ECE330: Power Circuits & Electromechanics

Lecture 21. Four-quadrant control of synchronous machines

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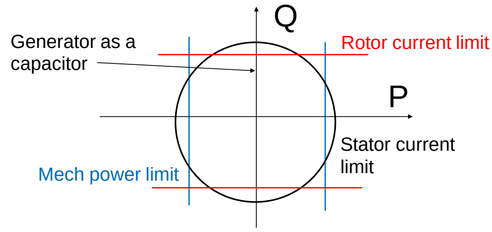
Four-quadrant operations



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Today

- Review: Synchronous machines
- Equivalent circuit model
- Four-quadrant control of P & Q
- Examples

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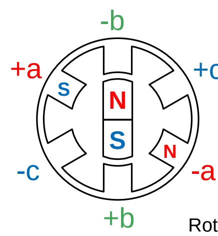
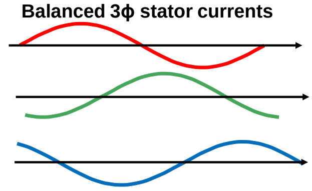
Last time: Rotating magnetic field



Balanced 3ϕ stator currents



Rotating field spins at synchronous speed
Drags rotor at same speed

Rotor construction?

Magnetic material → Reluctance machine
Permanent magnet → Brushless DC
Electromagnet → Synchronous machine

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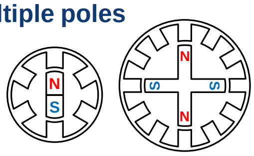
Recall: Multiple poles

2-pole @ 60 Hz
3600 rpm



4-pole @ 60 Hz
1800 rpm

Synchronous **speed**
= 60 π [rad/s]
= 1800 rpm

$$\omega_m = \frac{2}{p}\omega_s$$

Synchronous **frequency**
= 120 π [rad/s]
= 60 [Hz]

Torque [Nm] = Total 3p Power [W] / Mechanical speed [rad/s]

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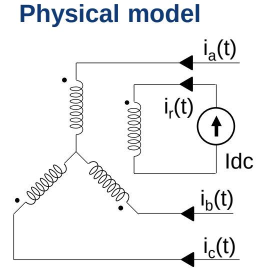
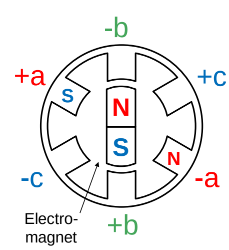
Physical model





$$v_a(t) = L_a\frac{di_a}{dt} + M_{ar}(\theta)\frac{di_r}{dt} + M'_{ar}(\theta)i_r(t)\frac{d\theta}{dt}$$

0
M cos(ωt)   Idc   ω

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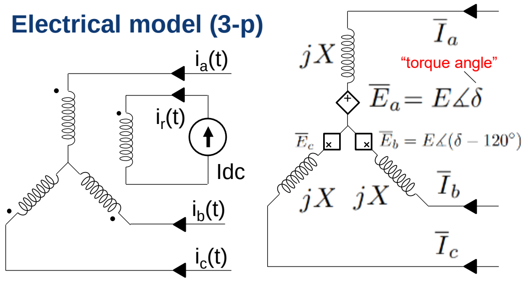
Electrical model (3-p)



$$v_a(t) = L_a \frac{di_a}{dt} + M_{ar}(\theta)\frac{di_r}{dt} + M'_{ar}(\theta) i_r(t) \frac{d\theta}{dt}$$

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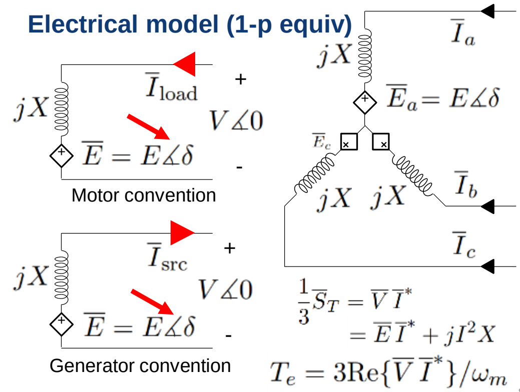
Electrical model (1-p equiv)



