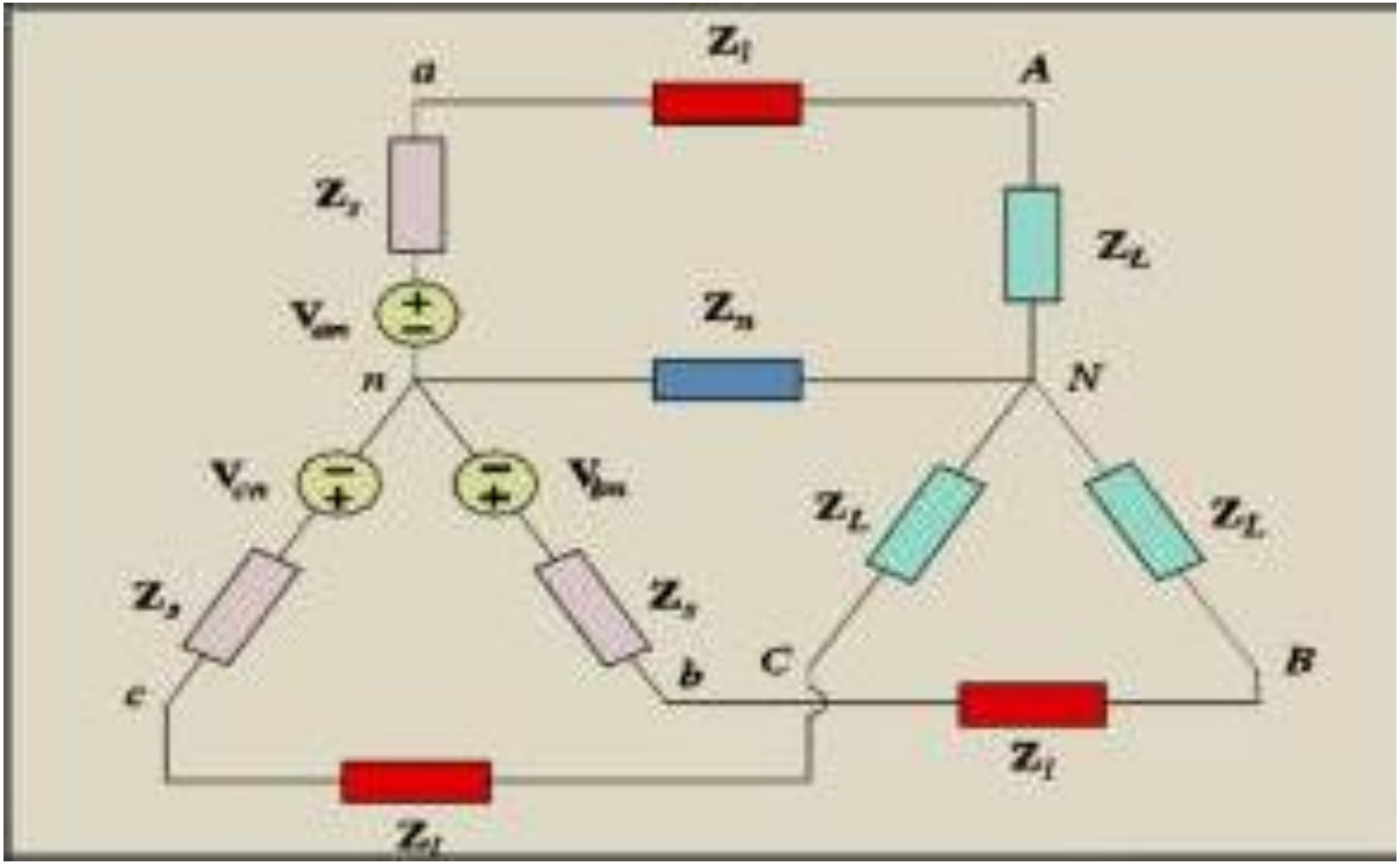
ECE 330
POWER CIRCUITS AND ELECTROMECHANICS

LECTURE 5
PER-PHASE CIRCUITS AND MAGNETICS (1)

Acknowledgment-These handouts and lecture notes given in class are based on material from Prof. Peter Sauer's ECE 330 lecture notes. Some slides are taken from Ali Bazi's presentations

Disclaimer- These handouts only provide highlights and should not be used to replace the course textbook.

