ECE 330 Exam #1, Fall 2014 Name: Solution
90 Minutes

Section (Check One) MWF 10am _______ MWF 2:00pm _______

1. ________ / 25 2. ________ / 25

3. ________ / 25 4. ________ / 25 Total ________ / 100

Useful information

$\sin(x) = \cos(x - 90^\circ)$ $\quad \overline{V} = \overline{Z}\overline{I}$ $\quad \overline{S} = \overline{V}\overline{I}^* = P + jQ$ $\quad \overline{S}_{3\phi} = \sqrt{3} V_L I_L \angle\theta$

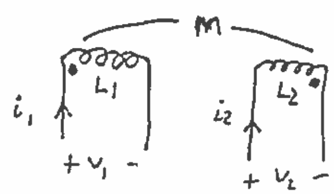
$0 < \theta < 180^\circ$ (lag) $\quad I_L = \sqrt{3} I_\phi$ (delta) $\quad \overline{Z}_Y = \overline{Z}_\Delta / 3 \quad \mu_0 = 4\pi \cdot 10^{-7}$ H/m

$-180^\circ < \theta < 0$ (lead) $\quad V_L = \sqrt{3} V_\phi$ (wye)

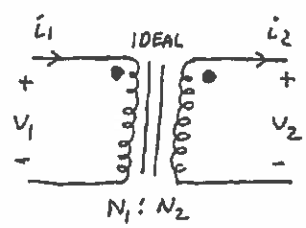
ABC sequence has A at zero, B at minus 120 degrees, and C at plus 120 degrees

$\int_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot \mathbf{n}\, da \quad \int_C \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot \mathbf{n}\, da \quad \Re = \frac{l}{\mu A} \quad MMF = Ni = \phi \Re$

$\phi = BA \quad \lambda = N\phi \quad k = \frac{M}{\sqrt{L_1 L_2}}$ $\quad$ 1 hp = 746 Watts

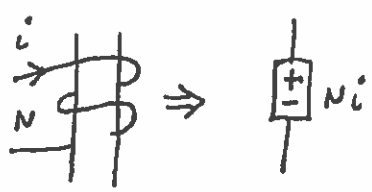


$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$



$a = \frac{N_1}{N_2} \quad N_1 i_1 = N_2 i_2$

$\frac{v_1}{v_2} = \frac{N_1}{N_2}$



Problem 1. (25 points)

A single phase ac voltage source of 240 Volts RMS, 60 HZ supplies a 5 kVA, 0.7 pf lagging load.

a) Find the complex power consumed by the load.
b) Find the complex impedance of the load.
c) Find the magnitude of the current supplied by the source.
d) How many VARS from a capacitor bank are required to improve the power factor to 0.9 lagging?
e) What is the magnitude of the current supplied by the source after addition of above capacitor bank?

a) $$S = 5000 \angle +\cos^{-1}(0.7) = 5000 \angle 45.57^\circ \text{ VA}$$
$$= 3500 + j3570 \text{ VA}$$

b) $$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{\bar{V}}{(\bar{S}/\bar{V})^*} = \frac{\bar{V}.\bar{V}^*}{\bar{S}^*} = \frac{|\bar{V}|^2}{\bar{S}^*} = \frac{240^2}{5000\angle -45.57} \Omega$$
$$= 11.52 \angle 45.57^\circ \Omega$$
$$= 8.064 + j8.227 \Omega$$

c) $$I = \left(\bar{S}/\bar{V}\right)^* = \left(\frac{5000 \angle 45.57}{240 \angle 0}\right)^*$$
$$= 20.83 \angle -45.57^\circ \text{ A} \Rightarrow |I| = 20.83 \text{ A}$$

d) $$\theta_{new} = \cos^{-1}(0.9) = 25.84^\circ$$
$$Q_{new} = (3570 - Q_{cap}) = P\tan(\theta) = 3500 \tan(25.84)$$
$$Q_{cap} = 3570 - 1695 = 1875 \text{ Vars}$$

e) $$\bar{S}_{new} = \frac{3500}{0.9} \angle \cos^{-1}(0.9) = 3889 \angle 25.84^\circ \text{ VA}$$
$$|I| = \frac{|S|}{|V|} = \frac{3889}{240} = 16.2 \text{ A}$$

Problem 2. (25 pts)

A balanced three-phase, 480 Volt (line to line), 60 Hz, ABC sequence power system supplies the following two loads that are connected in parallel:

Load #1 is a four-wire, wye-connected load which draws 84 Amps and 21kW of real power per phase.