$$\frac{1}{3}\bar{S}_T = \bar{V}\bar{I}^* = \bar{E}\bar{I}^* + jI^2X$$

$$T_e = 3\mathrm{Re}\{\bar{V}\bar{I}^*\}/\omega_m$$

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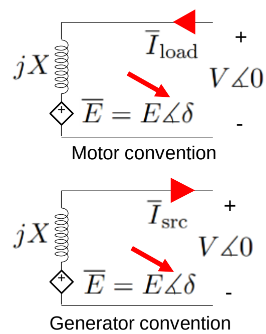
Equivalent circuit model



$$\frac{1}{3}\bar{S}_T = \bar{V}\bar{I}^*$$

$$T_e = 3\mathrm{Re}\{\bar{V}\bar{I}^*\}/\omega_m$$

Want Te > 0 as motor
Which current to use?
a) I_{load}; **b)** I_{src}

Generator.
Polarity of torque angle?
a) δ >0; **b)** δ <0; **c)** δ =0

$$\bar{E} = j\bar{I}_{src}\bar{X} + \bar{V}$$

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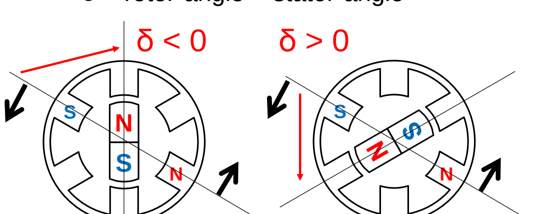
Polarity of the torque angle

δ = rotor-angle – stator-angle



Stator drags rotor
Motor

Rotor drags stator
Generator

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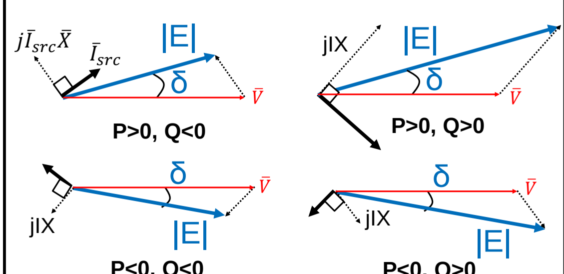
Phasor interpretation $\bar{E} = j\bar{I}_{src}\bar{X} + \bar{V}$



Underexcited: |E| cosδ < V
Inductor-like

Overexcited: |E| cosδ > V
Capacitor-like

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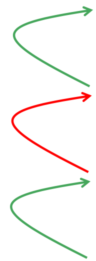
Arbitrary control of P & Q – Big picture



Decoupled: Real Power **P**, Reactive Power **Q**

Coupled: Real current I_d ("direct"), Imag current I_q ("quadrature")

Decoupled: Torque angle **δ**, Excitation voltage **|E|**

*Mech power input P_m, Rotor field current I_r

*Outside scope of ECE 330

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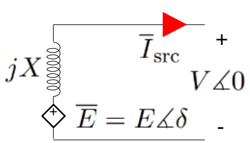
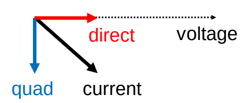
Direct and quadrature current



Generator convention

Take real and imag parts

$$\overline{I}_{\text{src}} = I_d + jI_q$$



Direct current
Aligned with voltage phasor
Proportional to real power

Quadrature current
Orthogonal to V phasor
Proportional to reactive power

Complex power (1-p)

$$\overline{V}\,\overline{I}^*_{\text{src}} = VI_d - jVI_q$$

Complex power (3-p)

$$P_{T,\text{src}} = 3VI_d, \quad Q_{T,\text{src}} = -3VI_q$$

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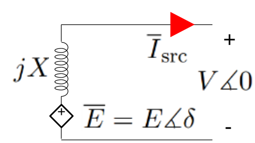
Controlling dq current via |E| and δ



