ECE 330 Exam #1, Fall 2015    Name: Solution
90 Minutes

Section (Check One)    MWF 10am ______    TR 2:00pm ______

1. ______ / 25    2. ______ / 25

3. ______ / 25    4. ______ / 25    Total ______ / 100

## Useful information

$\sin(x) = \cos(x - 90^\circ)$    $\bar{V} = \bar{Z}\bar{I}$    $\bar{S} = \bar{V}\bar{I}^* = P + jQ$    $\bar{S}_{3\phi} = \sqrt{3}V_L I_L \angle\theta$

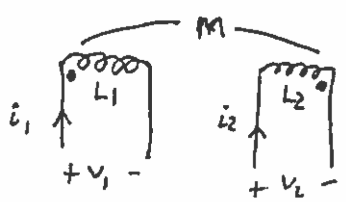
$0 < \theta < 180^\circ$ (lag)    $I_L = \sqrt{3} I_\phi$ (delta)    $\bar{Z}_Y = \bar{Z}_\Delta / 3$    $\mu_0 = 4\pi \cdot 10^{-7}$ H/m

$-180^\circ < \theta < 0$ (lead)    $V_L = \sqrt{3} V_\phi$ (wye)

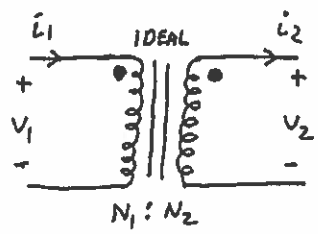
**ABC sequence has A at zero, B at minus 120 degrees, and C at plus 120 degrees**

$\int_C \mathbf{H} \cdot \mathbf{dl} = \int_S \mathbf{J} \cdot \mathbf{n} da$    $\int_C \mathbf{E} \cdot \mathbf{dl} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot \mathbf{n} da$    $\Re = \frac{l}{\mu A}$    $MMF = Ni = \phi\Re$

$\phi = BA$    $\lambda = N\phi$    $k = \frac{M}{\sqrt{L_1 L_2}}$    1 hp = 746 Watts

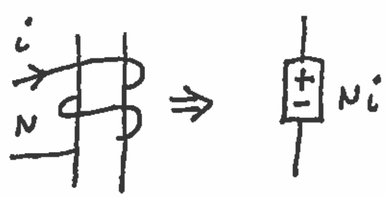



$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$




$a = \frac{N_1}{N_2}$    $N_1 i_1 = N_2 i_2$

$\frac{V_1}{V_2} = \frac{N_1}{N_2}$



**Problem 1. (25 points)**

A single phase ac voltage source of 240 Volts RMS, 60 HZ supplies a load that draws 13 Amps RMS and a real power of 2,700 Watts.

a) Find the power factor of the load – assume that it is lagging.

$$S = 240 \times 13 = 3120 \text{ VA} \qquad pf = \frac{2700}{3120} = 0.865 \text{ lag}$$

b) If it really is lagging, how many VARS from a capacitor bank are required to improve the power factor to 0.95 lagging?

$$\bar{S} = 3120 \angle 30.07^\circ = 2700 + j1563$$

$$\bar{S}_{new} = \frac{2700}{0.95} \angle +\cos^{-1} 0.95 = 2842 \angle 18.2^\circ = 2700 + j888$$

$$Q_{adj} = 1563 - 888 = 675 \text{ VARS}$$

c) What is the magnitude of the current supplied by the source after the addition of the above capacitor bank if it really is lagging power factor?

$$I = \frac{2842}{240} = 11.8 \text{ A}$$

d) What is the magnitude of the current supplied by the source after the addition of the above capacitor bank if the original load is actually leading power factor?

$$\bar{S}_{new} = 2700 - j1563 - j675 = 2700 - j2238$$

$$|\bar{S}| = 3507 \text{ VA} \qquad I = \frac{3507}{240} = 14.6 \text{ A}$$

**Problem 2. (25 points)**

A 120 (L-N)/208 (L-L) Volts RMS, 3-phase, 4 wire, wye-connected, ABC sequence, 60 HZ source supplies the following two lagging power factor loads:

Load #1 14 Amps (line current) 3500 Watts (3-phase), wye-connected
Load #2 8 Amps (phase current) 0.8 power factor, delta connected

a) What is the total line current (magnitude only) being supplied by the source?

$$|\bar{S}_1| = \sqrt{3} \times 14 \times 208 = 5043\,VA \quad pf = \frac{3500}{5043} = 0.694$$

$$\bar{S}_1 = 5043 \angle 46.05^\circ = 3500 + j3631 \leftarrow$$

$$\bar{S}_2 = 3 \times 8 \times 208 \angle +\cos^{-1} 0.8 = 3994 + j2995 \leftarrow$$

$$\bar{S}_{TOT} = 7494 + j6626 = 10{,}003 \angle 41.5^\circ$$

$$I_L = \frac{10{,}003}{\sqrt{3} \times 208} = 27.8\,A$$

b) How many capacitive VARS 3-phase need to be supplied at the load to reduce the total source line current to 25 Amps?

