ECE 330
POWER CIRCUITS AND ELECTROMECHANICS

LECTURE 6
MUTUAL INDUCTANCE

Acknowledgment-These handouts and lecture notes given in class are based on material from Prof. Peter Sauer's ECE 330 lecture notes. Some slides are taken from Ali Bazi's presentations

Disclaimer- These handouts only provide highlights and should not be used to replace the course textbook.

SOURCE ON PRIMARY SIDE

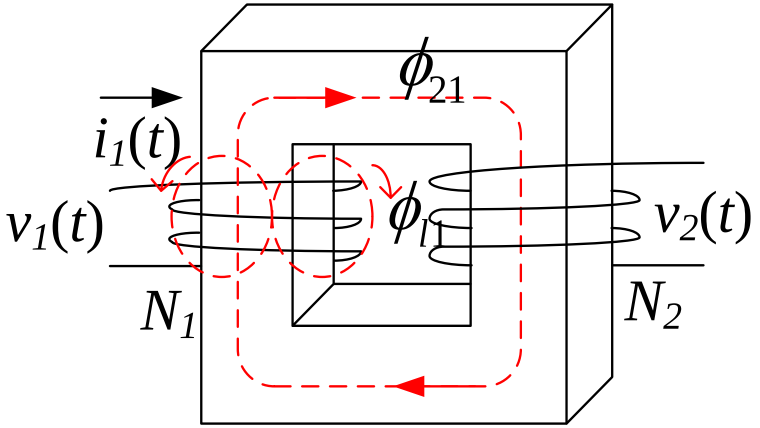
- When two coils are magnetically coupled there is "mutual inductance."

- Assuming an ac voltage source on one side and an open circuit on the other ($i_2 = 0$):

$$\phi_{11} = \phi_{\ell 1} + \phi_{21}.$$

$$\lambda_2 = N_2 \phi_{21}$$

$$v_2 = \frac{d\lambda_2}{dt} = N_2 \frac{d\phi_{21}}{dt}$$



ECE ILLINOIS

ILLINOIS

SOURCE ON PRIMARY SIDE

- Let the flux and current be linearly related (no saturation): $\lambda_2 = M_{21} i_1, \; \lambda_1 = L_1 i_1.$

- Then, $v_2 = M_{21} \dfrac{di_1}{dt},$ and $v_1 = L_1 \dfrac{di_1}{dt}.$

- M_{21} is the mutual inductance.

- L_1 is the self-inductance of coil 1.

SOURCE ON SECONDARY SIDE

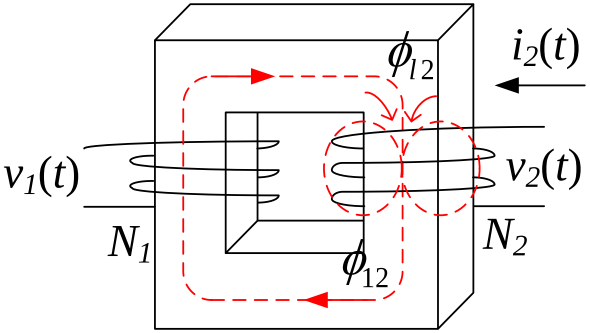
- Assuming an ac voltage source on one side and an open circuit on the other ($i_1 = 0$):

$$\phi_{22} = \phi_{l2} + \phi_{12}.$$

$$\lambda_1 = N_1 \phi_{12}, \quad v_1 = \frac{d\lambda_1}{dt} = N_1 \frac{d\phi_{12}}{dt}$$

- Let the flux and current be linearly related (no saturation): $\lambda_1 = M_{12} i_2, \; \lambda_2 = L_2 i_2.$

$$v_2 = L_2 \frac{di_2}{dt}, \text{ and } v_1 = M_{12} \frac{di_2}{dt}.$$



- M_{12} is the mutual inductance and
- L_2 is the self-inductance of coil 2.