PER-PHASE EQUIVALENTS

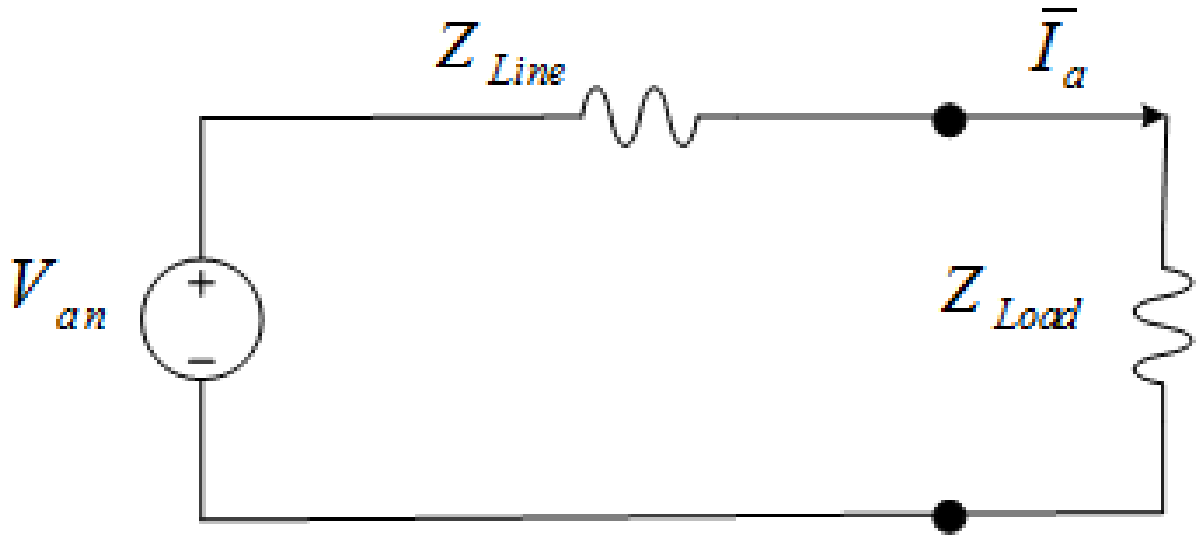


Source: blogspot.com

PER-PHASE EQUIVALENTS

In balanced three phase circuits, it is preferable to work with per-phase equivalents and then convert the variables to three-phase quantities.