Load #2 is a three-wire delta-connected load which draws 150kVA (3-phase) of apparent power at a power factor of 0.88 lag.

$$pf_1 = \frac{21k \times 3}{\sqrt{3} \times 480 \times 84} = .902$$

a) What is the source line current if Load #1 is lagging power factor?

$$\bar{S}_1 = \sqrt{3} \times 480 \times 84 \angle \pm 25.56, \quad \bar{S}_2 = 150k \angle \cos^{-1} 0.88 = 132K + j\,71{,}246$$

If $\theta > 0$, $\bar{S}_{TOT} = 132K + 63{,}000 + j\,71246 + j\,30{,}130$

$$= 195{,}000 + j\,101376 = 219{,}777 \angle 27.47^\circ$$

$219{,}777 = \sqrt{3} \times 480 \times I_L$ $\quad I_L = 264\ A$

b) What is the source line current if Load #1 is leading power factor?

If $\theta < 0$ $\quad \bar{S}_{TOT} = 132k + 63{,}000 + j\,71246 - j\,30{,}130$

$$= 195{,}000 + j\,41{,}160 = 199{,}296 \angle 11.92^\circ$$

$199{,}296 = \sqrt{3} \times 480 \times I_L$ $\quad I_L = 240\ A$

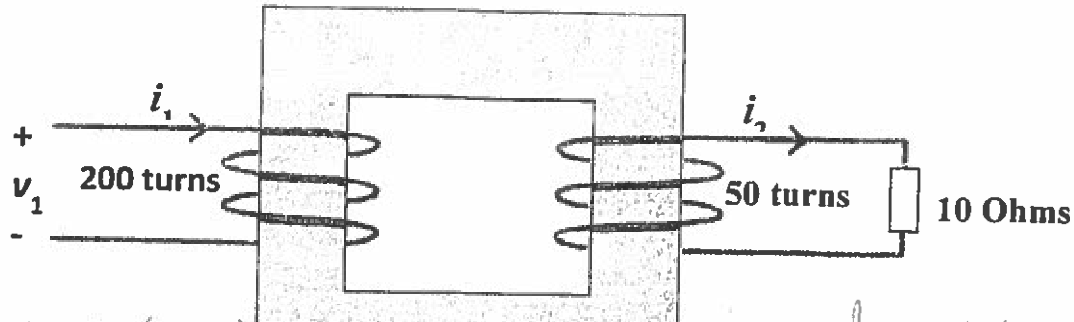
c) What would the source line current be if power factor correction capacitors were used to make the total power factor of both loads 1.0?

$$\bar{S} = 132k + 63{,}000 = 195{,}000 = \sqrt{3} \times 480 \times I_L$$

$I_L = 235\ A$

Problem 3. (25 points)

The iron core in the following circuit has a relative permeability of 1000, cross-sectional area of 20cm^2, and a mean length of 40cm. Neglect any leakage flux and all losses. v_1= 120 V rms at 60Hz (sinusoidal in time) is applied to the left coil.



$V_1 = 120\sqrt{2}\cos(2\pi .60.t)$

$\mathcal{R}_c = \dfrac{l_c}{\mu A} = \dfrac{0.4}{1000\mu_0 \times 20\times10^{-4}} = 159155 \ AT/Wb$

a) What is the peak flux and the peak flux density in the iron core?
b) How much current in the left coil (RMS magnitude) would be required to establish this flux?
c) What is the current, i_2, (RMS magnitude) through the 10 Ohm resistor?
d) What is the total current, i_1, (RMS magnitude) in the left coil?

a) $V_1 = \dfrac{d\lambda_1}{dt} \Rightarrow \lambda_1 = \int V_1 dt = \int 120\sqrt{2}\cos(2\pi.60.t)$