SOURCES ON BOTH SIDES

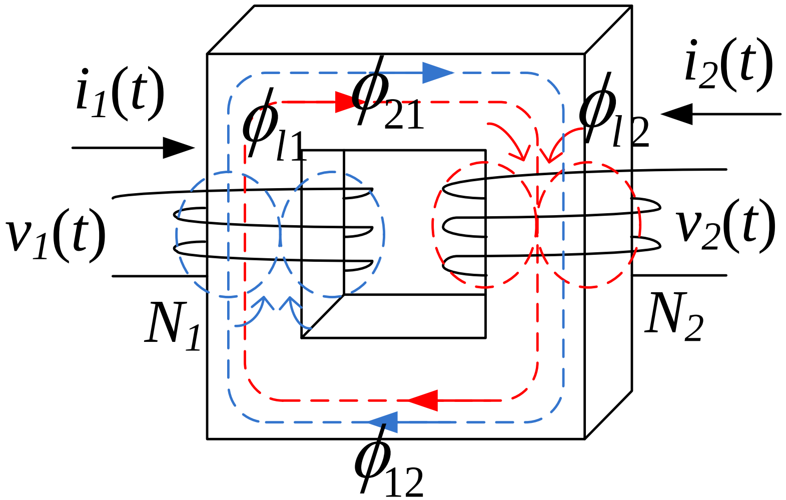
- Using superposition:

$$\phi_1 = \phi_{l1} + \phi_{21} + \phi_{12} = \phi_{11} + \phi_{12}$$

$$\phi_2 = \phi_{l2} + \phi_{21} + \phi_{12} = \phi_{22} + \phi_{21}$$

$$\lambda_1 = N_1\phi_{11} + N_1\phi_{12} = L_1 i_1 + M_{12} i_2$$

$$\lambda_2 = N_2\phi_{21} + N_2\phi_{22} = M_{21} i_1 + L_2 i_2$$



- Let $M_{12} = M_{21} = M$, then:

$$\lambda_1 = L_1 i_1 + M i_2$$

$$\lambda_2 = M i_1 + L_2 i_2$$

$$v_1 = \frac{d\lambda_1}{dt} = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = \frac{d\lambda_2}{dt} = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

COUPLING COEFFICIENT

- Define the coupling coefficient as: $k = \frac{M}{\sqrt{L_1 L_2}}$
- If $k = 0$, no coupling and $M = 0$.
- If $k = 1$, ideal coupling with zero leakage:

$$k = \frac{M}{\sqrt{L_1 L_2}} = \sqrt{\frac{\phi_{21}}{\phi_{11}}}\sqrt{\frac{\phi_{12}}{\phi_{22}}} = \sqrt{\frac{\phi_{21}}{\phi_{l1} + \phi_{21}}}\sqrt{\frac{\phi_{12}}{\phi_{l2} + \phi_{12}}}$$

- Therefore, $0 \le k \le 1$ and $0 \le M \le \sqrt{L_1 L_2}$.

M *is always* ≥ 0.