$$\bar{I}_a = \frac{\bar{V}_{an}}{Z_{Line} + Z_{Load}}$$



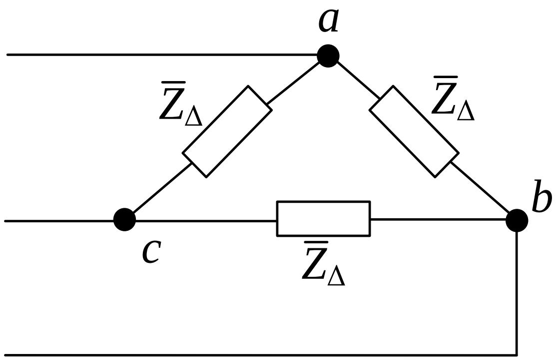
PER-PHASE EQUIVALENTS

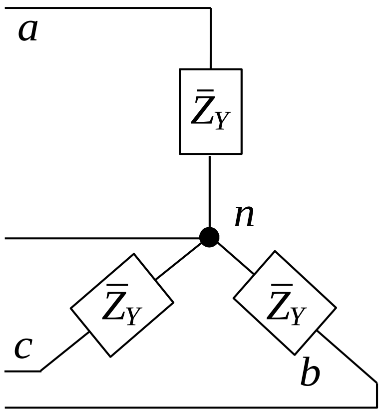
Δ-Y CONVERSION

- Per-Phase equivalent circuits are very convenient for analyzing three-phase circuits.
- For a Y-source the load could be either Y or Δ.
- The load seen between two phases, e.g., *a* and *b*, can be expressed as:

$$\bar{Z}_\Delta // 2\bar{Z}_\Delta = 2\bar{Z}_Y$$

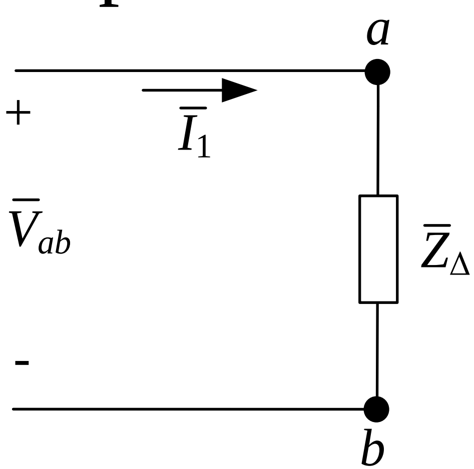
$$\Rightarrow \bar{Z}_\Delta = 3\bar{Z}_Y$$

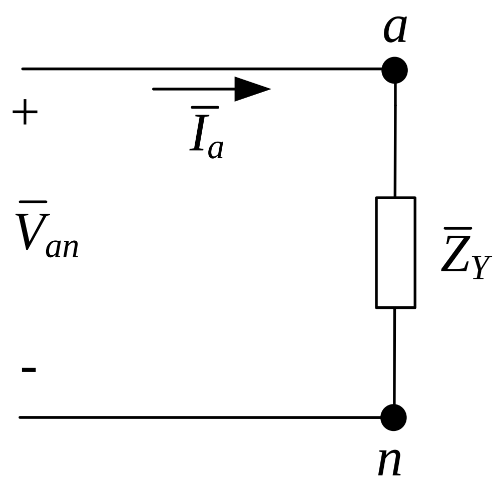




PER-PHASE EQUIVALENTS

- The per-phase circuits can then be shown as follows:





$$\bar{S}_{\Delta,1\varphi} = \bar{V}_{ab}\bar{I}_a^* = \frac{V_{ab}^2}{\bar{Z}_\Delta}$$

