ECE330: Power Circuits & Electromechanics

Lecture 20. Synchronous machines

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1

1

Last time: The world of electric machines

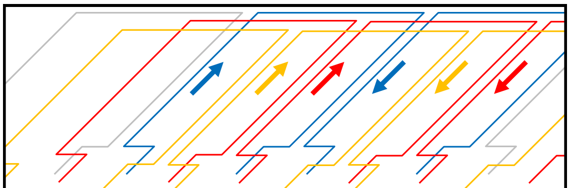
Switching DC fields
- Brushed DC motor
- ➔ Reluctance machine
- Stepper motor
- Brushless DC motor

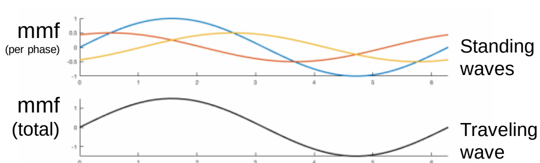
Rotating AC fields
- ➔ Synchronous machine
- ➔ Induction machine

Our focus in ECE 330

2

2



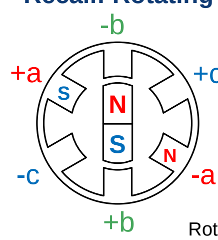


Three-phase 120 degrees apart in time and in space

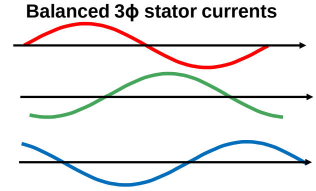
3

3

Recall: Rotating stator field



Balanced 3ϕ stator currents



Rotating field spins at synchronous speed
Drags rotor at same speed

Rotor construction?

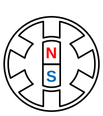
Magnetic material	➔	Reluctance machine
Permanent magnet	➔	Brushless DC
Electromagnet	➔	Synchronous machine

4

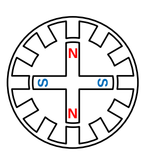
4

Important slide

Recall: Multiple poles



2-pole @ 60 Hz
3600 rpm



4-pole @ 60 Hz
1800 rpm

Synchronous **speed**
= 60 π [rad/s]
= 1800 rpm

$$\omega_m = \frac{2}{p}\omega_s$$

Synchronous **frequency**
= 120 π [rad/s]
= 60 [Hz]

Mechanical speed — # poles — Electrical frequency

5

5

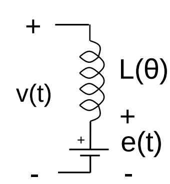
Today: Synchronous machine model

Flux linkage equation

$$\lambda(t) = \lambda(i, \theta)$$

Voltage of mechanical origin

$$v(t) = \frac{d\lambda}{dt} = \underbrace{\frac{\partial\lambda}{\partial i}}_{L(\theta)}\frac{di}{dt} + \underbrace{\frac{\partial\lambda}{\partial\theta}\frac{d\theta}{dt}}_{e(t)}$$



Energy conversion in a loop

$$0 = \underbrace{\oint_C i(\lambda', \theta')\, d\lambda'}_{\text{EFE}|_{\text{cycle}}} + \underbrace{\oint_C -\tau(\lambda', \theta')\, d\theta'}_{\text{EFM}|_{\text{cycle}}}$$

Average power $\quad \text{EFE}|_{\text{cycle}} \cdot \text{freq} = P_{\text{av}} = \text{Re}\{\overline{V}\,\overline{I}^*\}$

Average torque $\quad \text{EFM}|_{\text{cycle}} = -\tau_{\text{av}} \cdot \text{dist}$

6

6

Today

- Mechanical model (torque vs speed)
- Electrical model (current vs voltage)
- Example analysis

7

Average torque from average power

$$0 = \underbrace{\oint_C i(\lambda', \theta')\,\mathrm{d}\lambda'}_{\mathrm{EFE}|_{\mathrm{cycle}}} + \underbrace{\oint_C -\tau(\lambda', \theta')\,\mathrm{d}\theta'}_{\mathrm{EFM}|_{\mathrm{cycle}}}$$

$$\mathrm{EFE}|_{\mathrm{cycle}} \cdot \mathrm{freq} = P_{\mathrm{av}}$$

$$\mathrm{EFM}|_{\mathrm{cycle}} = -\tau_{\mathrm{av}} \cdot \mathrm{dist}$$

Pav = 1 W, Tav = ? Nm

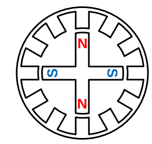
A) 2π / 60　B) 1 / 60π ←

C) 2π / 60　D) 1 / 120π

E) 1 / 240π

Freq = 60 Hz (as usual)

Dist = half-rotation = π



4-pole @ 60 Hz

1800 rpm

8

Average torque from average power

$$\mathrm{EFE}|_{\mathrm{cycle}} \cdot \mathrm{freq} = P_{\mathrm{av}}$$

$$\mathrm{EFM}|_{\mathrm{cycle}} = -\tau_{\mathrm{av}} \cdot \mathrm{dist}$$

$$\tau_{\mathrm{av}} = \frac{P_{\mathrm{av}}}{\mathrm{freq} \cdot \mathrm{dist}} = \frac{P_{\mathrm{av}}}{\mathrm{freq} \cdot 2\pi \cdot (2/p)} = \frac{P_{\mathrm{av}}}{\omega_m}$$