COUPLING COEFFICIENT

- What if i_2 is reversed? $i_2' = -i_2$

$$v_1 = \frac{d\lambda_1}{dt} = L_1 \frac{di_1}{dt} - M \frac{di'_2}{dt}$$

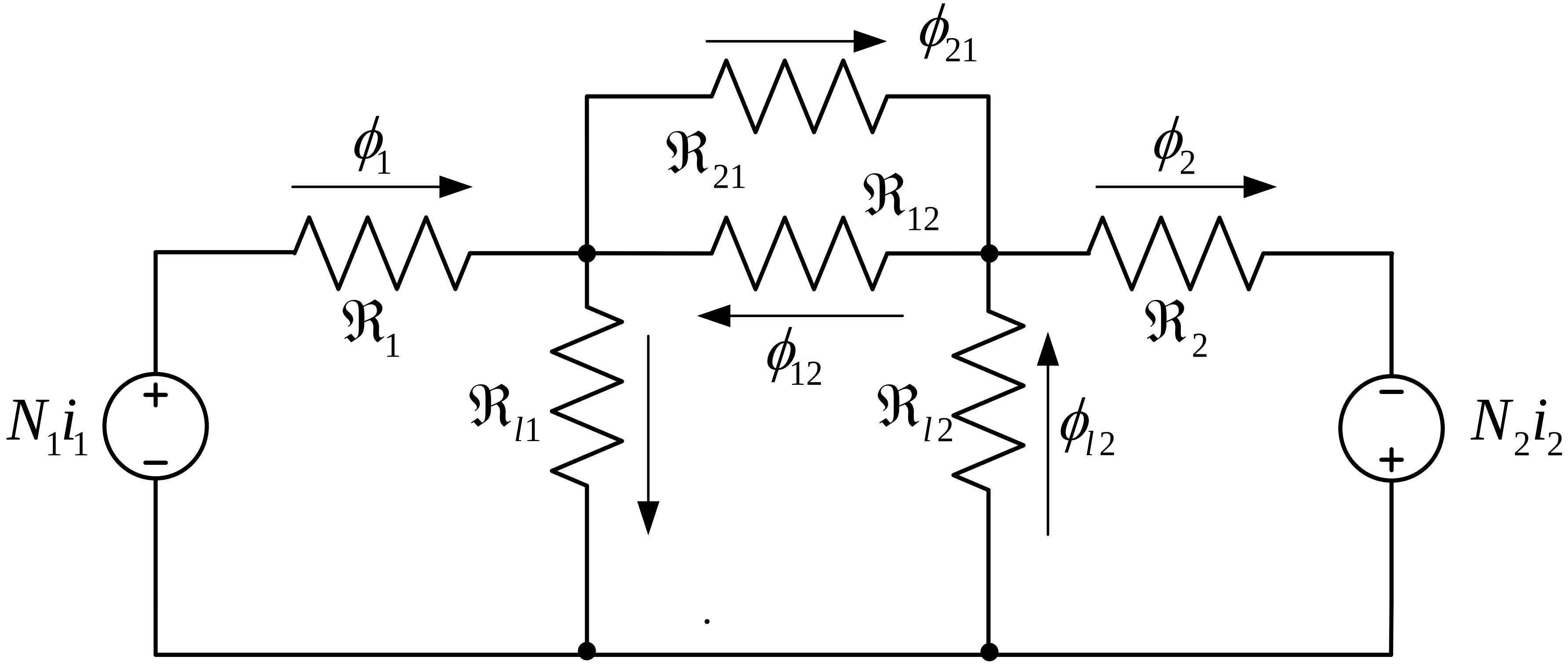
$$v_2 = \frac{d\lambda_2}{dt} = -L_2 \frac{di'_2}{dt} + M \frac{di_1}{dt}$$

- What if v_2 is reversed? $v_2' = -v_2$

$$v_1 = \frac{d\lambda_1}{dt} = L_1 \frac{di_1}{dt} - M \frac{di'_2}{dt}$$

$$v'_2 = \frac{d\lambda_2}{dt} = L_2 \frac{di'_2}{dt} - M \frac{di_1}{dt}$$

EQUIVALENT CIRCUIT



DOT MARKINGS

- Dots relate the flux direction between coils.
- If two fluxes are in the same direction, they add,

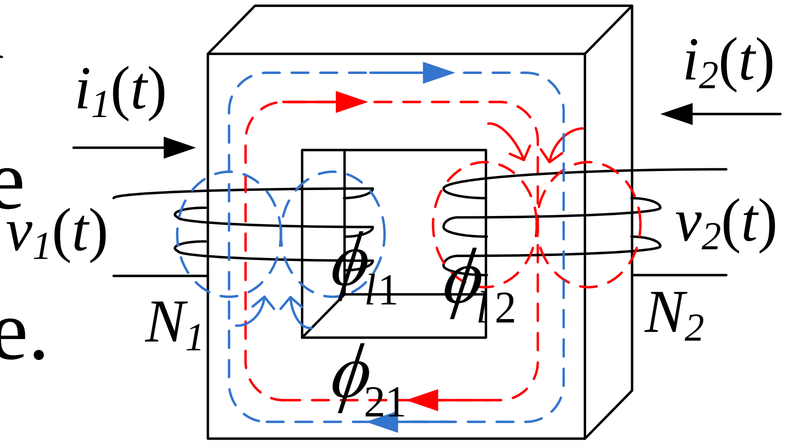
otherwise, they subtract.