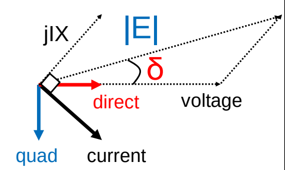
Generator convention

$$\overline{I}_{\text{src}} = I_d + jI_q$$

$$P_{T,\text{src}} = 3VI_d, \quad Q_{T,\text{src}} = -3VI_q$$

$$Q_{T,\text{src}} = \frac{3V(E\cos\delta - V)}{X}$$

$$P_{T,\text{src}} = \frac{3VE\sin\delta}{X}$$



"Feeder" circuit law

$$\overline{E} = \overline{V} + jX\overline{I}_{\text{src}}$$

Take real and imag parts

$$E\cos\delta = V - I_qX$$

$$E\sin\delta = 0 + I_dX$$

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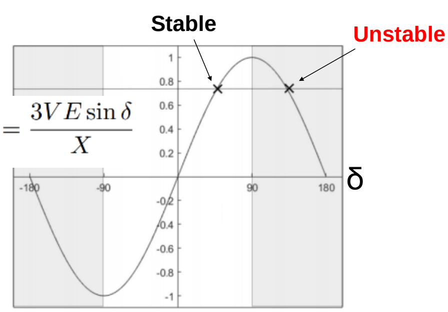
Stability requires |δ| < 90º

$$P_{T,\text{src}} = \frac{3VE\sin\delta}{X}$$



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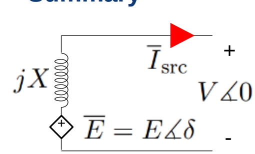
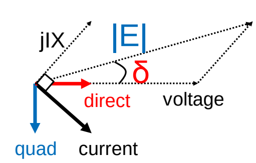
Summary



Generator convention



Phasor diagram

P, Q via d-q current (Decoupled)

$$P_{T,\text{src}} = 3VI_d, \quad Q_{T,\text{src}} = -3VI_q$$

$$\overline{I}_{\text{src}} = I_d + jI_q$$

d-q current via |E|, δ (Coupled)

$$\overline{E} = \overline{V} + jX\overline{I}_{\text{src}}$$

$$E\cos\delta = V - I_qX$$

$$E\sin\delta = 0 + I_dX$$

|E| via rotor current (Decoupled)

$$E = \text{const} \cdot I_r$$

Stability requires |δ| < 90º

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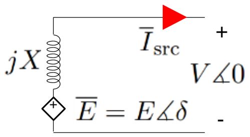
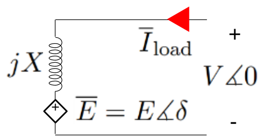
Warning: Polarity and sign errors!



Generator convention



Motor convention

$$Q_{T,\text{src}} = \frac{3V(E\cos\delta - V)}{X}$$

$$P_{T,\text{src}} = \frac{3VE\sin\delta}{X}$$

Formula sheet in previous exams

For $R_s = 0$: $P_T = \frac{3V_s|E_{ar}|}{X_s}\sin(-\delta)$

$$Q_T = \frac{3V_s^2}{X_s} - \frac{3V_s|E_{ar}|}{X_s}\cos(\delta)$$

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Example 6.5

Example 6.5 A three-phase, 60 Hz, six-pole, wye-connected synchronous generator is driven by a turbine that delivers 16910 W to the shaft. The friction and windage losses are 500 W. The field current is adjusted so that the voltage E_{ar} proportional to field current on a per phase basis is E_{ar} = 355 V. The generator supplies a load at 440 V (line-line). Find speed, $\overline{E}_{ar}$, $\overline{I}_a$, and real and reactive power output of the generator. Synchronous reactance $x_s = 5\Omega$.

40π mech. rad/sec

$$P_T = P_m = (16910 - 500)\text{ W} = 16410\text{ W}$$

Rephrase into a feeder problem

A three-phase source of voltage 355V (phase-to-neutral) is serving a single load connected through a feeder with impedance j5 Ω. The load draws 16410 W at an unknown PF. The voltage across the load is 440 V (line-to-line).
Find δ, current phasor, power.

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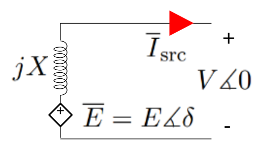
A three-phase source of voltage 355V (phase-to-neutral) is serving a single load connected through a feeder with impedance j5 Ω. The load draws 16410 W at an unknown PF. The voltage across the load is 440 V (line-to-line).
Find δ, current phasor, power.



$$P_{T,\text{src}} = 3VI_d,$$
$$Q_{T,\text{src}} = -3VI_q$$

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