$$\bar{S}_{Y,1\varphi} = \bar{V}_{an}\bar{I}_a^* = \frac{V_{an}^2}{\bar{Z}_Y}$$

$$\boxed{\bar{S}_{Y,1\varphi} = \bar{S}_{\Delta,1\varphi}}$$

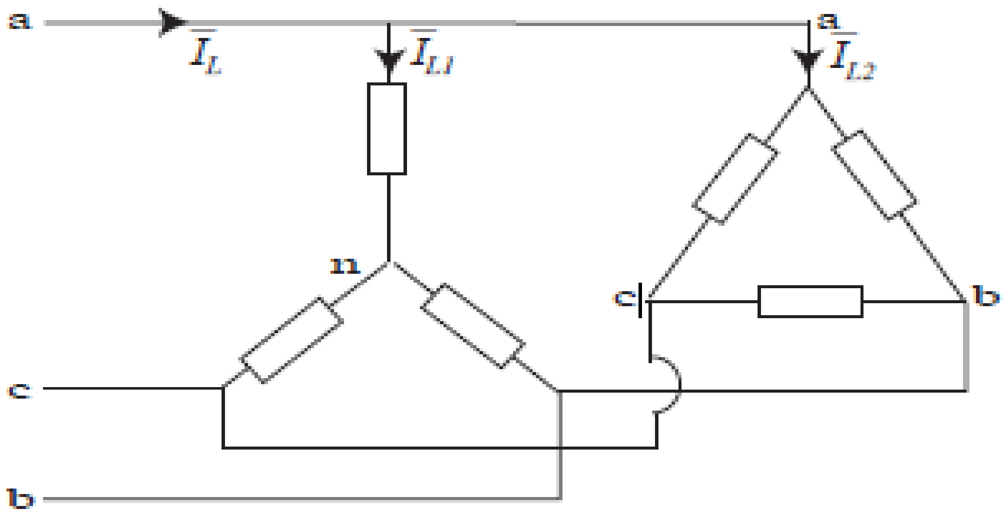
PER-PHASE EQUIVALENTS

- For either load, the voltage and load can be transformed to a Y-per-phase circuit, or Δ-per-phase circuit, and the S should be the same.
- Example: $\bar{Z}_{\Delta} = 10 + j5 \quad \Omega$, $|V_{ab}| = 208 \quad V$

$$\bar{S}_{Y,1\varphi} = \frac{V_{an}^2}{\bar{Z}_Y} = \frac{\left(\frac{V_{ab}}{\sqrt{3}}\right)^2}{\frac{\bar{Z}_{\Delta}}{3}} = \frac{V_{ab}^2}{\bar{Z}_{\Delta}} = 3.87\angle 26.6^o \, kVA$$

EXAMPLE 2.17

The following two three-phase loads are connected in parallel across a three-phase 480 V wye-connected supply.



EXAMPLE 2.17

- Load 1: 24 kW at 0.8 PF lag (wye-connected)
- Load 2: 30 kVA at 0.8 PF lead (delta-connected)

Find the line currents $\bar{I}_{L1}$ and $\bar{I}_{L2}$ for each of the two loads, total complex power $\bar{S}_T$ and total line current.

Take $\bar{V}_{an}$ as reference.

Triangle method:

EXAMPLE 2.17

$$\bar{S}_{T1} = \frac{24\times 10^3}{0.8}(0.8 + j\,0.6)\,VA \quad = 24,000 + j18,000\ VA$$

$$\bar{S}_{T2} = 30\times 10^3(0.8 - j\,0.6) = 24,000 - j\,18,000\,VA$$

$$\bar{S}_{T} = \bar{S}_{T1} + \bar{S}_{T2} = 48,000 \quad W$$

EXAMPLE 2.17

Line current:

$$(\sqrt{3})V_L I_{L1} \cos\theta = 24{,}000$$

$$I_{L1} = \frac{24{,}000}{(480)(\sqrt{3})(0.8)} = 36.08\ A$$

$$I_{L1} = 36.08\angle -36.78^\circ \ \textit{since the current is lagging}$$

$$(\sqrt{3})V_L I_{L2} = 30{,}000 \Rightarrow I_{L2} = 30{,}000/(\sqrt{3})480 = 36.08\ \ A$$

