ECE330
Exam #1
Spring 2004

Name SOLUTION
(Print Name)

Section: **(Circle One)** **10 MWF** **2 MWF**
(Sauer) **(Kimball)**

Problem 1 _______ **Problem 2** _______ **Problem 3** _______ **Problem 4** _______

TOTAL: _______

USEFUL INFORMATION

$\sin x = \cos(x - 90^\circ)$ $\quad \bar{Z}_Y = \frac{1}{3}\bar{Z}_\Delta$ $\quad \bar{S}_{3\phi} = \sqrt{3}\, V_L I_L \angle\theta$

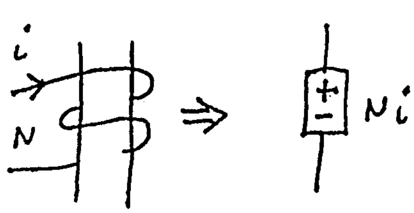
$\bar{V} = \bar{Z}\bar{I}$ $\quad \bar{S} = \bar{V}\bar{I}^*$ $\quad 1\,HP = 746\,W$ $\quad 0 \le \theta \le 180^\circ$ Lag

$-180^\circ \le \theta \le 0$ Lead

$\oint_C \underline{H}\cdot d\underline{l} = \int_S \underline{J}\cdot\underline{n}\,da$ $\quad \oint_C \underline{E}\cdot d\underline{l} = -\frac{d}{dt}\int_S \underline{B}\cdot\underline{n}\,da$ $\quad I_L = \sqrt{3}\, I_\phi$ (delta)

$\oint_S \underline{B}\cdot\underline{n}\,da = 0$ $\quad \mathcal{R} = \frac{l}{\mu A}$ $\quad MMF = Ni = \phi\mathcal{R}$

$\lambda = N\phi = Li$ $\quad \mu_0 = 4\pi \times 10^{-7}\ H/m$ $\quad k = \frac{M}{\sqrt{L_1 L_2}}$

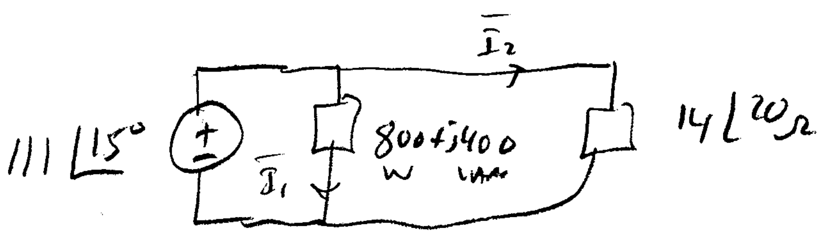


$\phi = BA$

Problem 1 (25 pts.)

A single-phase source with a voltage of 157 cos (377t + 15°) Volts is serving two single-phase loads in parallel. The first load consumes 800 Watts of real power and 400 Vars of reactive power. The second load is an impedance with magnitude 14 Ohms and angle 20 degrees.

a) Find the time-domain currents into each load.
b) Find the total complex power supplied by the source.
c) Find the value of a capacitor (in Farads) that you could add as a third load in parallel to provide all the reactive power consumed by the two other loads.
d) What would the time-domain source current be when the capacitor is added?



(a) $\bar{I}_1 = \dfrac{800-j400}{111\angle -15°} = \dfrac{894\angle -26.6°}{111\angle -15°} = \underline{8.06\angle -11.6°}$

$i_1 = 8.06\sqrt{2}\cos(377t - 11.6°)$

$\bar{I}_2 = \dfrac{111\angle 15°}{14\angle 20°} = 7.93\angle -5°$ $\quad i_2 = 7.93\sqrt{2}\cos(377t - 5°)$

(b) $\bar{S} = 800+j400 + 111\angle 15\,(7.93\angle -5°)^* = 800+j400 + 880\angle 20°$

