ECE 330 (Spring 2018) **Instructors:** S. Bose and H. Sowidan

Midterm 1

Duration: 90 minutes Total points: 100

Name: Solutions.

Section (Tick one): C (9:30am) ________ F (2:00pm) ________.

Scores (For official use only):

Problem 1: 0 /25, Problem 2: 0 /25,
Problem 3: 0 /25, Problem 4: 0 /25. **Total score:**______________/100.

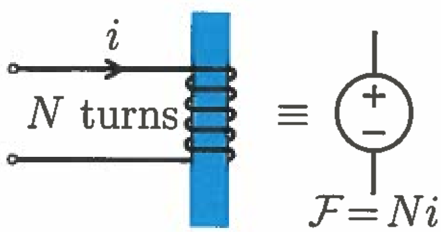
Relevant formulae

$\sin(x) = \cos(90^o - x)$ $\quad \bar{V} = \bar{I}\bar{Z}$ $\quad \bar{S} = \bar{V}\bar{I}^*$ $\quad \bar{S}_{3\phi} = \sqrt{3}V_L I_L \angle\theta$

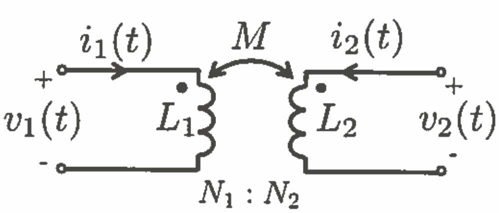
$$\begin{cases} 0^o < \theta < 180^o & : \text{lag} \\ -180^o < \theta < 0^o & : \text{lead} \end{cases} \qquad \begin{cases} I_L = \sqrt{3}I_\phi & : \text{delta} \\ V_L = \sqrt{3}V_\phi & : \text{wye} \end{cases} \qquad \bar{Z}_Y = \frac{\bar{Z}_\Delta}{3} \qquad \mu_0 = 4\pi \times 10^{-7} H/m$$

$$\int_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot \mathbf{n}\, da \qquad \int_C \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t}\int_S \mathbf{B} \cdot \mathbf{n}\, da \qquad \mathfrak{R} = \frac{l}{\mu A} \qquad \mathcal{F}(\text{mmf}) = Ni = \phi\mathfrak{R}$$

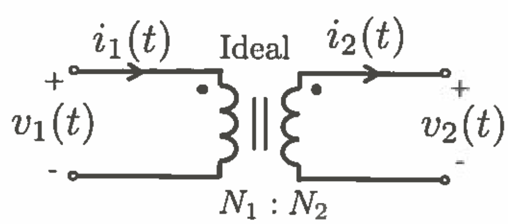
$$\phi = BA \qquad \lambda = N\phi \qquad v = \frac{d\lambda}{dt} \qquad k = \frac{M}{\sqrt{L_1 L_2}}$$



$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$



$$v_1(t) = L_1 \frac{d}{dt}[i_1(t)] + M\frac{d}{dt}[i_2(t)]$$



$$\frac{i_1(t)}{i_2(t)} = \frac{N_2}{N_1} = \frac{1}{a} \text{ and } \frac{v_1(t)}{v_2(t)} = \frac{N_1}{N_2} = a$$

Problem 1 [25 points]

A single-phase voltage source serves three loads connected in parallel at 60 Hz and 120 V (rms). The three loads are described as follows:

- Load 1 draws 3kVA at 0.5 power factor, lagging.
- Load 2 has an impedance of $(3 + j3)\Omega$.
- Load 3 is capacitive, and draws a current of magnitude 10 A (rms) and 1 kW of real power.

(a) Find the total complex power drawn by all three loads together. [12 points]
Hint: Capacitive load draws power at a leading power factor.

(b) Assuming a zero phase angle for the voltage source, compute the current supplied by the source as a function of time. [5 points]

(c) How much reactive VAR should be supplied by a capacitor in parallel to the three loads to make the overall power factor drawn from the source to be unity? What is the capacitance of such a capacitor? [3 + 5 points]

A. $\bar{S}_1 = |\bar{S}_1| \angle \cos^{-1}(pf) = 3000 \angle \cos^{-1}(0.5) = 3000 \angle 60^\circ$ VA   2

$1500 + j2598.07$ VA

$\bar{S}_2 = \dfrac{|V|^2}{Z^*} = \dfrac{(120)^2}{(3+j3)^*} = 3394 \angle +45^\circ$ VA $= 2399.92 + j2399.92$ VA

