ECE430
Spring 2006
Exam 1
March 1, 2006

Name SOLUTION

Section (C for Kimball MWF, F for Tate TR)____________

1:	
2:	
3:	
4:	
Total:	

Equations:

$$\overline{S}_{1\phi} = \overline{V}\overline{I}^* = \frac{|\overline{V}|^2}{\overline{Z}^*} = |\overline{I}|^2 \overline{Z}$$

$$\overline{S}_{3\phi} = 3\overline{V}_\phi \overline{I}_\phi^* = \sqrt{3} V_L I_L \measuredangle \theta$$

$$P_{3\phi} = \sqrt{3} V_L I_L \cos\theta$$

$$Q_{3\phi} = \sqrt{3} V_L I_L \sin\theta$$

$$pf = \cos\left(\measuredangle \overline{V} - \measuredangle \overline{I}\right)$$

$$\theta > 0 \rightarrow lagging, \theta < 0 \rightarrow leading$$

$$P^2 + Q^2 = S^2$$

$$X_c = -\frac{1}{\omega C}$$

$$X_L = \omega L$$

$$wye, abc\ sequence: \overline{V}_L = \overline{V}_\phi\left(\sqrt{3}\measuredangle 30^\circ\right), \overline{I}_\phi = \overline{I}_L$$

$$delta, abc\ sequence: \overline{V}_\phi = \overline{V}_L, \overline{I}_L = \overline{I}_\phi\left(\sqrt{3}\measuredangle -30^\circ\right)$$

$$\overline{Z}_\Delta = 3\overline{Z}_Y$$

$$\overline{Z}_1 \,||\, \overline{Z}_2 = \left(\overline{Z}_1^{-1} + \overline{Z}_2^{-1}\right)^{-1}$$

$$\Re = \frac{l}{\mu A}$$

$$\mu_0 = 4\pi \times 10^{-7}\ H/m$$

$$\lambda = N\Phi = Li$$

$$L = N^2 \wp = \frac{N^2}{\Re}$$

$$mmf\,(source) = Ni$$

$$mmf\,(drop) = \Phi\Re$$

$$\sum mmf = 0\ around\ loop$$

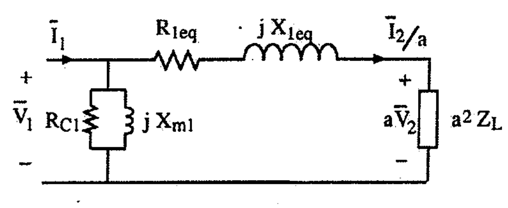
$$\oint \vec{H} \bullet d\vec{l} = \int \vec{J} \bullet \hat{n}\, da$$

$$\oint \vec{E} \bullet d\vec{l} = -\frac{\partial}{\partial t}\int \vec{B} \bullet \hat{n}\, da$$

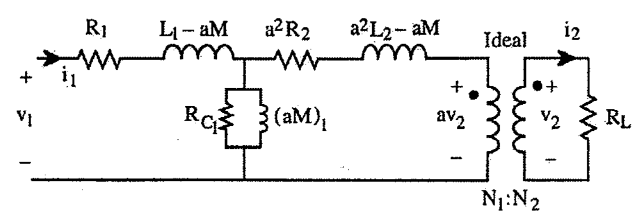
$$\oint \vec{B} \bullet \hat{n}\, da = 0$$

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

$$v = \frac{d\lambda}{dt}$$



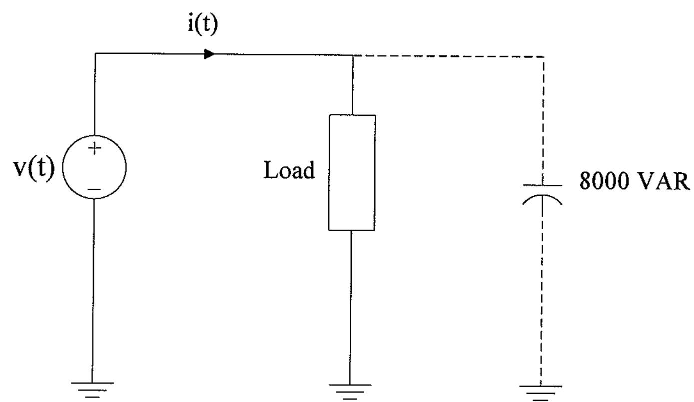
Transformer Approximate Equivalent Circuit