- Depending which ends you connect the load to the

secondary coil you either get an

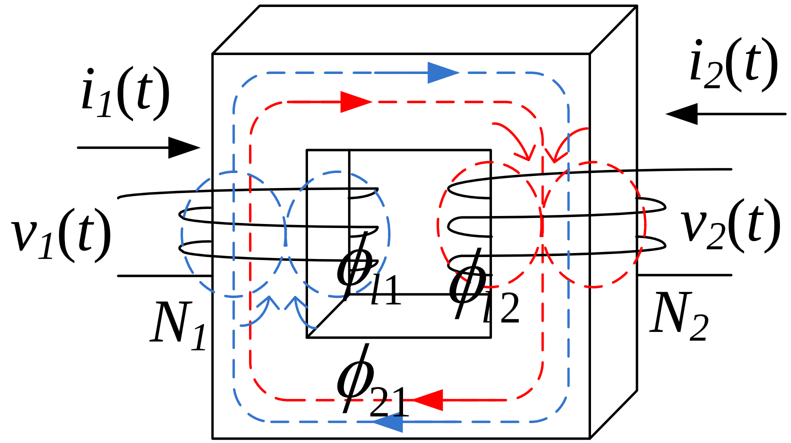
Output voltage in sync. With the

input voltage or in reverse phase.



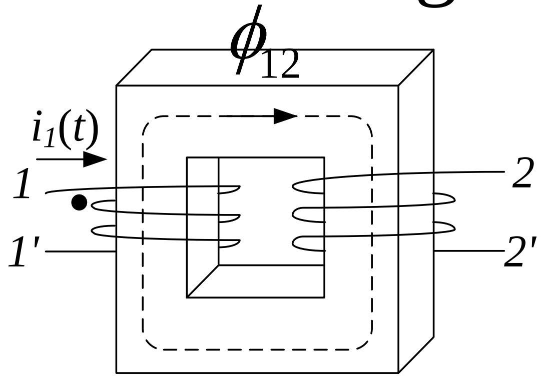
DOT MARKINGS

- The polarity markings are assigned such that a positively increasing current in the dotted terminal in one winding induces a positive voltage at the dotted terminal of the other winding



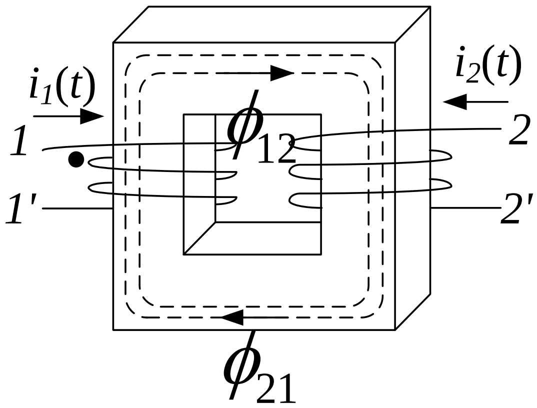
DOT MARKINGS

- Assume the following configuration.
  1) Select one coil and one terminal and place a dot on that terminal.
  2) Assume a current is flowing and determine the flux direction.



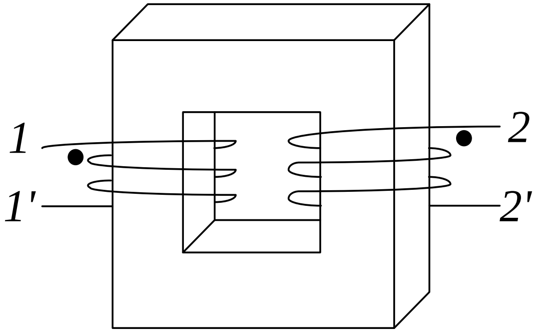
  3) Place a test current in the second coil and determine its flux direction.