$$|\bar{S}| = \sqrt{3} \times 208 \times 25 = 9006.4$$

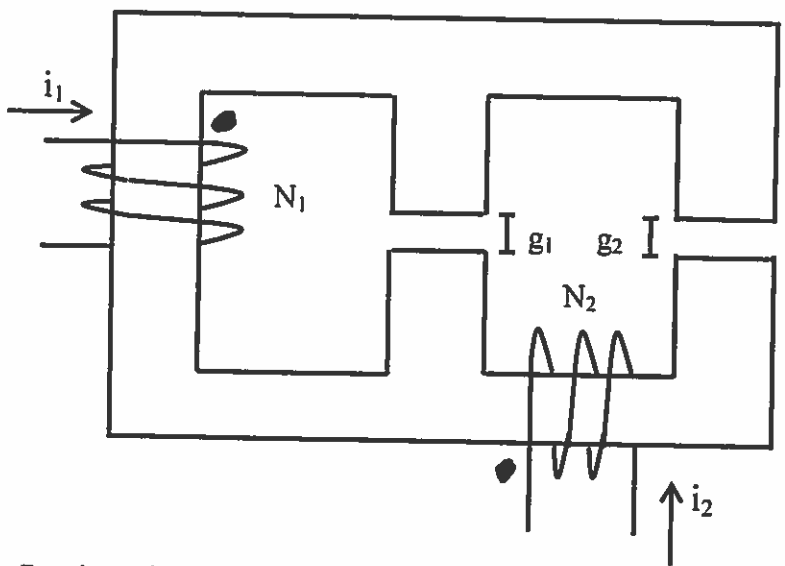
$$P = 7494 \quad \text{so} \quad pf = 0.832 \quad \theta = 33{,}69^\circ$$

$$Q = 9006.4 \sin 33.69^\circ = 4995\ VARs$$

$$\text{Add } 6626 - 4995 = 1631\ VARs$$

**Problem 3. (25 points)**

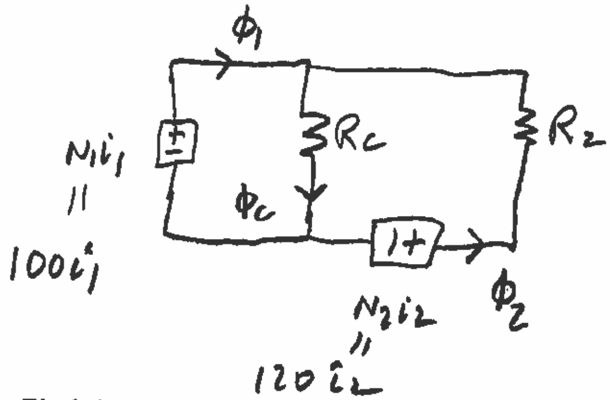
Consider the iron core geometry given in the figure below. Neglect fringing in the air gaps and assume the iron core has infinite permeability. The gaps are $g_1 = 0.10$ cm, $g_2 = 0.12$ cm and $A_{core} = 2\ cm^2$, $N_1 = 100$, and $N_2 = 120$ turns.




a) Put the polarity dots on the two coils.

*See figure*

b) Draw the equivalent magnetic circuit and label everything you need to solve for the magnetic fluxes in the iron core segments.




$$R_c = \frac{0.001}{4\pi\times10^{-7}\times2\times10^{-4}} = 39.8\times10^{5}$$

$$R_2 = \frac{0.0012}{4\pi\times10^{-7}\times2\times10^{-4}} = 47.7\times10^{5}$$

c) Find the positive mutual inductance (M) between the two coils

$$100 i_1 = 39.8\times10^{5}\,\phi_c \qquad \phi_c = 25\times10^{-6}\, i_1$$

$$100 i_1 = -47.7\times10^{5}\,\phi_2 + 120 i_2 \qquad \phi_2 = -21\times10^{-6}\, i_1 + 25.2\times10^{-6}\, i_2$$

