ECE 330
POWER CIRCUITS AND ELECTROMECHANICS

LECTURE 7
TRANSFORMERS (1)

Acknowledgment-These handouts and lecture notes given in class are based on material from Prof. Peter Sauer's ECE 330 lecture notes. Some slides are taken from Ali Bazi's presentations

Disclaimer- These handouts only provide highlights and should not be used to replace the course textbook.

IDEAL TRANSFORMER

- The transformer transfers electrical energy from one circuit to another through the changing magnetic field that links both circuits.
- In the transmission, distribution, and utilization of electrical energy, the transformer changes voltage levels at a fixed frequency, such as 50 Hz or 60 Hz, and the power to be handled can vary from a few hundred watts to several hundred megawatts.

IDEAL TRANSFORMER

In the communications field, the use of the transformer is based on different considerations.

Some of the applications:

Impedance matching, DC isolation, and changing voltage levels with power handling capacities of the order of a few watts but with the capability of operating satisfactorily over a very wide frequency range.

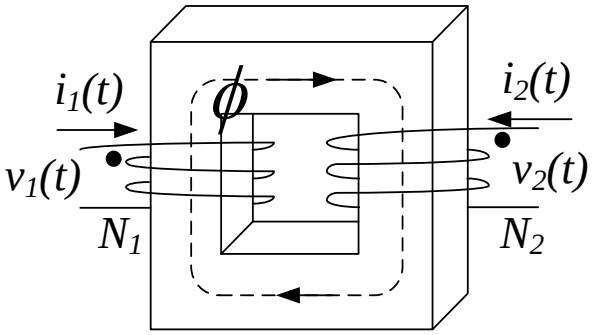
IDEAL TRANSFORMER

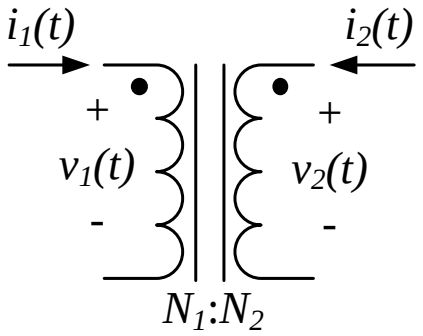
- An ideal transformer is assumed to have no losses, stray capacitance, and leakage flux.

$$v_1(t) = N_1 \frac{d\phi(t)}{dt}$$

$$v_2(t) = N_2 \frac{d\phi(t)}{dt}$$

$$\Rightarrow \frac{v_1(t)}{v_2(t)} = \frac{N_1}{N_2} = a$$





- If the core has a reluctance $\Re$

$$N_1 i_1 + N_2 i_2 = \phi\, \Re$$

IDEAL TRANSFORMER

- For an infinite core permeability:

$$N_1 i_1 + N_2 i_2 = 0$$

$$\Rightarrow \frac{i_1}{i_2} = \frac{-N_2}{N_1} = \frac{-1}{a}$$

- If $i_1(t)$ or $i_2(t)$ is in the other direction: $\frac{i_1}{i_2} = \frac{N_2}{N_1} = \frac{1}{a}$

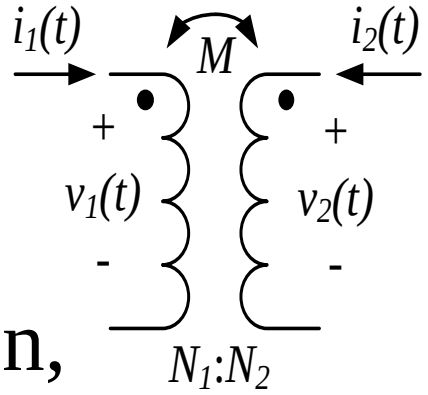
- Then, $\frac{i_1}{i_2} = \frac{N_2}{N_1} = \frac{1}{a}$ and $\frac{v_1}{v_2} = \frac{N_1}{N_2} = a$

- Also, $v_1 i_1 = v_2 i_2$.

STORED ENERGY

- With self and mutual inductance, the energy stored can be shown to be:

$$W = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M i_1 i_2$$



- But $k = \frac{M}{\sqrt{L_1 L_2}}$ and $k - 1 = \frac{M - \sqrt{L_1 L_2}}{\sqrt{L_1 L_2}}$ then,

$$W = \frac{1}{2}\left(\sqrt{L_1} i_1 + \sqrt{L_2} i_2\right)^2 + (k-1)\sqrt{L_1 L_2} i_1 i_2 .$$

- In an ideal transformer, $k = 1$, and the stored energy is zero:

$$\frac{i_1}{i_2} = \frac{-\sqrt{L_2}}{\sqrt{L_1}} = \frac{-v_2}{v_1} = \frac{-1}{a}$$