DOT MARKINGS



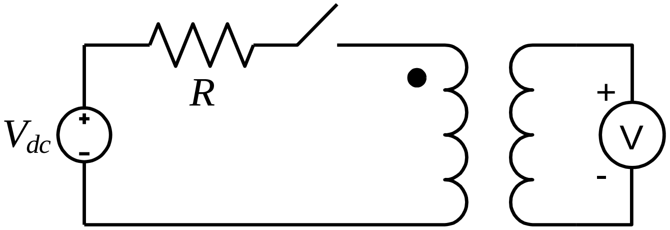
4) If ϕ_1 and ϕ_2 add, then place dot on 2 (where test current enters).

5) If they subtract, then place dot on 2' (where test current leaves).



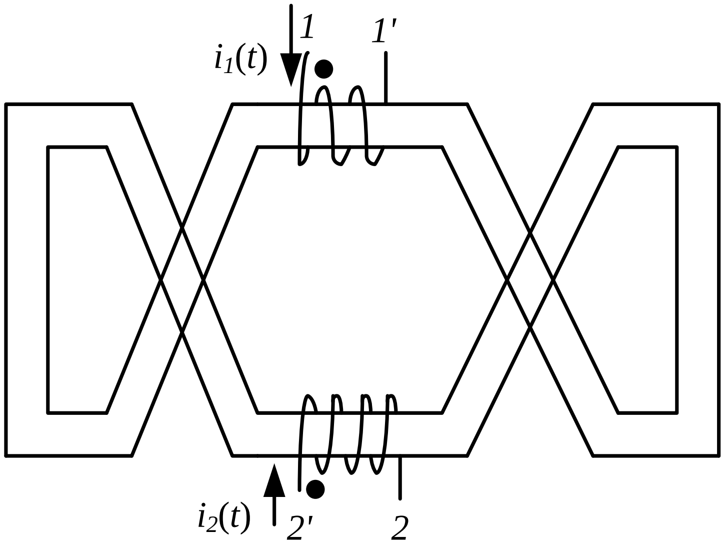
DOT MARKINGS

- Practical determination of dot locations:
  1) Build the following circuit.
    - Turn the switch on => pulse is generated where di/dt is not zero on the secondary side.
    - If the pulse causes the meter (V) to read positive, then the dot on the secondary is on the top terminal.
    - If (V) reads a negative pulse, then the dot is on the lower side.



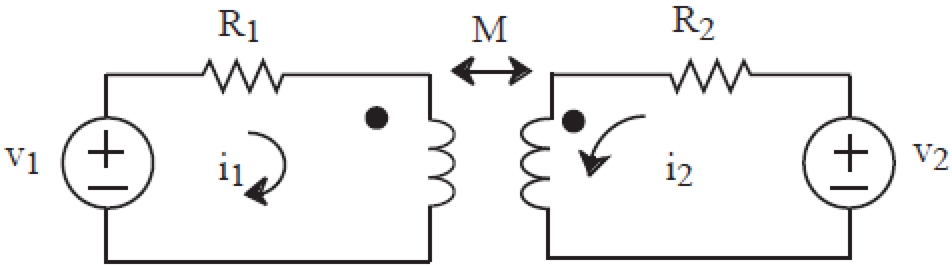
EXAMPLES

The dots in the circuit below are as shown.



WRITING EQUATIONS WITH MUTUALLY COUPLED COILS

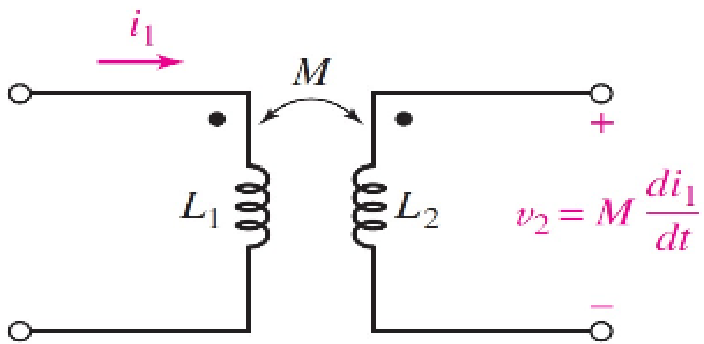
Suppose we have two mutually coupled coils and the dot markings are as shown