$= \dfrac{120\sqrt{2}}{2\pi.60}\sin(2\pi.60.t)$

$\Phi_{peak} = \dfrac{\lambda_{1peak}}{N_1} = \dfrac{120\sqrt{2}}{2\pi.60\times200} = 0.00225$ Webers

$B_{peak} = \dfrac{\Phi_{peak}}{Area} = \dfrac{0.00225}{20\times10^{-4}} = 1.125 \ T$

b) $NI = \Phi\mathcal{R} \Rightarrow I_{magnetizing} = \dfrac{\Phi_{rms}.\mathcal{R}_c}{N_1} = \dfrac{0.00225/\sqrt{2}\times159155}{200}$

$= 1.27 \ A$ rms

c) No leakage $\Rightarrow \dfrac{V_1}{V_2} = \dfrac{N_1}{N_2} \Rightarrow V_2 = \dfrac{120\times50}{200} = 30V$

$I_2 = \dfrac{V_2}{Z_2} = \dfrac{30}{10} = 3A$

d. I_2 referred to primary, $I_2' = \dfrac{N_2 I_2}{N_1} = \dfrac{50\times3}{200}$

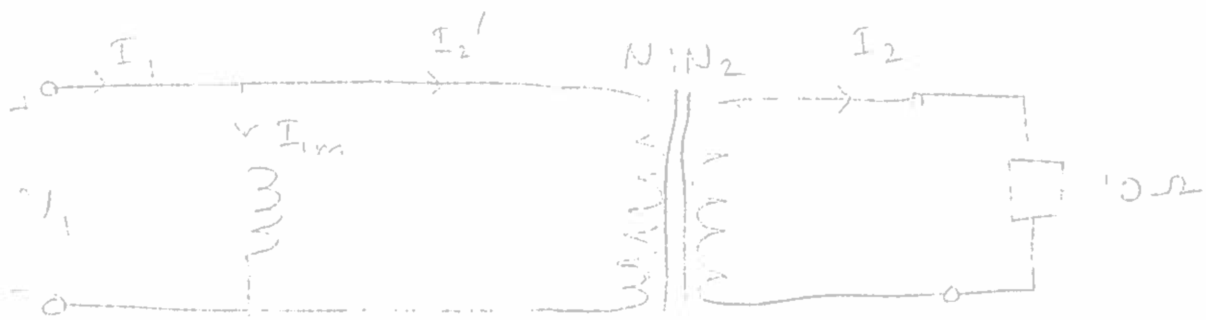
$= 0.75\,A$

$I_{1\,total} = jI_m + I_2'$

↑ current through inductor

current through resistor

$|I_1| = \sqrt{0.75^2 + 1.27^2} = 1.47\,A$



Ideal transformer

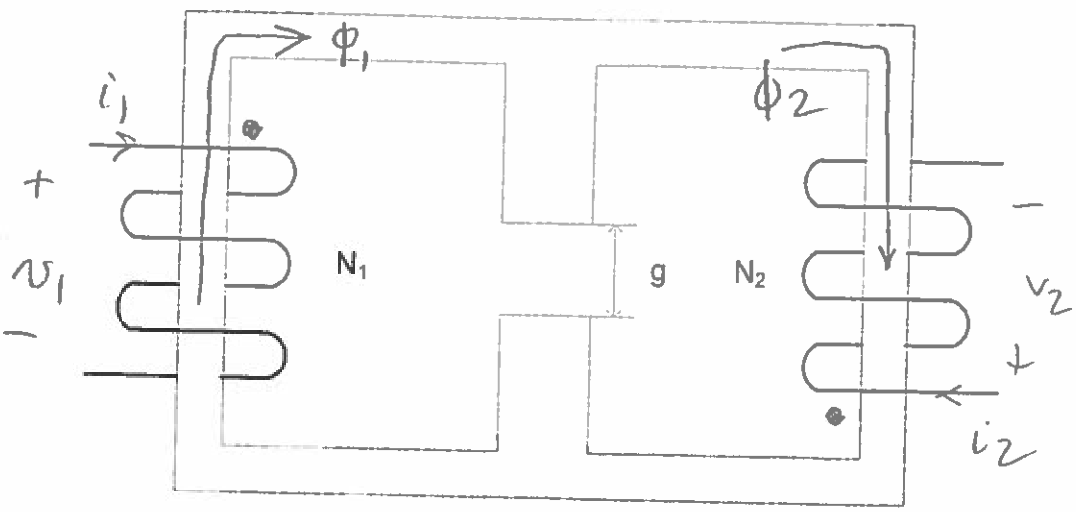
Problem 4. (25 points)