Transformer Equivalent Circuit

Problem 1 (25 points)

A single-phase source, generating a voltage $v(t) = 120\sqrt{2}\cos(377t)$, is connected to a single load. The power factor of the load is measured to be 0.8 lagging. A 8000 VAR capacitor is then added in parallel to the load, and the power factor of the load and capacitor combination is measured to be 0.9 leading.



a) Find the average (real) power (P) and reactive power (Q) of the initial load (10 pts)
b) Determine the complex impedance ($\bar{Z}_{load}$) of the initial load (5 pts)
c) Determine $i(t)$ both before and after the capacitor is added to the system (10 pts)

a) p.f. orig = 0.8 lag $\Rightarrow \measuredangle \bar{S}_{orig} = 36.87°$ $\qquad \bar{S}_{orig} = P_{orig} + j\,Q_{orig}$

p.f. w/ cap = 0.9 lead $\Rightarrow \measuredangle \bar{S}_{cap} = -25.84°$ $\qquad \bar{S}_{cap} = P_{orig} + j\,Q_{cap}$

$\tan(36.87°) = \dfrac{Q_{orig}}{P_{orig}} \qquad \tan(-25.84°) = \dfrac{Q_{orig} - 8000}{P_{orig}} \qquad \bar{S}_{cap} = P_{orig} + j\,(Q_{orig} - 8000)$

$\tan(-25.84°) = \dfrac{Q_{orig}}{P_{orig}} - \dfrac{8000}{P_{orig}} = \tan(36.87°) - \dfrac{8000}{P_{orig}} \Rightarrow P_{orig} = \dfrac{8000}{\tan(36.87°) - \tan(-25.84°)}$

$Q_{orig} = P_{orig}(\tan(36.87°))$

$$\boxed{\begin{array}{l} P_{orig} = 6481\ W \\ Q_{orig} = 4861\ VAR \end{array}}$$

b) $\bar{Z}_{load} = \dfrac{|\bar{V}|^2}{\bar{S}^*_{orig}} \qquad \bar{V} = 120\angle 0° \qquad \bar{Z}_{load} = \dfrac{120^2}{6481 - j4861}$

$$\boxed{\begin{array}{l} \bar{Z}_{load} = 1.42 + j1.07\ \Omega \\ \text{-or-}\ \ 1.77\angle 37°\ \Omega. \end{array}}$$

c) $\bar{I}_{orig} = \dfrac{\bar{S}^*_{orig}}{\bar{V}^*} = \dfrac{6481 - j4861}{120\angle 0°} = 67.5\angle -36.87° \Rightarrow \boxed{i(t)_{orig} = \sqrt{2}\cdot 67.5\cos(377t - 36.87°)}$

$\bar{I}_{cap} = \dfrac{\bar{S}^*_{cap}}{\bar{V}^*} = \dfrac{(6481 + j4861 - j8000)^*}{120\angle 0°} = \dfrac{6481 + j3139}{120\angle 0°} = 60\angle 25.84°$

$$\boxed{i(t)_{cap} = \sqrt{2}\cdot 60\cdot\cos(377t + 25.84°)}$$

Problem 2 (25 points)

For distributed generation, UL1741 specifies a particular test to verify that the generator will shut down when the grid is disconnected. In this problem, you must design the test. The specifications of the generator are:

480 V (rms line-to-line three-phase)
5 kVA
60 Hz

The requirements of the test are:

Overall power factor = 1.0
Resistive load equal to rating of the generator ($P_R = S_{gen}$)
Inductive load with $Q_L = 2.5\ S_{gen}$
Capacitive load with $Q_C = -Q_L$

a) Determine the rated line current (5 points)

b) Determine the resistance, assuming a delta configuration (6 points)

c) Determine the inductance and capacitance, assuming a wye configuration (8 points)

d) Draw the circuit and label all impedances (4 points)

e) If the frequency were changed to 50 Hz, what would need to change? (2 points)

a) $S = \sqrt{3}\, V_L I_L$ +2 $\underline{S = VI^*}$

$I_L = \frac{S}{\sqrt{3}\, V_L} = \frac{5000}{\sqrt{3} \times 480} = 6.014\ A$ (+5)

b) $P_{1R} = \frac{1}{3} S = 1667 W = \frac{480^2}{R}$ (+3)

$R = \frac{480^2}{1667} = 138.24$ (+3)

c) $Q_{1L} = 2.5 \times P_{1R} = 4167\ VAR = \frac{V^2}{X}$ (+3)

$V = \frac{480}{\sqrt{3}} = 277$

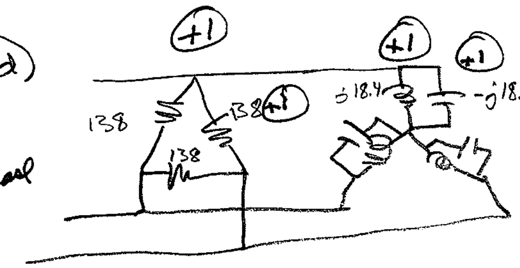
$X_L = \frac{277^2}{4167} = 18.43$ (+2)

$X_C = -X_L = -18.43$ (+2)

$L = \frac{X_L}{\omega} = 48.89\,mH$ (+1)

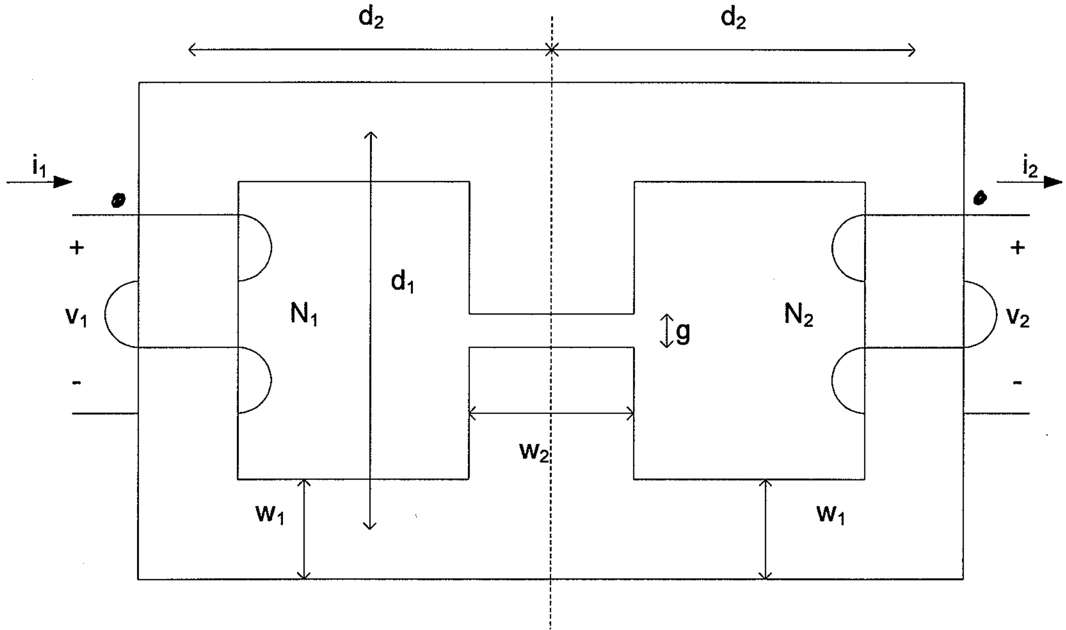
$-\frac{1}{\omega C} = X_C \Rightarrow C = -\frac{1}{\omega X_C} = 143.9\,\mu F$

d)



e) X_L, X_C unchanged but
L bigger, C smaller
$\left(L = \frac{X_L}{\omega},\ C = -\frac{1}{\omega X_C}\right)$
(+2)

Problem 3 (25 points)
Consider the structure below.