3

$|\bar{S}_3| = |V| \cdot |\bar{I}_3| = (120)(10) = 1200$ VA, $pf_3 = \dfrac{P_3}{|\bar{S}_3|} = \dfrac{1000}{1200} = 0.833$

$\bar{S}_3 = |\bar{S}_3| \angle -\cos^{-1}(pf_3) = 1200 \angle \cos^{-1}(0.833) = 1200 \angle -33.56^\circ$ VA

leading (capacitive)

$1000 - j663.37$ 3

$\boxed{\bar{S}_T = \bar{S}_1 + \bar{S}_2 + \bar{S}_3 = 6542 \angle 41.50^\circ \text{ VA} = 4900 + j4335 \text{ VA}}$ 4

B. $\bar{I}_T = \left(\dfrac{\bar{S}_T}{\bar{V}_T}\right)^* = \left(\dfrac{6542 \angle 41.5^\circ}{120 \angle 0^\circ}\right)^* = \boxed{54.52 \angle -41.50^\circ \text{ A}}$ $= 40.83 - j36.12$ 3 2

$\boxed{i(t) = 77.1 \cos(2\pi 60 t - 41.50^\circ) \text{ A}}$

C. $P = Re[\bar{S}_T] = 4900$ W

$pf_{new} = 1 \leftarrow$ (unity)

$Q_{old} = Im[\bar{S}_T] = 4335$ VAR

$|\bar{S}_{new}| = \dfrac{P}{pf_{new}} = \dfrac{4900}{1} = 4900$ VA, $Q_{new} = \sqrt{|\bar{S}_{new}|^2 - P^2} = 0 \leftarrow$ (unity pf)

$Q_{cap} = Q_{new} - Q_{old} = 0 - 4335 = -4335$ VAR $\rightarrow$ $\boxed{4335 \text{ VAR supplied}}$ 3

1.C. (cont.)

$$\bar{S} = P + jQ = \frac{|V|^2}{R + jX} \longrightarrow |Q_c| = \frac{|V|^2}{|X_c|} = \frac{|V|^2}{\left(\frac{1}{\omega C}\right)} = \omega C |V|^2$$

$$C = \frac{|Q_c|}{\omega |V|^2} = \frac{4335}{(2\pi 60)(120)^2} = \boxed{7.985 \cdot 10^{-4}\ F}$$ 5

Problem 2 [25 points]

Suppose a positive sequence three-phase generator (source) supplies two loads connected in parallel through a lossless transmission line. The line-to-line voltage across the load is 4.16 kV.

- Load 1 is <u>wye-connected</u> with a per phase impedance of $60\angle 25°\ \Omega$.
- Load 2 is <u>delta-connected</u> with a per phase impedance of $90\angle 10°\ \Omega$.

(a) Draw the per phase equivalent of the circuit for phase a. [6 points]
Hint: Transform the delta-connected load to an equivalent wye-connected load.

(b) Compute the line current phasors $\bar{I}_a, \bar{I}_b, \bar{I}_c$ and the phase voltage phasors $\bar{V}_{an}, \bar{V}_{bn}, \bar{V}_{cn}$ at the generator end. Assume $\angle\bar{V}_{an} = 0$. [4 + 4 points]

(c) Complete the phasor diagram provided by sketching and labeling the line current and phase voltage phasors computed in part (b). [5 points]

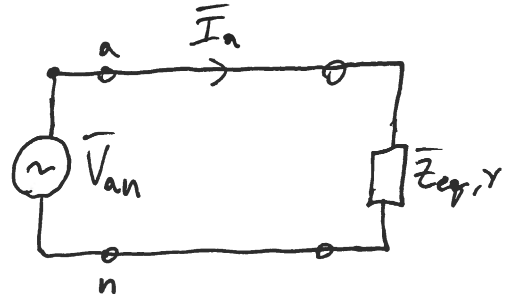
(d) Describe how the phasor diagram in part (c) would change if the three-phase generator was *negative* sequence. [2 points]

