ECE330: Power Circuits & Electromechanics

Lecture 3. Power Factor Correction

Prof. Richard Y. Zhang
Univ. of Illinois at Urbana-Champaign
ryz@illinois.edu

1

1

Schedule

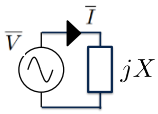
- ~~Wed 1/22: Phasors~~
- ~~Fri 1/24: Complex power~~ ← **Ready for HW1**
- Mon 1/27: Power factor correction
- **Wed 1/29: Quiz 1 + Review**
- Fri 1/31: Three-phase power

2

2

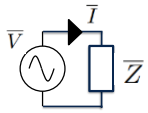
Important

Last time: Reactance and impedance



"Imaginary Ohm's law"

$$\overline{V} = (jX)\overline{I}$$
$$\overline{I} = \overline{V}/(jX)$$



"Complex Ohm's law"

$$\overline{V} = \overline{Z}\,\overline{I}$$
$$\overline{I} = \overline{V}/\overline{Z}$$

Impedance [Ω] (Complex number) $\overline{Z} = R + jX$ **Reactance [Ω]** (Real number)

Reactance = **Im**{ Impedance }

3

3

Important

Last time: Complex power

Phasors		Waves
$\overline{V}_{rms} = V\angle\theta_v$ [V]	Equivalence	$v(t) = \sqrt{2}V\cos(\omega t + \theta_v)$ [V]
$\overline{I}_{rms} = I\angle\theta_i$ [A]		$i(t) = \sqrt{2}I\cos(\omega t + \theta_i)$ [A]
↓ product		↓ product
Complex Power [VA]		Instantaneous Power [W]
$\overline{S} = \overline{V}_{rms}\overline{I}^{*}_{rms}$		$p(t) = VI[\cos(2\omega t + \theta_v + \theta_i) + \cos(\theta_v - \theta_i)]$
↓ real part		↓ average
Real Power [W]	Equal	Average Power [W]
$P = \mathrm{Re}\{\overline{V}_{rms}\overline{I}^{*}_{rms}\}$		$\langle p(t)\rangle = VI\cos(\theta_v - \theta_i)$
Simple complex arithmetic		Complicated trig ids & integrals

4

4

Important

Last Time: Power Triangle

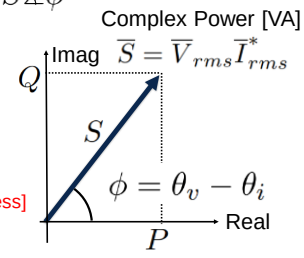
$$\overline{S} = P + jQ = S\angle\phi$$

Reactive Power [VAR]
$Q = \mathrm{Im}\{\overline{V}_{rms}\overline{I}^{*}_{rms}\}$

Apparent Power [VA]
$S = |\overline{S}|$

Power Factor [Dimensionless]
$\cos\phi = P/S$

$\phi > 0$ Lagging
$\phi < 0$ Leading

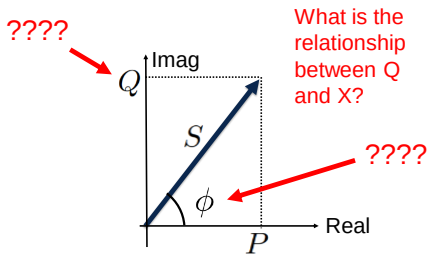


Real Power [W]
$P = \mathrm{Re}\{\overline{V}_{rms}\overline{I}^{*}_{rms}\}$

5

5

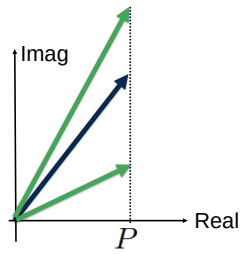
Today: Reactive power and power factor



6

6

Goal: Improve the power factor



Which one is "improved"?

(Why does it matter?)

7

7

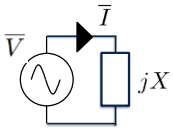
Today

- Reactance and reactive power
- Power factor correction ← "why?"
- Example problem ← "how?"

8

8

Reactance and reactive power



$$\overline{I} = I\angle\theta, \qquad \text{Complex Ohm's law}$$

$$\overline{V} = (jX)(I\angle\theta) = (X\angle 90°)(I\angle\theta)$$
$$= IX\angle(\theta + 90°)$$

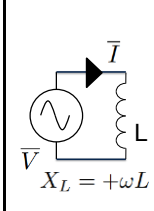
$$\overline{S} = \overline{V}\overline{I}^{*} = (IX\angle(\theta + 90°))(I\angle\theta)^{*}$$
$$= I^2X\angle(\theta + 90° - \theta) = I^2X\angle 90°$$
$$= 0 + jI^2X = 0 + j\frac{V^2}{X}$$