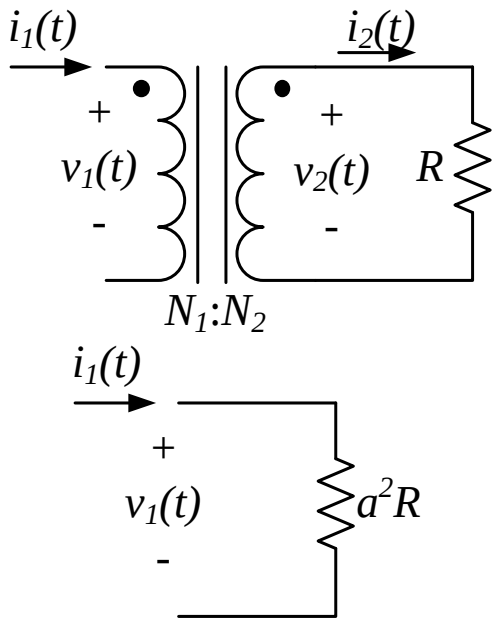
IMPEDANCE

- An impedance on one side can be reflected or referred to the other side.

$$R = \frac{v_2}{i_2}$$

$$\Rightarrow R = \frac{v_1}{i_2}\frac{N_2}{N_1} = \frac{v_1}{i_1}\left(\frac{N_2}{N_1}\right)^2 = \frac{1}{a^2}\frac{v_1}{i_1}$$

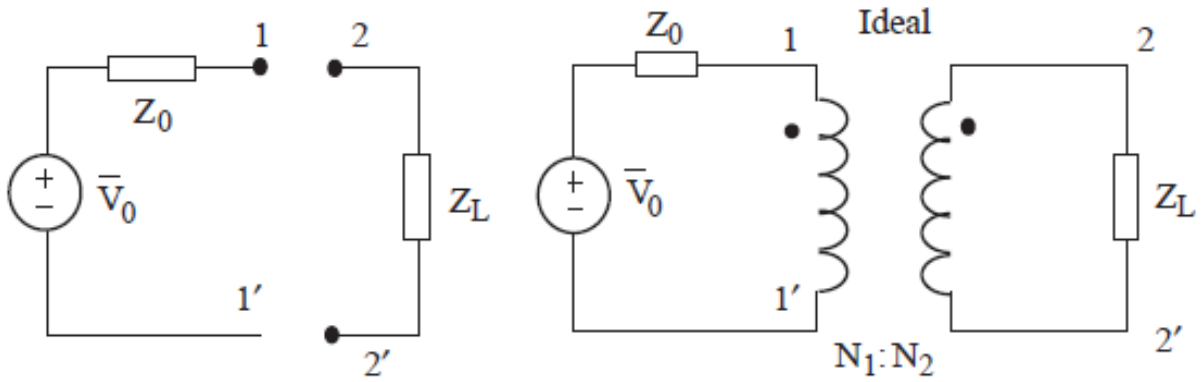
$$\Rightarrow \frac{v_1}{i_1} = a^2 R$$



- In phasor domain, and having a complex impedance:

$$\frac{\bar{V}_1}{\bar{I}_1} = a^2 \bar{Z}$$

IMPEDANCE MATCHING

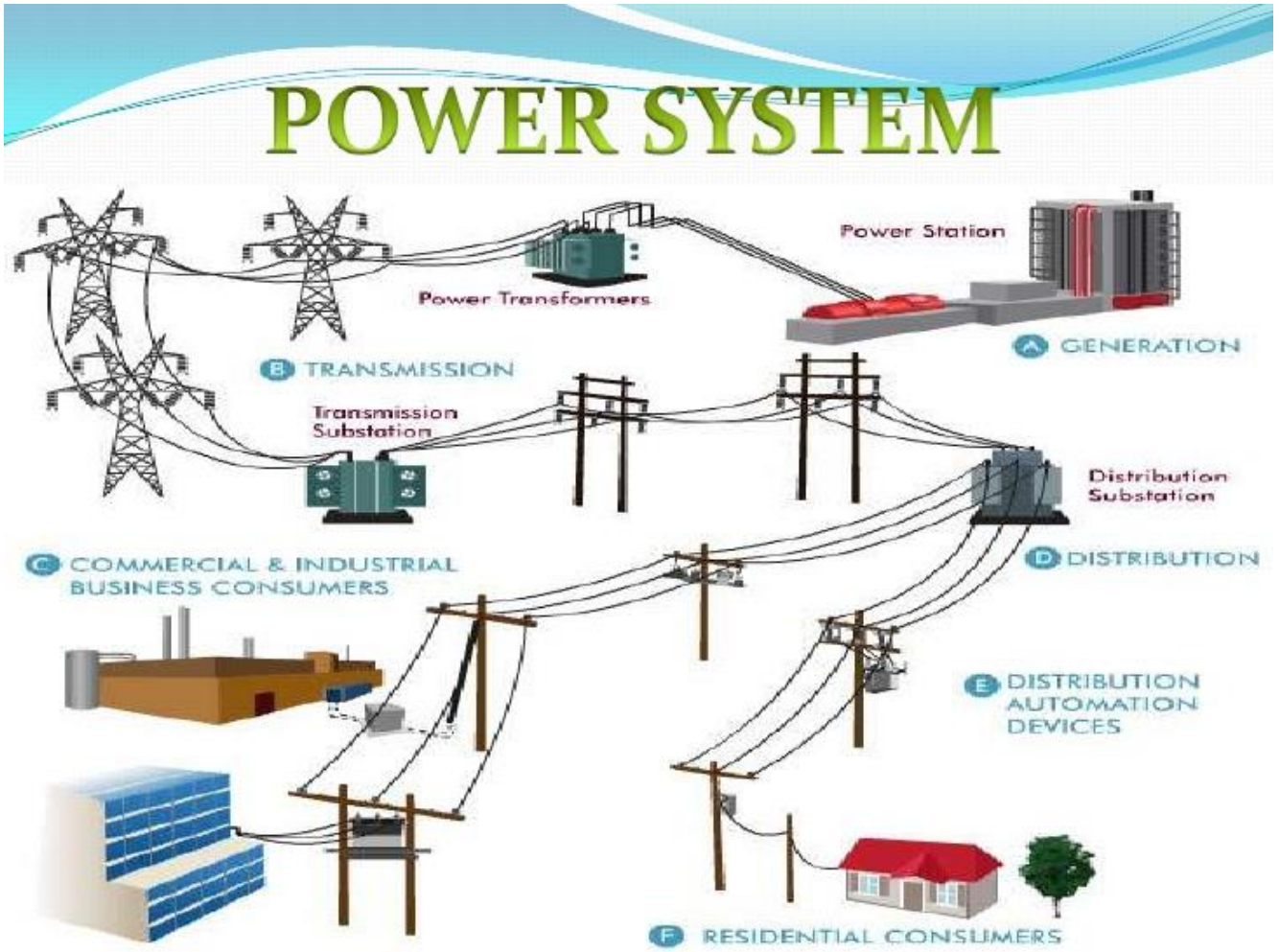


- We know that maximum power can be transferred to the load if either (1) $Z_L = Z_o^*$ or (2) $|Z_L| = |Z_o|$. The magnitudes of Z_L and Z_o can be made almost equal by interposing an ideal transformer between the source and the load.
- Turns ratio is so selected that $|Z_o| \approx (N_1/N_2)^2 |Z_L|$, which is equal to the impedance looking into terminals 1 1' .

POWER TRANSFORMER

The power transformer is an essential component of the power system. It steps up the voltage at the generating point, transmits it over long distances, and then steps it down at the sub-transmission and distribution levels for use by both commercial units and individual homes.

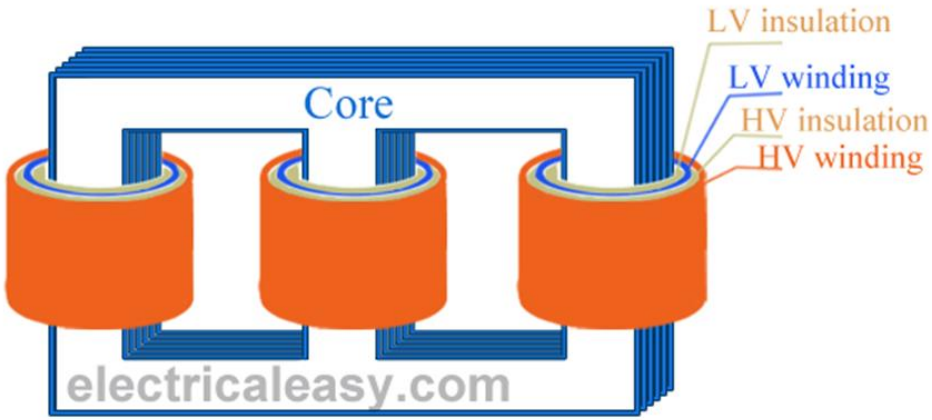
Source: torbosquid.com



POWER TRANSFORMER

A physical transformer consists of two windings mounted on a core, generally of a ferromagnetic material of high permeability.

Source: electronichub.org



Core type three phase transformer

Source: electricaleasy.com