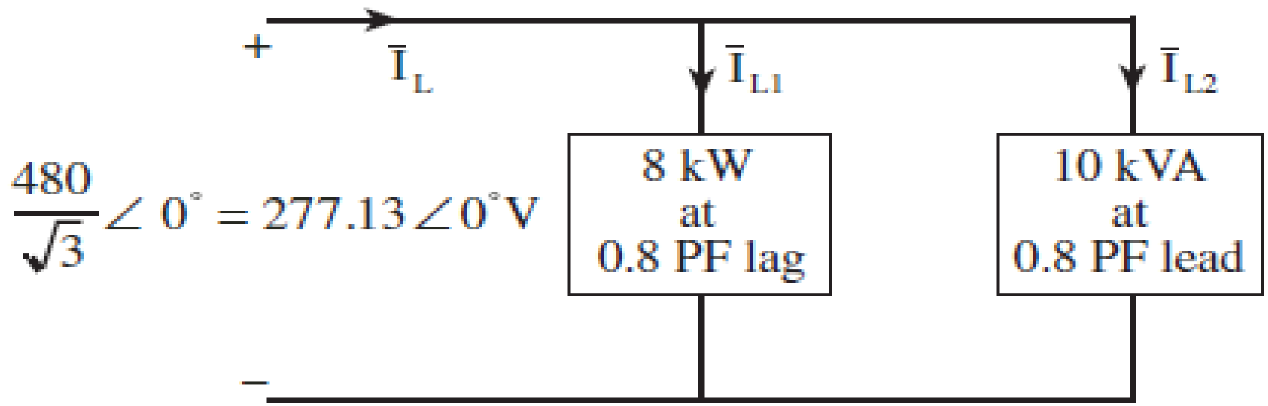
EXAMPLE 2.17

$$I_{L2} = 36.08\angle 36.78^\circ \;\; \textit{since the current is leading}.$$

$$Q_T = Q_{T1} + Q_{T2} = 18000 - 18000 = 0$$

$$(\sqrt{3})V_L I_L = 48,000 \Rightarrow I_L = \frac{48,000}{(480)(\sqrt{3})} = 57.7 \quad A$$

Per-phase equivalent method



EXAMPLE 2.17

Phase-to-neutral voltage is $480/\sqrt{3} = 277.13 \quad V$

$$8\times10^3 = (277.13)I_{L1}(0.8) \Rightarrow I_{L1} = 36.08\ A$$

$$I_{L1} = 36.08\angle-36.78^\circ \ \textit{since the PF is lag}$$

$$(10\times10^3)(0.8) = (277.13)(I_{L2})(0.8) \Rightarrow I_{L2} = 36.08\ A$$

$$\bar{I}_{L2} = 36.08\angle36.78^\circ\ A \ \textit{since the PF is leading}$$

$$\bar{I}_L = \bar{I}_{L1} + \bar{I}_{L2} == 36.08\angle-36.78^\circ + 36.08\angle36.78^\circ = 57.7\angle0^\circ\ A$$

MAGNETIC CIRCUITS

- Maxwell's Equations:
  - Ampere's Law: $\oint_C \underline{H}\,.d\underline{l} = \int_S \underline{J}\,.\,\underline{n}\,da = N\ I$

The magnetic field in any closed circuit is proportional to the electric current flowing through the loop.

  - Faraday's Law: $\oint_C \underline{E}\,.d\underline{l} = -\int_S \frac{d\underline{B}}{dt}.\underline{n}\ da$

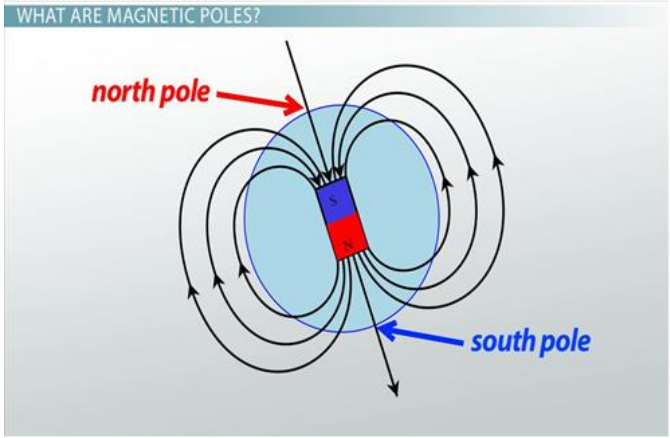
The line integral of the electric field around a closed loop is equal to the negative of the rate of change of the magnetic flux through the area enclosed by the loop. .

MAGNETIC CIRCUITS

- Conservation of Charge: $\oint_S \underline{J}.\underline{n}\ da = 0$
- Gauss's Law: $\oint_S \underline{B}.\underline{\mathrm{n}}\, da = 0$

The net magnetic flux out of any closed surface is zero.