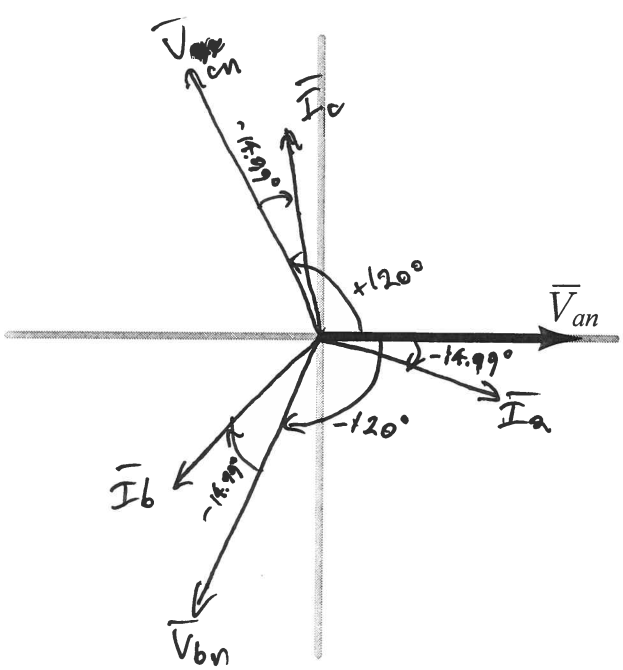
(e) Calculate the total complex power drawn by the two loads together. [4 points]

A. $\bar{Z}_{1,Y} = 60\angle 25°\ \Omega$

$\bar{Z}_{2,Y} = Z_{2,\Delta}\cdot\frac{1}{3} = (90\angle 10°)\cdot\frac{1}{3} = 30\angle 10°\ \Omega$

$\bar{Z}_{eq,Y} = Z_{1,Y} \,||\, Z_{2,Y} = 20.15\angle 14.99°\ \Omega$





B. $|\bar{V}_{an}| = \frac{V_{\ell\text{-}\ell}}{\sqrt{3}} = \frac{4.16\ kV}{\sqrt{3}} = 2.402\ kV$

$\bar{V}_{an} = |\bar{V}_{an}|\angle 0° = 2.402\angle 0°\ kV$

$\bar{V}_{bn} = \bar{V}_{an}\cdot(1\angle -120°) = 2.402\angle -120°\ kV$

$\bar{V}_{cn} = \bar{V}_{an}\cdot(1\angle +120°) = 2.402\angle +120°\ kV$

$\bar{I}_a = \frac{\bar{V}_{an}}{\bar{Z}_{eq,Y}} = \frac{(2.402\angle 0°)\ kV}{(20.15\angle 14.99°)}$

$= 0.1192\angle -14.99°\ kA = 119.2\angle -14.99°\ A$

$\bar{I}_b = \bar{I}_a\cdot(1\angle -120°) = 119.2\angle -134.99°\ A$

$\bar{I}_c = \bar{I}_a\cdot(1\angle +120°) = 119.2\angle +105.01°\ A$

2.C. Switch $\bar{I}_b$ + $\bar{I}_c$

Switch $\bar{V}_{cn}$ + $\bar{V}_{bn}$

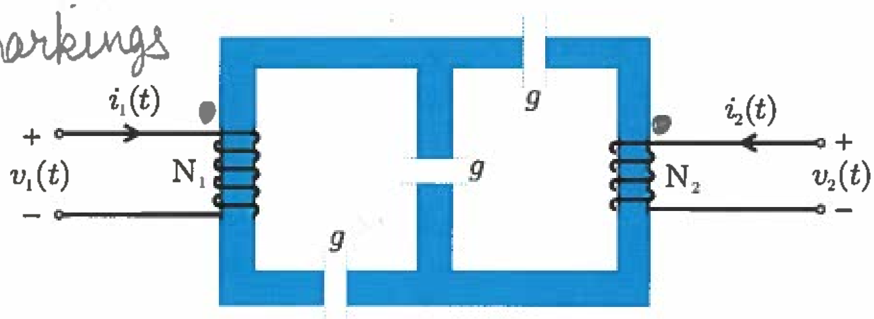
D. $\bar{S}_T = 3 \cdot \bar{V}_{an} \cdot \bar{I}_a^* = 3 \cdot (2.402\angle 0° \text{ kV}) \cdot (119.2\angle -14.99° \text{ A})^*$

$= \boxed{858.7\angle 14.99° \text{ kVA}}$

Problem 3 [25 points]

The magnetic circuit shown below is immersed in a gas with a relative permeability, $\mu_r = 2$, which completely fills the gaps in the core. Assume the iron core has infinite permeability, i.e., $\mu_c = \infty$. Neglect fringing effects in the gaps. Each gap in the core has length $g = 5$ mm. The cross-sectional area for all parts of the core is $A = 20\text{ cm}^2$. Assume that the windings have no resistance, and neglect any core losses. The permeability of free space is given by $\mu_0 = 4\pi \times 10^{-7}$ H/m. The number of turns on each coil is $N_1 = 20$ and $N_2 = 40$.