Average torque = Average power / Mechanical rad/s

Half the speed, same power, then twice the torque

Sacrifice speed to increase torque

9

Today

- Mechanical model (torque vs speed)
- Electrical model (current vs voltage)
- Example analysis

10

Preliminary: Interlocking loops

$-i_2$ $-i_1$ $+i_1$ $+i_2$

⊙ ⊙ ⊗ ⊗

$$\lambda_1 = +L_1 i_1 + |M| i_2$$
$$\lambda_2 = +|M| i_1 + L_2 i_2$$

$-i_2$ $+i_1$ $-i_1$ $+i_2$

⊙ ⊗ ⊙ ⊗

$$\lambda_1 = +L_1 i_1 - |M| i_2$$
$$\lambda_2 = -|M| i_1 + L_2 i_2$$

⊗ $+i_1$

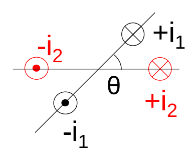
$-i_2$ ⊙　⊗ $+i_2$

$-i_1$ ⊙

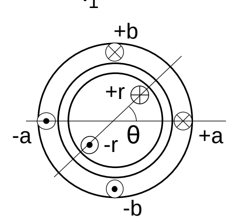
$$\lambda_1 = +L_1 i_1$$
$$\lambda_2 = \qquad + L_2 i_2$$

11

Inductance model



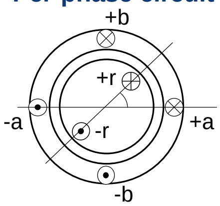
$$\lambda_1 = +L_1 i_1 + |M|\cos\theta\, i_2$$
$$\lambda_2 = +|M|\cos\theta\, i_1 + L_2 i_2$$



$$\lambda_a = +L_a i_a + |M|\cos\theta\, i_r$$
$$\lambda_b = +L_b i_b + |M|\sin\theta\, i_r$$
$$\lambda_r = +L_r i_r + |M|\cos\theta\, i_a + |M|\sin\theta\, i_b$$

12

Per-phase circuit model



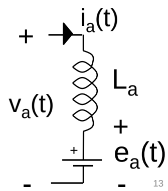
$\lambda_a = + L_a\, i_a + |M|\cos\theta\, i_r$
$\lambda_b = + L_b\, i_b + |M|\sin\theta\, i_r$
$\lambda_r = +L_r\, i_r + |M|\cos\theta\, i_a + |M|\sin\theta\, i_b$

fix $i_r(t) = I_r$, let $\theta = \omega t$

$$v_a = L_a \frac{\mathrm{d}i_a}{\mathrm{d}t} - \omega M I_r \sin\omega t$$

90^o shift

$$v_b = L_b \frac{\mathrm{d}i_b}{\mathrm{d}t} + \omega M I_r \cos\omega t$$



13

13

Per-phase phasor model

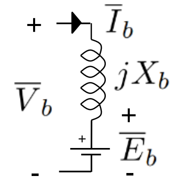
Time-domain

$$v_b = L_b \frac{\mathrm{d}i_b}{\mathrm{d}t} + \omega M I_r \cos\omega t$$

Phasor representation

$$\overline{V}_b = j\omega L_b \cdot \overline{I}_b + \overline{E}_b$$

where $|\overline{E}| = \omega_s M I_r / \sqrt{2}$ ← Always true



4-pole, 60 Hz, M = 1 H, Ir = 1 A.
| Eb | = ? V

A) 2π / 60 B) 60π / sqrt(2)
C) 60π D) 120π / sqrt(2)
E) 120π

electrical domain ignores number of poles

14

14

Per-phase phasor model

Phasor representation

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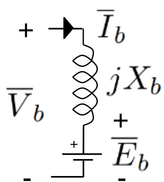
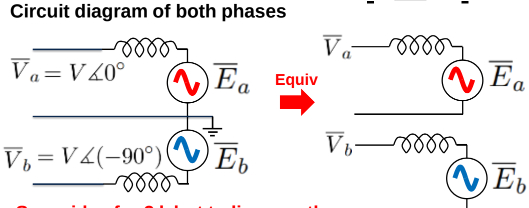
where $|\overline{E}| = \omega_s M I_r / \sqrt{2}$



Circuit diagram of both phases



Same idea for 3ϕ but tedious math

15

15

Today

- Mechanical model (torque vs speed)
- Electrical model (current vs voltage)
- Example analysis

16

16

Example 6.4

Example 6.4 A two-pole, three-phase, 60 Hz, wye-connected synchronous machine has synchronous reactance $x_s = 2\Omega$ per phase. The machine is operating as a generator delivering power at a voltage of 1905 V per phase. The current is 350 A and the PF of the load is 0.8 lagging. Find E_{ar}, δ, and the torque of electric origin.

↓ Rephrase into a feeder problem

A three-phase source is serving a single load connected through a feeder with impedance j2 Ω. The load draws 350 A per phase at a lagging PF of 0.8. The voltage across the load is 1905V (phase-to-neutral).

a) Compute the source voltage as a complex phasor.

b) Compute the power consumed at the load and divide it by 120π, the mech speed of a 2-pole machine.

17

17

Example 6.4

A three-phase source is serving a single load connected through a feeder with impedance j2 Ω. The load draws 350 A per phase at a lagging PF of 0.8. The voltage across the load is 1905V (phase-to-neutral).

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18

18