9 lead (total for all loads).

a) $\bar{S}_{TOT} = 100k\angle 25.8^\circ + \frac{60k}{0.7}\angle -45.6^\circ + 3\times 480\times 75\angle 25.8^\circ$

$= 90k + j43.5k + 60k - j61k + 97.2k + j47k$

$= 247.2\,kW + j29.5\,kVAR = 249k\angle 6.8^\circ$

b) $\sqrt{3}\times 480\times I_L = 249k \qquad I_L = 300A$

c) $\bar{S}_{new} = \frac{247.2k}{0.9}\angle -25.9 = 247.2kW - j120kVAR$

$Q_{add,3\phi} = -120k - 29.5k = -149.5kVAR$

$Q_{add\ per\ phase} = -50\,kVAR$

3. **(25 points)**

Two coils are wound around a laminated rectangular iron core that has a mean path length of 0.4 meters. The cross-sectional area of the iron core is 0.005 m^2. Coil number 1 has 100 turns and coil number 2 has 25 turns. The relative permeability of the iron path is 1000. The conductivity of the coil wires can be assumed to be infinity. You may neglect leakage flux but you may not neglect the iron reluctance.

If a 120 V (RMS), 60Hz sinusoidal voltage source is applied to coil number 1, and a 2 Ohm resistor is connected across the terminals of coil number 2, find the following things using **time domain**:

a) The magnetic flux ($\Phi(t)$) in the iron (give units).
b) The magnetic flux density ($B(t)$) in the iron (give units).
c) The magnetic field intensity ($H(t)$) in the iron (give units).
d) The current $i_1(t)$
e) The current $i_2(t)$

You may use any phase angle on the source that you like and any assumed polarities and directions that you like.

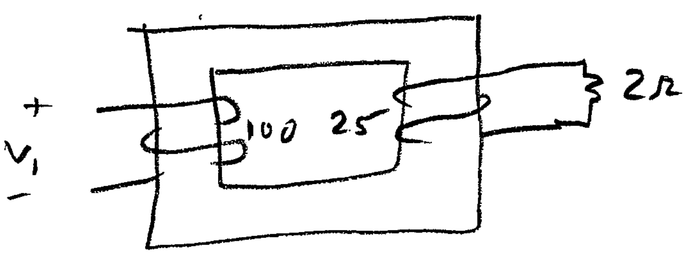
a) $120\sqrt{2}\cos 2\pi 60t = 100\frac{d\phi}{dt}$ $\quad \phi = 0.0045 \sin 2\pi 60t$ (wb)

b) $B = \frac{\phi}{A} = 0.9 \sin 2\pi 60t$ (T) $\quad$ c) $H = \frac{B}{\mu} = 716 \sin 2\pi 60t$ (At/m)

d) + e)

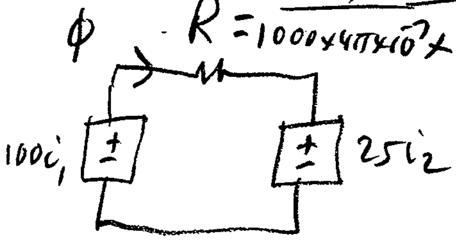
$v_2 = \frac{25}{100} 120\sqrt{2}\cos 2\pi 60t$ (V)

$= 30\sqrt{2}\cos 2\pi 60t$ (V)



$i_2 = \frac{v_2}{2} = 15\sqrt{2}\cos 2\pi 60t$ (A)

$R = \frac{0.4}{1000 \times 4\pi \times 10^{-7} \times .005}$



$100 i_1 = \phi R + 25 i_2$

$= \frac{0.4}{1000 \times 4\pi \times 10^{-7} \times .005} \times .0045 \sin 2\pi 60t + 25 \times 15\sqrt{2}\cos 2\pi 6$

$i_1 = 2.86 \sin 2\pi 60t + 3.75\sqrt{2}\cos 2\pi 60t$

4. (25 points)

An ideal single phase 240:120V, 60 Hz transformer is being used to supply rated voltage to a complex impedance load drawing 1kW at 0.8 power factor lagging.

Find the complex power being supplied to the load

$$\bar{S}_L = \frac{1000}{0.8} \angle \cos^{-1}(0.8) = 1250\angle 36.9° \text{ VA}$$
$$= 1000 + j750 \text{ VA}$$

Find the load impedance as seen from the low voltage side

$$\bar{S}_L = \frac{|\bar{V}|^2}{\bar{Z}_L^*} \Rightarrow \bar{Z}_L = \left(\frac{|\bar{V}|^2}{\bar{S}_L}\right)^* = \frac{120^2}{1000 - j750} = 11.52\angle 36.9° \,\Omega$$
$$= 9.216 + j6.9 \,\Omega$$

Find the magnitudes of the currents $\bar{I}_1$ and $\bar{I}_2$ (using 1 as the high voltage side)

$$|\bar{I}_2| = \frac{1250 \text{ VA}}{120 \text{ V}} = 10.41 \text{ A} \qquad |\bar{I}_1| = \frac{1}{a}|\bar{I}_2| = 5.205 \text{ A}$$

Find the load impedance as seen from the high voltage side

$$\bar{Z}_{L1} = a^2 \bar{Z}_{L2} = (2)^2(11.52\angle 36.9°) = 46.08\angle 36.9° \,\Omega$$
$$= 36.9 + j27.65 \,\Omega$$

What value of reactive power (Q) will bring the load to unity power factor?

$$Q = -750 \text{ VAR}$$

What value of capacitance C does this require if it is placed in shunt on the low voltage side? (recall that the impedance of a capacitor is $\bar{Z} = 1/(j\omega C)$)

$$\frac{|\bar{V}|^2}{(1/j\omega C)^*} = -j750 \Rightarrow -j750 = -j377\, C_2 |120|^2$$
$$C_2 = 1.38 \cdot 10^{-4} \text{ F}$$

What value of capacitance C does this require if it is placed in shunt on the high voltage side?

$$-j750 = -j377\, C_1 |240|^2$$
$$C_1 = 3.45 \cdot 10^{-5} \text{ F}$$

Assuming that larger capacitance values are more expensive, where should the capacitor be placed to minimize costs?

$C_1 < C_2 \Rightarrow$ H.V. Side