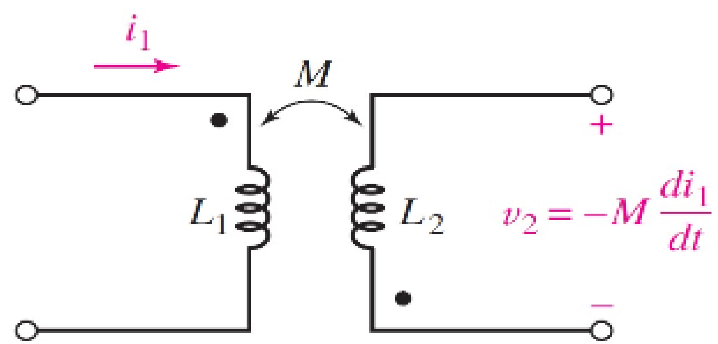


The self induced voltage due to the self inductance is in the direction of the current and is a voltage drop. The polarity of the mutually induced voltage depends on the dot marking

WRITING EQUATIONS WITH MUTUALLY COUPLED COILS

If the current enters the dotted terminal of one coil, the voltage will be positive at the dot on the second coil

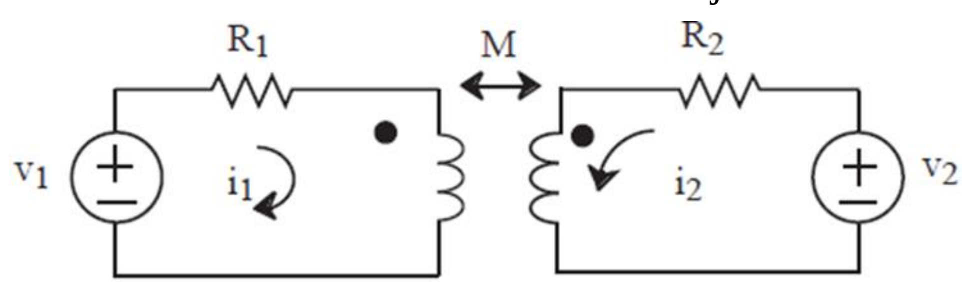




$$v_1 = i_1 R_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

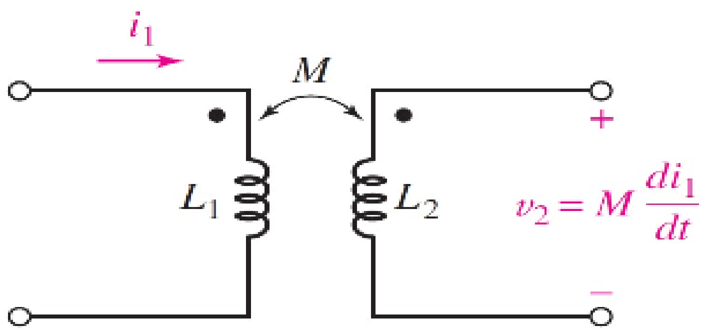
$$v_2 = i_2 R_2 + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

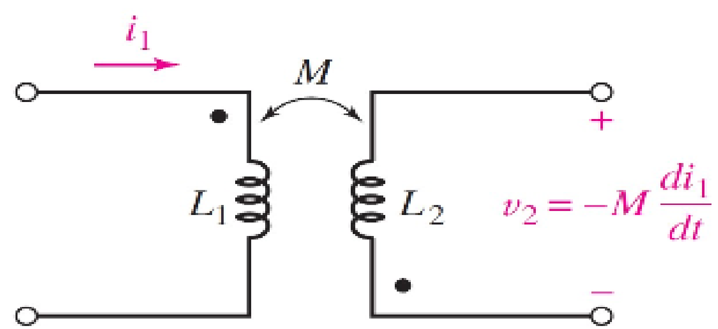
Source: jacobs-university.de



WRITING EQUATIONS WITH MUTUALLY COUPLED COILS

If the current enters the dotted terminal of one coil, the voltage will be positive at the dot on the second coil