+5

Given: N_1=1000, N_2=100; d_1 = 10 cm, d_2 = 6 cm; w_1 = 1 cm, w_2 = 2 cm; g = 2 mm.
Depth into page is 2 cm. Permeability of the iron is $1000\mu_0$.
HINT: There are three important magnetic paths. One is d_1 long, two are (d_1+2d_2) long.

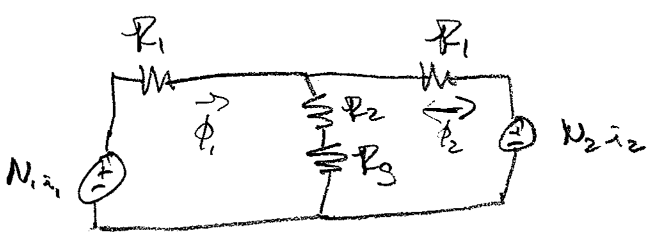
a) Label dots on the windings (5 points)

b) Draw the magnetic equivalent circuit and determine the reluctances (neglect fringing and leakage) (7 points)

c) Solve the magnetic equivalent circuit to find the flux linkage in the two windings, λ_1 and λ_2, as functions of i_1 and i_2 (9 points)

d) Determine L_1, L_2, M, and k (4 points)

b)



+3

(+2) $\mathcal{R}_1 = \frac{d_1+2d_2}{\mu A_1}$

$A_1 = w_1 \cdot \text{depth} = 2\,cm^2$

$\mathcal{R}_1 = \frac{(10+12)\times 10^{-2}}{1000\times 4\pi\times 10^{-7}\times 2\times 10^{-4}} = 875352 = 875\times 10^3$

(+1) $\mathcal{R}_2 = \frac{d_1-g}{\mu A_2}$

$A_2 = w_2 \cdot \text{depth} = 4\,cm^2$

$\mathcal{R}_2 = \frac{(10-0.2)\times 10^{-2}}{1000\times 4\pi\times 10^{-7}\times 4\times 10^{-4}} = 194965 = 195\times 10^3$

(+1) $\mathcal{R}_g = \frac{g}{\mu_0 A_2} = 3978874 = 3.98\times 10^6$

c) $N_1 i_1 - \phi_1 \mathcal{R}_1 - (\phi_1 - \phi_2)(\mathcal{R}_2 + \mathcal{R}_g) = 0$

$N_2 i_2 + \phi_2 \mathcal{R}_1 - (\phi_1 - \phi_2)(\mathcal{R}_2 + \mathcal{R}_g) = 0$ +4

$N_1 i_1 = (\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_g)\phi_1 - (\mathcal{R}_2 + \mathcal{R}_g)\phi_2$

$N_2 i_2 = (\mathcal{R}_2 + \mathcal{R}_g)\phi_1 - (\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_g)\phi_2$

$$\begin{bmatrix} \mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_g & -(\mathcal{R}_2 + \mathcal{R}_g) \\ \mathcal{R}_2 + \mathcal{R}_g & -(\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_g) \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \begin{bmatrix} N_1 & 0 \\ 0 & N_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$\begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \mathcal{R}^{-1} \begin{bmatrix} N_1 & 0 \\ 0 & N_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 625.6e{-6} & -51.72e{-6} \\ 517.2e{-6} & -62.56e{-6} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$ +3

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} N_1 & 0 \\ 0 & N_2 \end{bmatrix} \begin{bmatrix} & \\ & \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 625.6\,mH & -51.7\,mH \\ 51.7\,mH & -6.256\,mH \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$\lambda_1 = 625.6\,mH \cdot i_1 - 51.7\,mH \cdot i_2$