(for a magnetic dipole ,in any closed surface the magnetic flux inward toward the south pole will equal the flux outward from the north pole).



Source:study.com

MAGNETIC CIRCUITS

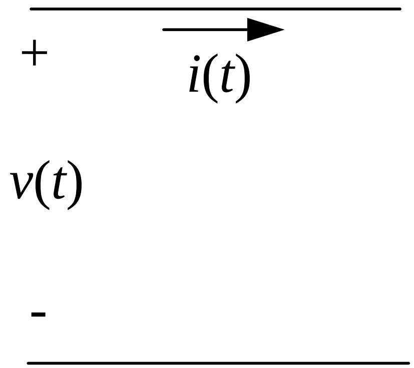
- What do these symbols mean?
  - Integral over a closed contour *C*: $\oint_C$
  - Surface *S* define by *C*: $\int_S$
  - Integral over a closed surface *S*: $\oint_S$
  - Length of the contour *C*: $\int_C dl$
  - *H* is the magnetic field intensity (A.turns/m)
  - *B* is the magnetic flux density (Tesla or Wb/m^2)
  - *E* is the electric field (V/m)
  - *J* is the current density (A/m^2)
  - n is the normal vector to *S*.

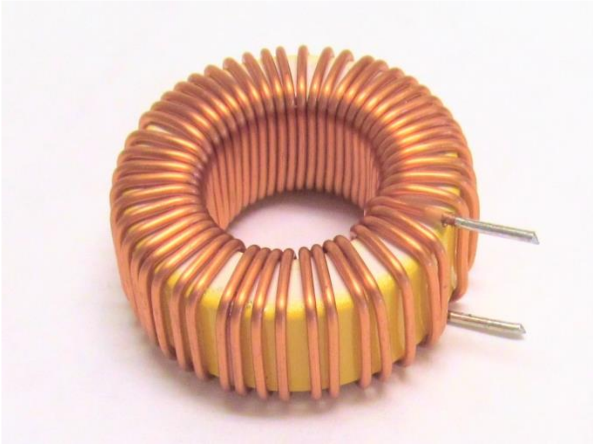
STATIC MAGNETIC CIRCUIT

In static magnetic circuits, there are no moving members. The most important device in this category is the transformer. We use Ampere's current law (ACL) and Gauss's law (GL) for magnetic fields to derive useful flux-current terminal relations. The analysis is helpful in the design of inductors, and study of transformers.

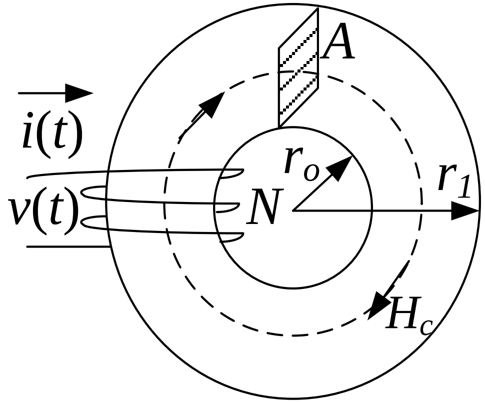
MAGNETIC CIRCUIT - TOROID

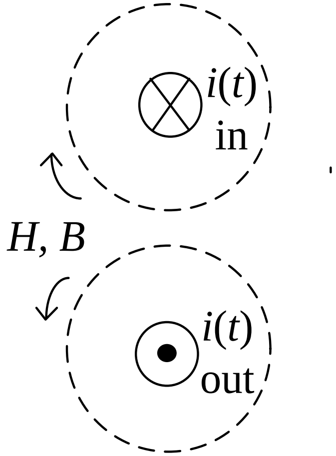
- Current flowing in a conductor produces a magnetic field.
- Voltage produces an electric field.
- Common example:





- *Source: electronicshub.org*





ECE ILLINOIS

ILLINOIS

MAGNETIC CIRCUIT - TOROID

- Applying Ampere's law $\oint_c H.d\ell = Ni$ we get $H_c \ell_c = Ni$

where ℓ_c is the mean length of the core.