b) → dot markings



(a) Calculate the reluctance of each gas-filled gap. [2 points]

(b) Draw the magnetic circuit for the iron core and the two windings. [5 points]

(c) Utilize the magnetic circuit to solve for the flux linkages $\lambda_1(t)$ and $\lambda_2(t)$ in the two coils in terms of the currents $i_1(t)$ and $i_2(t)$. [8 points]

(d) Find numerical values for the self inductances L_1 and L_2, and the mutual inductance M of the two coils. [3 points]

(e) Using the previously determined values of L_1, L_2, and M, calculate the coupling coefficient $k = \frac{M}{\sqrt{L_1 L_2}}$. Comment whether the coupling is tight or loose. [2 points]

(f) Draw the polarity markings on the coils to indicate the nature of the coupling among the two coils on the above figure. [3 points]

(g) Express an equation for the voltage, $v_1(t)$, in terms of the currents $i_1(t)$ and $i_2(t)$ (and/or their derivatives). [2 points]

a) $\mu_r = 2$, $g = 5\times10^{-3}$ m $\quad A = 20\times10^{-4}\,m^2$

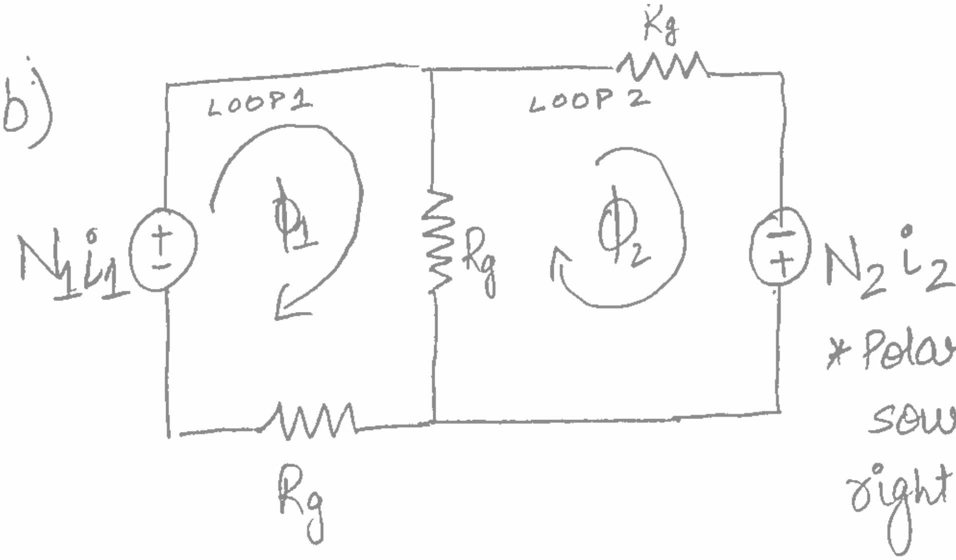
$\mu_0 = 4\pi\times10^{-7}$ H/m $\quad N_1 = 20$, $N_2 = 40$

$$R_g = \frac{g}{\mu A} = \frac{g}{\mu_r \mu_0 A_g} = \frac{5\times10^{-3}}{2\times4\pi\times10^{-7}\times20\times10^{-4}} = 994718.4\,\frac{At}{Wb}$$

or

$$= 9.94718\times10^{5}\,\frac{At}{Wb}$$

b)



* R_g, N_1, N_2 values defined in the previous parts

* Polarity of the MMF sources are found using right hand thumb rule.

c) We write KVL equations for the 2 loops in the magnetic circuit

$$N_1 i_1 = [\phi_1 - \phi_2] R_g + \phi_1 R_g \quad \text{———} \; ①$$

$$N_2 i_2 = [\phi_2 - \phi_1] R_g + \phi_2 R_g. \quad \text{———} \; ②$$

$$\begin{bmatrix} N_1 i_1 \\ N_2 i_2 \end{bmatrix} = R_g \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \frac{1}{R_g} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} N_1 i_1 \\ N_2 i_2 \end{bmatrix} = \frac{1}{3R_g} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} N_1 i_1 \\ N_2 i_2 \end{bmatrix}$$

$$\phi_1 = \frac{2}{3R_g} N_1 i_1 + \frac{1}{3R_g} N_2 i_2 \, , \quad \lambda_1 = N_1 \phi_1$$

$$\phi_2 = \frac{1}{3} R_g N_1 i_1 + \frac{2}{3R_g} N_2 i_2 \quad \lambda_2 = N_2 \phi_2$$