$\lambda_2 = 51.7\,mH \cdot i_1 - 6.256\,mH \cdot i_2$ +2

d) $L_1 = 625.6\,mH$ +1

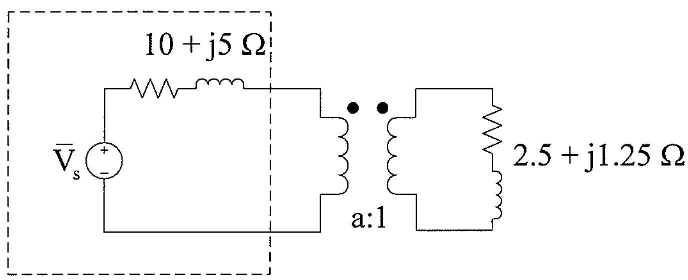
$L_2 = 6.256\,mH$ +1

$M = 51.7\,mH$ +1

$k = \frac{M}{\sqrt{L_1 L_2}} = 0.826$ +1

Problem 4

A load is being supplied by a single-phase source (represented here as an ideal voltage source behind a series impedance of 10 + j5 Ω) and transformer, as shown in the following diagram. The load, connected to the LV side of the transformer, has a complex impedance of 2.5 + j1.25 Ω.



Source

a) Assuming the transformer is ideal, determine the turns ratio that maximizes the average power delivered to the load (7 pts)

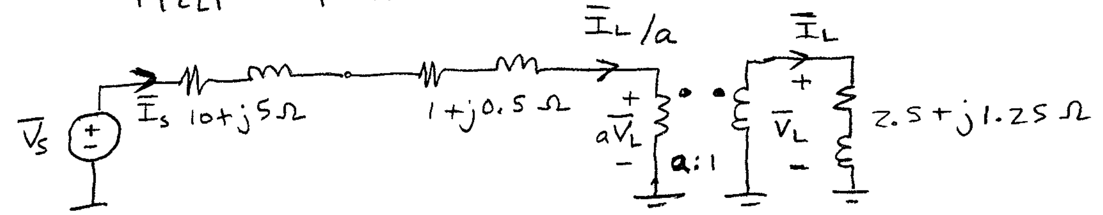
After performing a short-circuit test, it is determined that the transformer is not ideal, but instead has parameters $R_{1eq} = 1\ \Omega$, $X_{1eq} = 0.5\ \Omega$. The open-circuit test is not performed, therefore R_C and X_m are neglected. Using these parameters, and the turns ratio calculated in part (a):

b) If the load is to be supplied at 120 V, find the RMS magnitude of the current leaving the source. (7 pts)
c) If the load is to be supplied at 120 V, what is the required RMS voltage magnitude $|\bar{V}_s|$ of the ideal voltage source? (11 pts)

a) $|\bar{Z}_o| = a^2 |\bar{Z}_L| \qquad |\bar{Z}_o| = \sqrt{10^2 + 5^2} \approx 11.18 \qquad |\bar{Z}_L| = \sqrt{2.5^2 + 1.25^2} \approx 2.75$

$a = \sqrt{\dfrac{|\bar{Z}_o|}{|\bar{Z}_L|}} = \sqrt{\dfrac{11.18}{2.75}} = \sqrt{4} = 2 \qquad \boxed{a = 2}$

b)



$\bar{V}_L = 120\angle 0° \qquad \bar{I}_L = \dfrac{120\angle 0°}{2.5 + j1.25} = 38.4 - j19.2 \qquad \bar{I}_s = \dfrac{\bar{I}_L}{a} = \dfrac{38.4 - j19.2}{2} = 19.2 - j9.6$

$|\bar{I}_s| = |19.2 - j9.6| = \boxed{21.47\ A}$

c) $\bar{V}_s = (10 + j5)\bar{I}_s + (1 + j0.5)\bar{I}_s + 240\angle 0° \qquad \bar{I}_s = 19.2 - j9.6$

$= (10 + j5)(19.2 - j9.6) + (1 + j0.5)(19.2 - j9.6) + 240\angle 0°$

$\bar{V}_s = 504\angle 0°\ V \qquad |\bar{V}_s| = \boxed{504\ V}$