$= 800+j400 + 827 + j301 = 1627 + j701 = 1771\,\underline{\angle 23.3°}$

(c) $701 = 111^2 \times 377C \qquad C = 151\,\mu F$

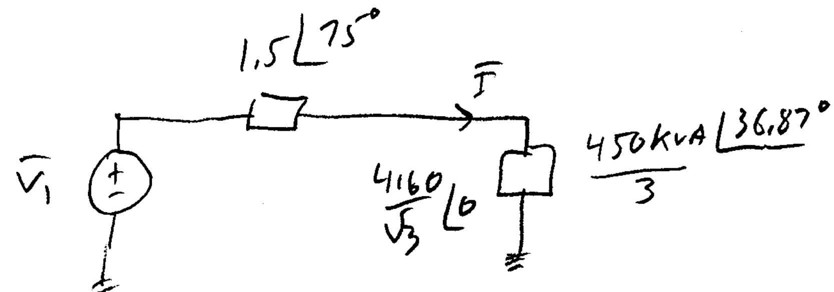
(d) $\bar{I} = \left(\dfrac{1627}{111\angle 15°}\right)^* = 14.65\angle 15°$

$i = 14.65\sqrt{2}\cos(377t + 15°)$

Problem 2 (25 pts)

A three-phase power system consists of a wye-connected generator connected to a delta connected load through a transmission line having a per-phase impedance magnitude of 1.5 Ohms and angle 75 degrees. The delta-connected load consumes a total three-phase apparent power of 450 kVA at 0.8 power factor lag when the generator voltage is set so that the voltage at the load is 4,160 Volts (line to line).

a) Find the magnitude of the generator line current for this system.
b) Find the magnitude of the line-line voltage of the generator.
c) Find the total three-phase complex power supplied by the generator.



a\) $\bar{I} = \left(\frac{\bar{S}}{\bar{V}}\right)^* = \left(\frac{150\text{k}\angle -36.87°}{2402\angle 0}\right) = \underline{62.45\angle -37°}$

b\) $\bar{V}_1 = (1.5\angle 75)(62.45\angle -37°) + 2402\angle 0$

$= 93.68\angle 38° + 2402\angle 0 = 73.8 + j57.68 + 2402$

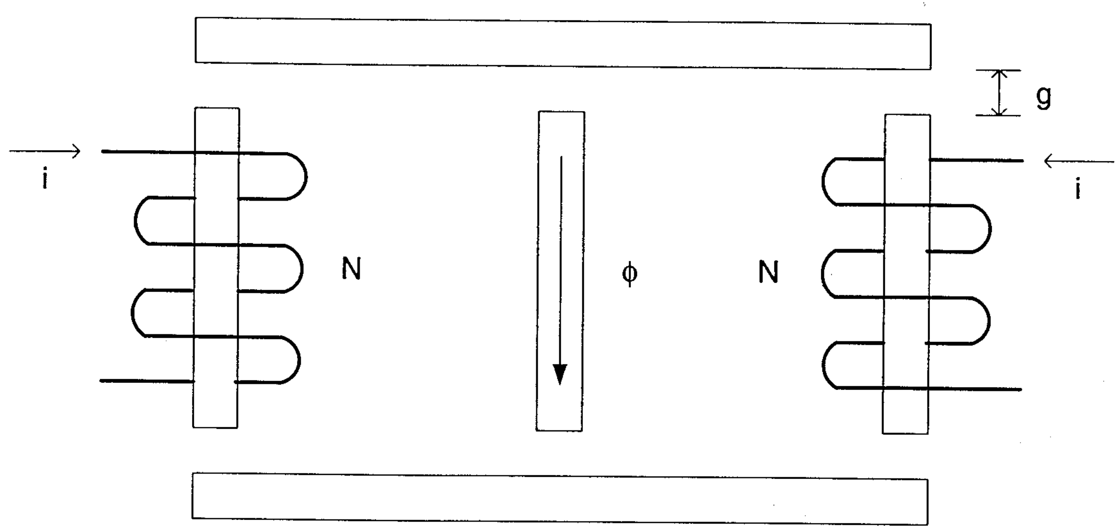
$= 2476 + j58 = 2477\angle 1.3°$

$V_{LL} = 2477\sqrt{3} = 4290\text{ V}$

c\) $\bar{S} = 3 \times 2477\angle 1.3° (62.45\angle -37°)^* = 464{,}065\angle +38°$

Problem 3 (25 pts)

In the magnetic structure below, $\mu = \infty$ in the iron. All legs have the same cross sectional area, A = 1 cm^2. All gaps are equal in length, g = 1 mm. Neglect fringing. N = 100. The coils are connected in series so that both currents are equal.