Continued on the next page

$\lambda_2 = 120\phi_2 = -0.00252 i_1 + 0.00302 i_2$

$$m = 0.00252 \text{ H}$$

d) Find the magnetic flux density in each of the three the iron core segments (left, center and right) when the currents are $i_1$ =7 Amps (DC) and $i_2$ =6 Amps (DC)

$$\phi_1 = \phi_c - \phi_2 = 25\times10^{-6} i_1 + 21\times10^{-6} i_1 - 25.2\times10^{-6} i_2$$

$$= 46\times10^{-6}\times7 - 25.2\times10^{-6}\times6$$

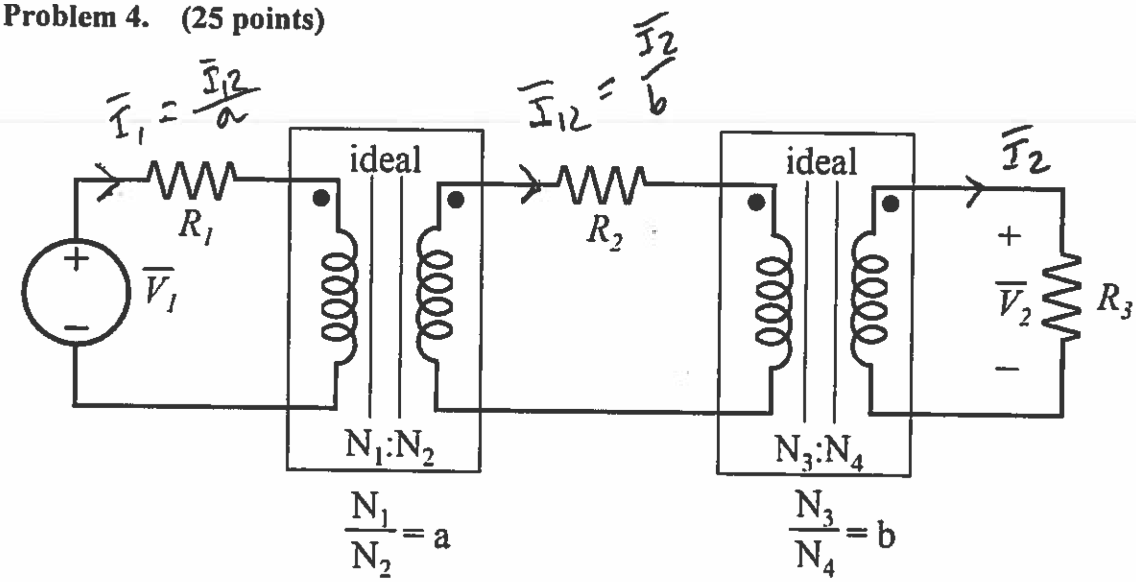
$$= 322\times10^{-6} - 151.2\times10^{-6} = 170.8\times10^{-6}\ \text{wb}$$

$$B_1 = \frac{\phi_1}{2\times10^{-4}} = \frac{170.8\times10^{-6}}{2\times10^{-4}} = 0.854\ T$$

$$B_2 = \frac{\phi_2}{2\times10^{-4}} = \frac{-21\times10^{-6}\times7 + 25.2\times10^{-6}\times6}{2\times10^{-4}} = \frac{4.2\times10^{-6}}{2\times10^{-4}} = 0.021\ T$$

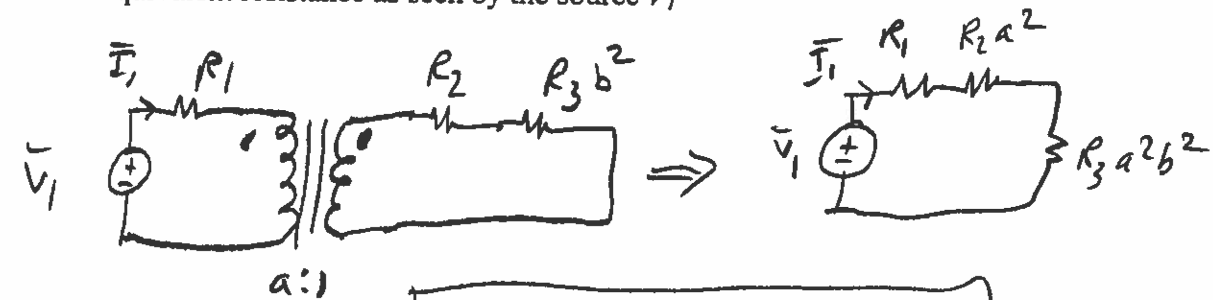
$$B_c = \frac{\phi_c}{2\times10^{-4}} = \frac{25\times10^{-6}\times7}{2\times10^{-4}} = 0.875\ T$$

**Problem 4. (25 points)**




For the circuit showed above:

a) Find the equivalent resistance as seen by the source $\overline{V_1}$




$\bar{V}_1 = \bar{I}_1 R_{eq}$

$$\boxed{R_{eq} = R_1 + R_2 a^2 + R_3 a^2 b^2}$$

b) Find the voltage $\overline{V_2}$ as a function of $\overline{V_1}$

$$\bar{V}_2 = \bar{I}_2 R_3 = b R_3 \bar{I}_{12} = a b R_3 \bar{I}_1 = a b R_3 \frac{\bar{V}_1}{R_{eq}}$$

$$= \frac{a b R_3}{R_1 + R_2 a^2 + R_3 a^2 b^2} \bar{V}_1$$