$\longrightarrow$

$\therefore \lambda_1 = \frac{2}{3R_g} N_1^2 i_1 + \frac{1}{3R_g} N_1 N_2 i_2 = 2.68\times10^{-4} i_1 + 2.68\times10^{-4} i_2$ — (A)

$\lambda_2 = \frac{1}{3R_g} N_1 N_2 i_1 + \frac{2}{3R_g} N_2^2 i_2 = 2.68\times10^{-4} i_1 + 10.72\times10^{-4} i_2$. — (B)

d) $\left.\begin{array}{l} \lambda_1 = L_1 i_1 + M i_2 \\ \lambda_2 = L_2 i_2 + M i_1 \end{array}\right\} \rightarrow$ So, from (A) & (B)

$L_1 = 2.68\times10^{-4}$ H or 0.268 mH

$L_2 = 10.72\times10^{-4}$ H or 1.072 mH

$M = 2.68\times10^{-4}$ H or 0.268 mH.

e) $K = \frac{M}{\sqrt{L_1 L_2}} = 0.5$

since, K is right in the middle, the coupling is neither tight nor close.

b) marked on the figure in the problem.

g) $= V_1(t) = +\frac{d\lambda_1}{dt} = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$

$V_1(t) = 2.68\times10^{-4} \frac{di_1}{dt} + 2.68\times10^{-4} \frac{di_2}{dt}$.

Problem 4 [25 points]

The equivalent circuit model of a non-ideal transformer is given in Figure 1. It is supplying a load of $R = 5\ \Omega$. Suppose the turns ratio is $a = N_1/N_2 = 2$.

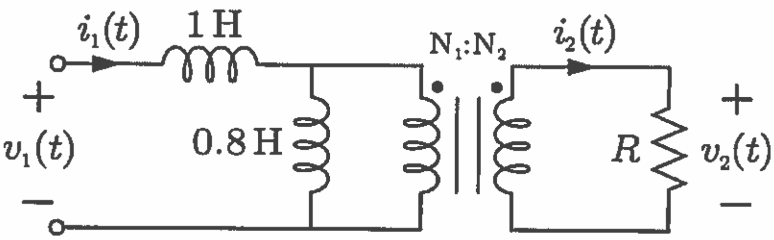


Figure 1: Equivalent circuit representation of a non-ideal transformer.

(a) Write down the "loop equations" (Kirchhoff's voltage law) for the circuit in Figure 1 in time-domain, i.e., express all voltages and currents as functions of time. [7 points]

(b) If $v_1(t) := 120\sqrt{2}\cos(\omega t)$ V is supplied across the first coil, compute the voltage phasor $\bar{V}_2$ across the load. Assume an operating frequency of $\omega = 10$ rad/s, and zero phase angle for the source. [10 points].
Hint: Express the loop equations in part (a) in phasor notation and solve.

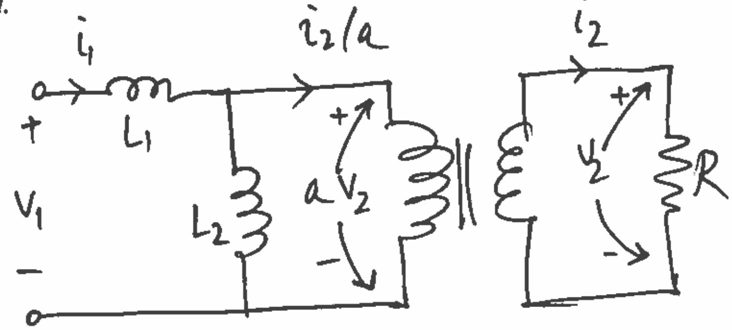
(c) What is the efficiency of the transformer in Figure 1? Does the figure describe a realistic transformer model? [2 points]

(d) Does the equivalent circuit of the transformer in Figure 1 model copper losses, i.e., resistances in the wires of the coils? How would you modify the circuit to include such losses? [2 points]

(e) Does the equivalent circuit of the transformer in Figure 1 model core losses, i.e., losses due to hysteresis and eddy currents? How would you modify the circuit to include such losses? [2 points]

(f) If you were to perform open and short circuit tests on this transformer, what will the watt-meter read in each case? [2 points]
Hint: A watt-meter measures real power consumed.

a: $L_1 = 1H$, $L_2 = 0.8H$.