ℓ_c can be approximated as $2\pi(r_o+r_1)/2$.

- Assuming a linear relationship between B and H

where $B = \mu H$ and μ is the permeability (H/m).

- $\mu = \mu_r \mu_o$ where μ_r is the relative permeability and μ_o is the permeability of free space. $\mu_o = 4\pi \times 10^{-7}$ H/m.

MAGNETIC CIRCUIT - TOROID

- The flux density in the core is

$$B_c = \mu H_c = \mu \frac{N\ i}{\ell_c}.$$

- Since B is the flux density (Wb/m^2), then the flux is

$$\phi_c = A_c B_c = \frac{\mu A_c}{\ell_c} N\ i\,.$$

- Define the magnetomotive force (mmf) as $mmf = N\ i$

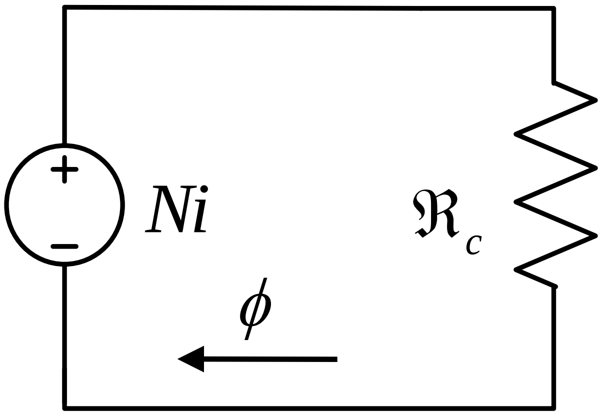
- Define the reluctance to be

$$\Re = \frac{Ni}{\phi_c} = \frac{\ell_c}{\mu A_c} \quad ampere-turns\ /weber$$

Magnetic Circuit Equivalent

- Then, $\phi_c \Re_c = Ni = mmf$
- The permeance is

$P = 1/\Re \quad Wb/ampere-turn$



It has similarity to a resistive circuit with the following equivalences

Electric Circuit	Magnetic Circuit
Voltage	*mmf*
Current	*Flux*
Resistance	*Reluctance*
Conductance	*Permeance*

MAGNETIC VS. ELECTRIC CIRCUITS

- The following analogies hold:
- Differences:
  - Leakage.
  - μ is not perfectly constant.
  - Saturation
- KVL and KCL analogous to total MMFs across a loop add up to zero, and total flux entering or leaving a node is zero.

INDUCTANCE

- Faraday's law can be written as: $\oint_C E . d\ell = \frac{d}{dt}\int_S B . n \; da.$
- E is (V/m) and ℓ is (m) => left side is voltage.
- Define the flux linkage as $\lambda = N \; B A.$
- Then, $v(t) = \frac{d\lambda}{dt} = \frac{d \; NBA}{dt} = N \frac{d\phi}{dt} = N^2 \frac{\mu A}{\ell} \frac{di(t)}{dt}.$
- Define the inductance $L = N^2 \frac{\mu A}{l}$, then $\boxed{v(t) = L \frac{di(t)}{dt}.}$