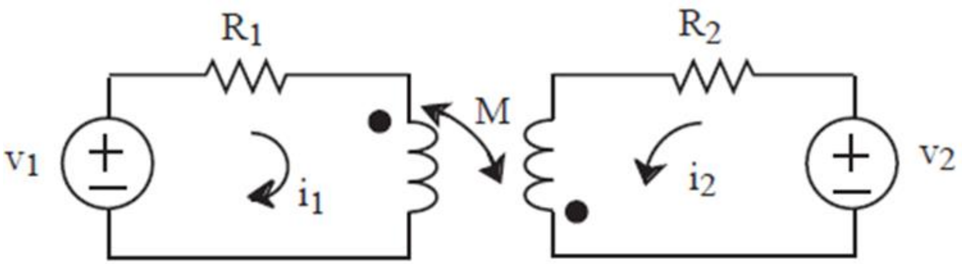




$$v_1 = i_1 R_1 + L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

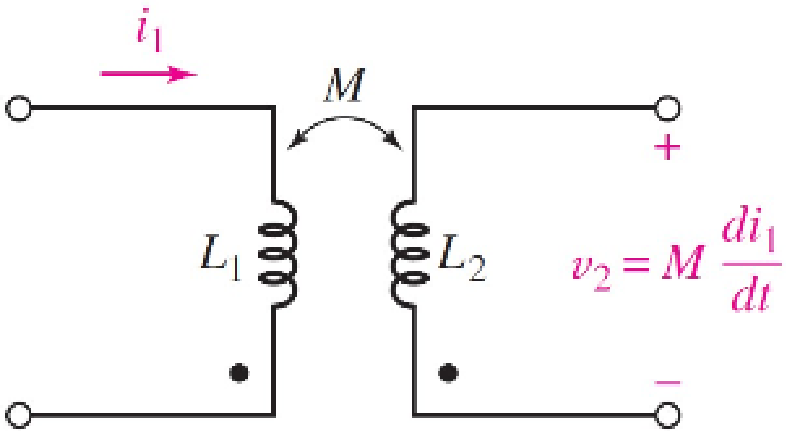
$$v_2 = i_2 R_2 + L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$

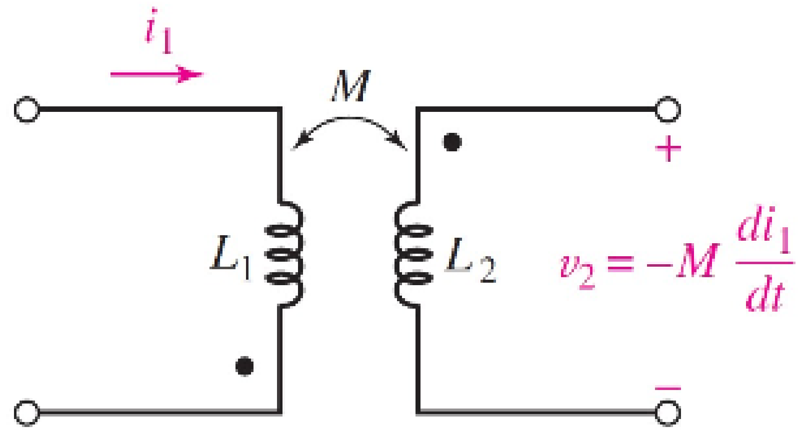
Source: jacobs-university.de



WRITING EQUATIONS WITH MUTUALLY COUPLED COILS

A current entering the undotted terminal of one coil provides a voltage that is positively sensed at the undotted terminal of the second coil





Source: jacobs-university.de

WRITING EQUATIONS WITH MUTUALLY COUPLED COILS

- If the reference current in a coil leaves the dotted (undotted) terminal, then the voltage induced at the dotted (undotted) terminal of the other coil has a negative sign.