- $v_1(t) - L_1 \dfrac{di_1(t)}{dt} - L_2 \dfrac{d}{dt}\left(i_1(t) - \dfrac{i_2(t)}{a}\right) = 0.$
- $v_1(t) - L_1 \dfrac{di_1(t)}{dt} - a v_2(t) = 0.$
- $v_2(t) = i_2(t) \cdot R.$

b. Transforming the equations to the phasor domain, we get

- $\bar{V}_1 = j\omega L_1 \bar{I}_1 + j\omega L_2 (\bar{I}_1 - \bar{I}_2/a)$.
- $\bar{V}_1 = j\omega L_1 \bar{I}_1 + a\bar{I}_2 R$ (we have used $\bar{V}_2 = \bar{I}_2 R$).

$$\Rightarrow \begin{pmatrix} j\omega L_1 + j\omega L_2 & -j\omega L_2/a \\ j\omega L_1 & aR \end{pmatrix} \begin{pmatrix} \bar{I}_1 \\ \bar{I}_2 \end{pmatrix} = \begin{pmatrix} \bar{V}_1 \\ \bar{V}_1 \end{pmatrix},$$

where $\bar{V}_1 = 120\angle 0^\circ$ Volts.

$$\Rightarrow \begin{pmatrix} \underbrace{j10 + j8}_{=j18} & -j4 \\ j10 & 10 \end{pmatrix} \begin{pmatrix} \bar{I}_1 \\ \bar{I}_2 \end{pmatrix} = \begin{pmatrix} 120\angle 0^\circ \\ 120\angle 0^\circ \end{pmatrix}.$$

$$\Rightarrow \begin{pmatrix} \bar{I}_1 \\ \bar{I}_2 \end{pmatrix} = \frac{1}{(j18)10 - (j10)(-j4)} \cdot \begin{pmatrix} 10 & +j4 \\ -j10 & j18 \end{pmatrix} \begin{pmatrix} 120\angle 0^\circ \\ 120\angle 0^\circ \end{pmatrix}.$$

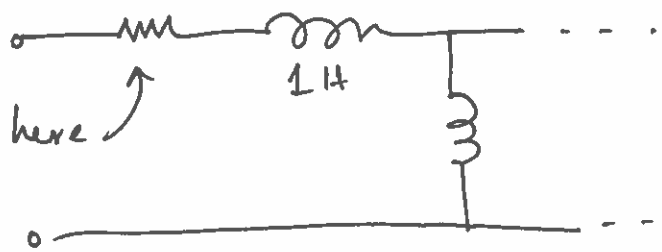
$$\bar{V}_2 = R\cdot\bar{I}_2 = 5 \cdot \frac{1}{(j18)\cdot 10 - (j10)(-j4)} \cdot \left[(-j10)(120\angle 0^\circ) + (j18)(120\angle 0^\circ)\right] \text{ Volts}$$

$$= \frac{5}{j180 - 40} \cdot j8 \times 120\angle 0^\circ \text{ volts}.$$

$$= 26.03\angle -12.5^\circ \text{ Volts}.$$

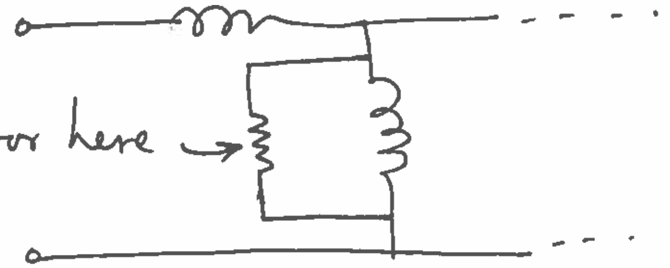
c. Efficiency = 100%, because the equivalent ckt of the transformer has no resistors, and hence, no losses! It is not a realistic ~~mo~~ transformer model as you always expect to have some losses.

d. No.



Add a resistor here

e. No.



Add a resistor here

f. $P_{sc} = 0\,W$, $P_{oc} = 0\,W$

Wattmeter reads real power consumed by the transformer. Our equiv. ckt model has no resistors, implying zero power loss!