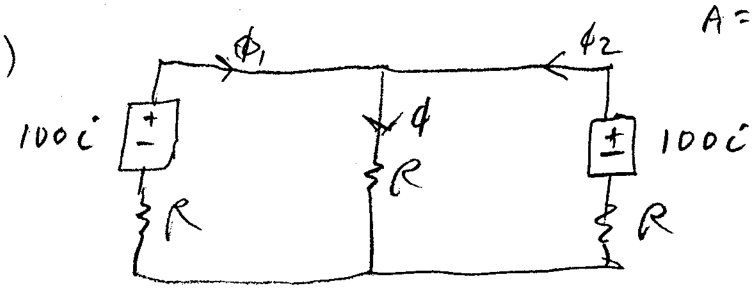
a) Find the current necessary to generate a flux density in the center post of 1.0 T.
b) Find the total flux, ϕ, in the center post.
c) Find the flux linkage, λ_L, in the left-hand coil.

$A = 10^{-4}$

$R = \dfrac{2\times10^{-3}}{4\pi\times10^{-7}\times10^{-4}}$

$= 1.59\times10^{7}$

a)



b) $\phi = BA = 1\times10^{-4}$

$$\left.\begin{aligned}-100i + (1\times10^{-4})(1.59\times10^{7}) + 1.59\times10^{7}\phi_1 &= 0\\ -100i + (1\times10^{-4})(1.59\times10^{7}) + 1.59\times10^{7}\phi_2 &= 0\end{aligned}\right\} \quad \phi_1 - \phi_2 = 0$$

But $\phi_1 + \phi_2 = 1\times10^{-4}$

So $\phi_1 = \phi_2 = 0.5\times10^{-4}$

$-100i + (1\times10^{-4})(1.59\times10^{7}) + 1.59\times10^{7}(0.5\times10^{-4}) = 0$

$\boxed{i = 23.85A}$

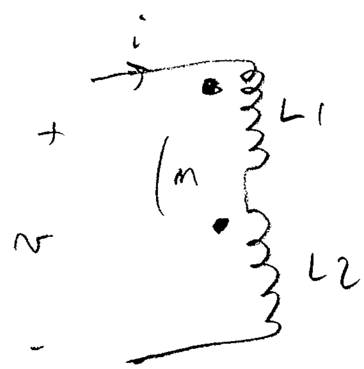
c) $\lambda_L = 100\phi_1 = \boxed{50\times10^{-4}}$

Problem 4 (25 pts)

Two identical but mutually coupled coils are connected in two ways, and inductance is measured in each case.

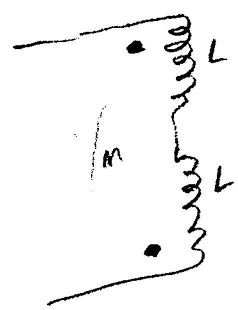
a) In series with the undotted terminal of one coil connected to the dotted terminal of the other coil. Leq = 0.06 H
b) In series with the undotted terminal of one coil connected to the undotted terminal of the other coil. Leq = 0.02 H

Find L, M, and k



$$v = L_1\frac{di}{dt} + m\frac{di}{dt} + L_2\frac{di}{dt} + m\frac{di}{dt}$$

$$(L_1 = L_2) \quad = \underbrace{(L_1 + L_2 + 2m)}_{0.06}\frac{di}{dt}$$



$$v = L\frac{di}{dt} - m\frac{di}{dt} + L\frac{di}{dt} - m\frac{di}{dt}$$

$$= \underbrace{(2L - 2m)}_{0.02}\frac{di}{dt}$$

$$2L + 2m = .06$$
$$2L - 2m = .02$$
$$4L = .08 \qquad \boxed{L = .02}$$

$$k = \frac{.01}{\sqrt{.02 \times .02}} = 0.5 \qquad .04 + 2m = .06 \qquad 2m = .02 \qquad m = .01$$