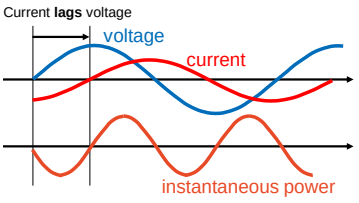
Think P = I²R from ECE 210

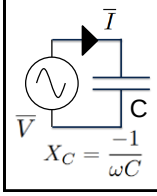
Reactive power law $Q = I^2X = V^2/X$

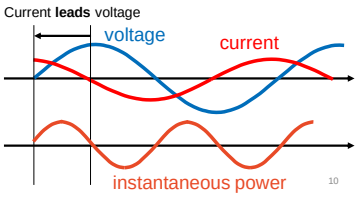
9

9







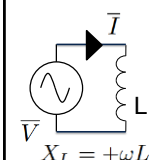


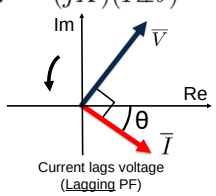
10

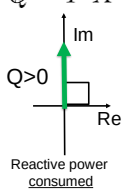
10

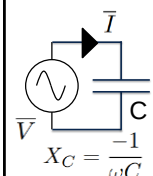
Important

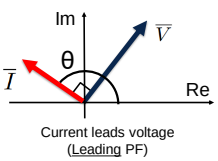
$$\overline{V} = (jX)(I\angle\theta) \qquad Q = I^2X$$

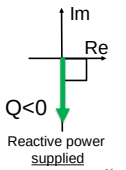








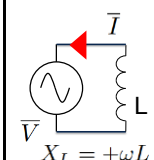


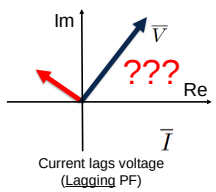


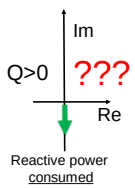
11

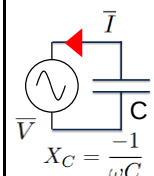
11

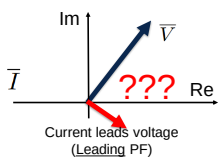
Extreme warning!! Direction of current!!

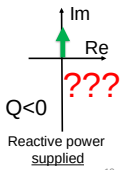












12

12

Important

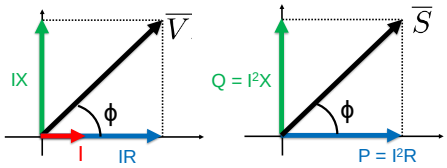
Impedance and complex power

$\overline{I} = I\angle 0°$

$\overline{Z} = R + jX$

$$\overline{V} = \overline{ZI} = (R + jX) \cdot I = IR + jIX$$

$$\overline{S} = \overline{VI}^* = (IR + jIX) \cdot I = I^2R + jI^2X$$



Impedance load: Z angle = PF angle

13

13

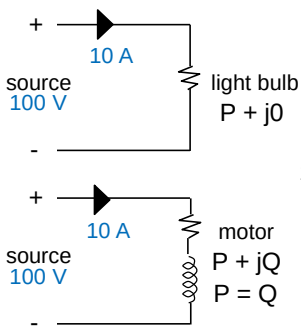
Today

- ~~Reactance and reactive power~~
- Power factor correction
- Example problem

14

14

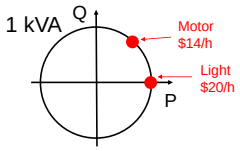
Apparent power limit



Constraints
- Load voltage = 100 V
- Wire rating = 10 A
- Revenue: 20c per kWh

Just the magnitude

VI = S
= $(P^2 + Q^2)^{1/2}$

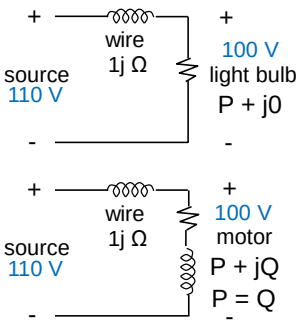


Fine the motor for poor PF!

15

15

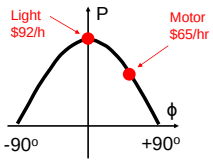
Voltage headroom



Constraints
- Source voltage = 110V
- Load voltage ≥ 100V
- Revenue: 20c per kWh

Skipping to lecture 26

$$P \approx \frac{V_s V_l}{X} \cos\phi$$



Fine the motor for poor PF!

16

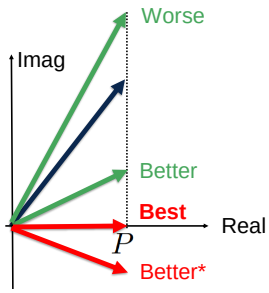
16

Goal: Improve the power factor

Which one is better?

Which one is better?

Why is P unchanged?