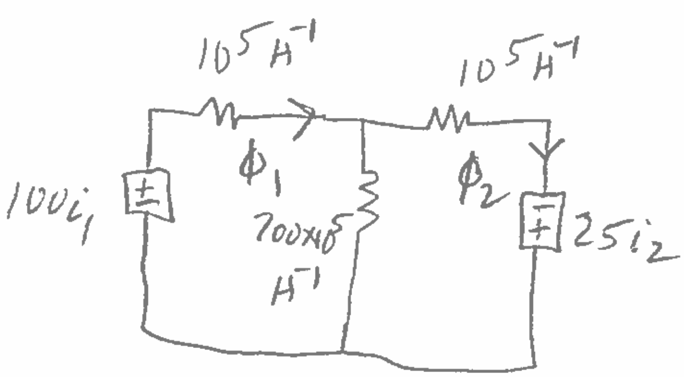
In the figure below, $N_1 = 100$, $N_2 = 25$. The reluctances of the left and right branches of the core are each $1 \times 10^5\ H^{-1}$. The gap, g, in the center post is 1 cm, and the cross-sectional area of the center post is 4 cm^2. Neglect fringing and the effect of the steel in the center post. The resistance of the left coil (primary) is 1 ohm and the resistance of the right coil (secondary) is 0.05 ohm.

a) Label voltages v_1 and v_2 and currents i_1 and i_2 so that all self and mutual inductance terms are positive. Assign dot polarities. Assign v_1 and i_1 to the primary and v_2 and i_2 to the secondary.

b) Find L_1 (self inductance of the primary), L_2 (self inductance of the secondary), and M (mutual inductance between coils).

c) Treating this structure as a non-ideal transformer, draw the equivalent circuit with all impedances referred to the primary (assume a 60Hz supply and use the frequency domain).





$$R_{gap} = \frac{.01}{4\pi \times 10^{-7} \times 4 \times 10^{-4}} = 200 \times 10^5\ H^{-1}$$

$$\left.\begin{aligned} -100 i_1 + 10^5 \phi_1 + 200\times10^5(\phi_1 - \phi_2) = 0 \\ -25 i_2 + 10^5 \phi_2 + 200\times10^5(\phi_2 - \phi_1) = 0 \end{aligned}\right\}$$

$$201\times10^5 \phi_1 - 200\times10^5 \phi_2 = 100 i_1$$
$$-200\times10^5 \phi_1 + 201\times10^5 \phi_2 = 25 i_2$$

$$\phi_2 = .1244\times10^{-5} i_2 + .995\phi_1$$

$$201\times10^5 \phi_1 - 199\times10^5 \phi_1 = 100 i_1 + 24.88 i_2$$

$$2\times10^5 \phi_1 = 100 i_1 + 24.88 i_2$$

$$\phi_1 = .0005 i_1 + .000124 i_2$$

$$\lambda_1 = N_1\phi_1 = .05 i_1 + .0124 i_2$$

$$\phi_2 = .1244\times10^{-5} i_2 + .995(.0005 i_1 + .000124 i_2)$$

$$= .0004975 i_1 + .000125 i_2$$

$$\lambda_2 = N_2\phi_2 = .0124 i_1 + .003125 i_2$$

$$\boxed{L_1 = .05H \quad m = .0124H \quad L_2 = .003125H}$$

$L_{m_1} = .0124\times\frac{100}{25} = .0496H$ $\quad L_{\ell_1} = L_1 - L_{m_1} = .0004H$

$L_{m_2} = .0124\frac{25}{100} = .0031H$ so $L_{\ell_2} = L_2 - L_{m_2} = .000025H$