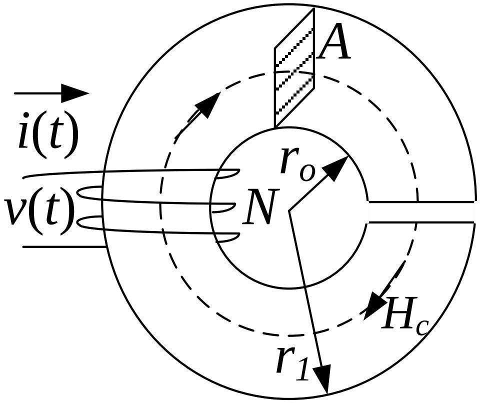
MAGNETIC CIRCUIT – TOROID WITH AIR GAP

- Some magnetics have air gaps that store energy.
- Back to the Toroid example but with air gap.
- The gap length is ℓ_g and its permeability is

$$\mu_o = 4\pi x\, 10^{-7} \quad H\ /\mathrm{m}$$

$$\oint_c H.d\ell = Ni$$

$$\Rightarrow H_c \ell_c + H_g \ell_g = Ni$$



Source: softsolder.com

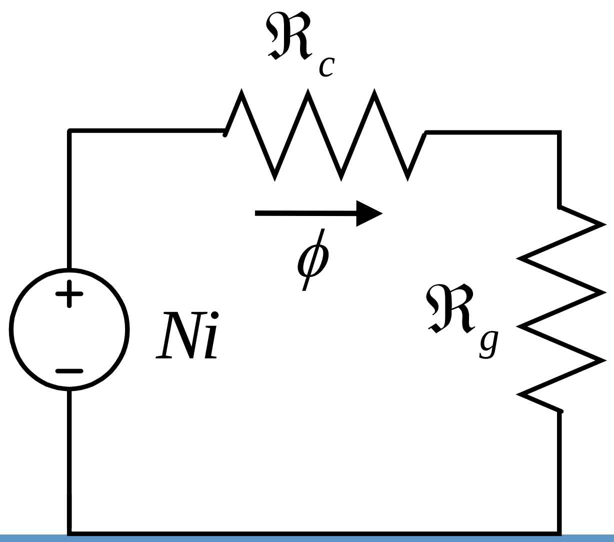
MAGNETIC CIRCUIT – TOROID WITH AIR GAP

- Therefore, $B_c = \mu H_c = \mu \frac{Ni}{\ell_c}$ and $B_g = \mu_o H_g = \mu_o \frac{Ni}{\ell_g}$.
- Assuming all flux passes through the air gap,

$$\phi_c = \phi_g = A_g B_g = A_c B_c = \phi.$$

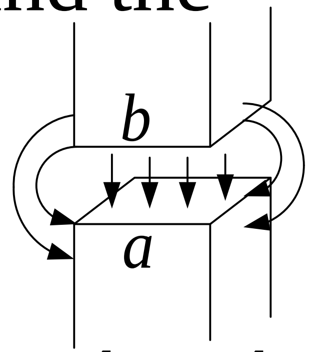
- Then,

$$Ni = \phi \frac{\ell_g}{\mu_o A_g} + \phi \frac{\ell_c}{\mu_o A_c} = \phi(\Re_g + \Re_c)$$



FRINGING

- Fringing occurs when the flux jumps around the air gap to join the other side of the core.



- Fringing can be simply modeled by having $A_g > A_c$
- Two methods to account for this:

- Empirical approximation $\begin{cases} A_c = ab \\ A_g = (a + \ell_g)(b + \ell_g) \end{cases}$

- A_g is given as percentage times A_c

$$A_g = kA_c,\ k \geq 1$$

EXAMPLE

$$Ni = 400 \;\; At, \;\; \ell_c = 6 \;\; cm, \;\; \ell_g = 0.1 \; cm, \;\; A_c = 1 \; cm^2$$

$$A_g = 1.1 A_c \;\; , \;\; \mu_r = 10^4 \;\; \text{H/ m},$$

- Find the flux.

$$H_c \ell_c + H_g \ell_g = Ni$$

$$\phi \frac{\ell_c}{\mu_1 A_c} + \phi \frac{\ell_g}{\mu_o A} = Ni$$

$$\phi = 5.5 \; x \, 10^{-5} \quad \text{Wb}$$