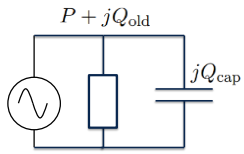
Holy grail: Unity power factor (PF = 1)

17

17

Important

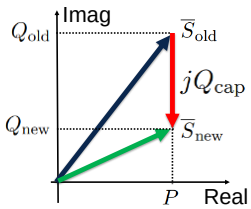
Power factor correction



$$\overline{S}_{old} = P + jQ_{old}$$

$$\overline{S}_{new} = P + j(Q_{old} + Q_{cap})$$

$$Q_{cap} = Q_{new} - Q_{old} < 0$$



Requires conservation of complex power (Pai Sec 2.4)

Report as: Add $|Q_{cap}|$ VARs of capacitance

18

18

Today

- ~~Reactance and reactive power~~
- ~~Power factor correction~~
- Example problem

19

19

Conservation laws

$\overline{I}_{\text{tot}}$ $\overline{I}_1$ $\overline{I}_2$

$\overline{V}$

$\overline{V} = \sqrt{2}\angle 45°$ [V]

$\overline{I}_1 = \frac{1}{\sqrt{2}}\angle 45°$ [A]

$\overline{I}_2 = \sqrt{2}\angle -45°$ [A]

What is $\overline{S}_{\text{tot}} = \overline{V}\overline{I}^*_{\text{tot}}$?

Aside: Conservation of current

$$\overline{I}_1 = 0.5 + j0.5$$
$$\overline{I}_2 = 1 - j1$$
$$\overline{S}_{\text{tot}} = \overline{V}\,\overline{I}^*_{\text{tot}}$$
$$= (1 + j1)(1.5 + j0.5)$$
$$= (1.5 - 0.5) + j(1.5 + 0.5)$$

Conservation of complex power

$$\overline{S}_1 = \overline{V}\overline{I}^*_1$$
$$= (\sqrt{2}\angle 45°)\left(\tfrac{1}{\sqrt{2}}\angle 45°\right)^*$$
$$= 1\angle 0° = 1 + 0j$$
$$\overline{S}_2 = \overline{V}\overline{I}^*_2$$
$$= (\sqrt{2}\angle 45°)\left(\sqrt{2}\angle -45°\right)^*$$
$$= 2\angle 90° = 0 + 2j$$

$\overline{S}_{\text{tot}} = \overline{S}_1 + \overline{S}_2 = 1 + 2j$

Unit: Volt-Amps [VA]

20

20

PF correction

$\overline{I}_{\text{tot}}$ $\overline{I}_1$ $\overline{I}_2$

$\overline{V}$

$\overline{V} = \sqrt{2}\angle 45°$ [V]

$\overline{I}_1 = \frac{1}{\sqrt{2}}\angle 45°$ [A]

$\overline{I}_2 = \sqrt{2}\angle -45°$ [A]

$\overline{S}_{\text{tot}} = \overline{S}_1 + \overline{S}_2 = 1 + 2j$

Unit: Volt-Amps [VA]

What is the power factor?

$$\cos\phi = \frac{\text{Re}\{\overline{S}\}}{|\overline{S}|} = \frac{1}{\sqrt{1^2 + 2^2}}$$

≈ 0.447 lagging

How much capacitance to improve to unity?

2 VARs of capacitance (-2 VAR of reactive load)

How much capacitance to improve PF to 0.95?

$\phi = +\cos^{-1}(0.95),$

$Q_{\text{new}} = P\tan\phi \approx 0.329.$

$P/|\overline{S}_{\text{new}}| = \cos\phi$

$P^2 + Q^2_{\text{new}} = |\overline{S}_{\text{new}}|^2$

$Q_{\text{new}} = \sqrt{\frac{P^2}{\cos^2\phi} - P^2}$

$2 - 0.329 = 1.671$ VARs

10-20% cost savings

21

21

Reduction in line current

$\overline{I}_{\text{tot}}$ $\overline{I}_1$ $\overline{I}_2$

$\overline{V}$

$\overline{V} = \sqrt{2}\angle 45°$ [V]

Line current

$\overline{S} = \overline{V}\overline{I}^*$

$|\overline{S}| = |\overline{V}|\,|\overline{I}|$

$\overline{S}_{\text{old}} = 1 + 2j$ [VA] (PF = 0.447 lag)

$\overline{S}_{\text{new}} = 1 + 0.329j$ [VA] (PF = 0.95 lag)

$\overline{S}_{\text{unity}} = 1 + 0j$ [VA] (PF = 1)

$\overline{I}_{\text{old}} = |\overline{S}_{\text{old}}|/|\overline{V}| \approx 1.581$ [A]

-0.837 A

$\overline{I}_{\text{new}} = |\overline{S}_{\text{new}}|/|\overline{V}| \approx 0.7443$ [A]

-0.037 A

$\overline{I}_{\text{unity}} = |\overline{S}_{\text{unity}}|/|\overline{V}| \approx 0.7071$ [A]

22

22

Schedule

- ~~Wed 1/22: Phasors~~
- ~~Fri 1/24: Complex power~~ ← **Ready for HW1**
- ~~Mon 1/27: Power factor correction~~
- **Wed 1/29: Quiz 1 + Review**
- Fri 1/31: Three-phase power

23

23