ECE 330
POWER CIRCUITS AND ELECTROMECHANICS

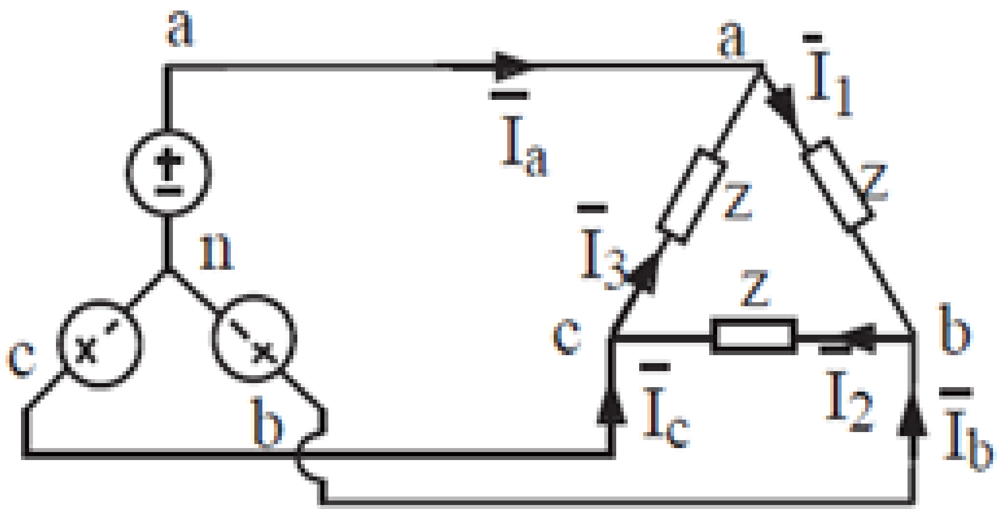
LECTURE 4

THREE-PHASE CONNECTIONS (2)

Acknowledgment-These handouts and lecture notes given in class are based on material from Prof. Peter Sauer's ECE 330 lecture notes. Some slides are taken from Ali Bazi's presentations

Disclaimer- These handouts only provide highlights and should not be used to replace the course textbook.

Y-Δ Connection



- V_{ab} is assumed to be at 0^o(reference).

$$\bar{V}_{ab} = V_{ll} \angle 0^o$$

$$\bar{V}_{bc} = V_{ll} \angle -120^o$$

$$\bar{V}_{ca} = V_{ll} \angle +120^o$$

Y-Δ Connection

- Assuming $Z = |Z| \angle \theta$, the currents $\bar{I}_1$, $\bar{I}_2$, and $\bar{I}_3$ in the three phases or legs of the delta connection lag the respective voltages by θ and called phase currents.

$$\bar{I}_1 = I_\varphi \angle -\theta$$
$$\bar{I}_2 = I_\varphi \angle -120^o - \theta$$
$$\bar{I}_3 = I_\varphi \angle 120^o - \theta$$

- The line currents are obtained by applying KCL:

$$\bar{I}_a = \bar{I}_1 - \bar{I}_3$$
$$\bar{I}_b = \bar{I}_2 - \bar{I}_1$$
$$\bar{I}_c = \bar{I}_3 - \bar{I}_2$$

Y-Δ Connection

- Using I_a as an example:

$$\bar{I}_a = I_\varphi\angle-\theta - I_\varphi\angle 120^o - \theta$$

$$= I_\varphi \cos(-\theta) + jI_\varphi \sin(-\theta) - I_\varphi \cos(120^o - \theta) - jI_\varphi \sin(120^o - \theta)$$

$$= I_\varphi[\cos(-\theta) - \cos(120^o - \theta)] + jI_\varphi[\sin(-\theta) - \sin(120^o - \theta)]$$

$$= -2I_\varphi \sin\left(\frac{-2\theta + 120^o}{2}\right)\sin(-60^o) + 2jI_\varphi \cos\left(\frac{-2\theta + 120^o}{2}\right)\sin(-60^o)$$

$$= \sqrt{3}I_\varphi \sin(60^o - \theta) - \sqrt{3}jI_\varphi \cos(60^o - \theta)$$

$$\Rightarrow j\bar{I}_a = j\sqrt{3}I_\varphi \sin(60^o - \theta) + \sqrt{3}I_\varphi \cos(60^o - \theta)$$

$$= \sqrt{3}I_\varphi\angle 60^o - \theta$$

$$\therefore \bar{I}_a = \sqrt{3}I_\varphi\angle 60^o - \theta - 90^o = \sqrt{3}I_\varphi\angle-\theta - 30^o$$

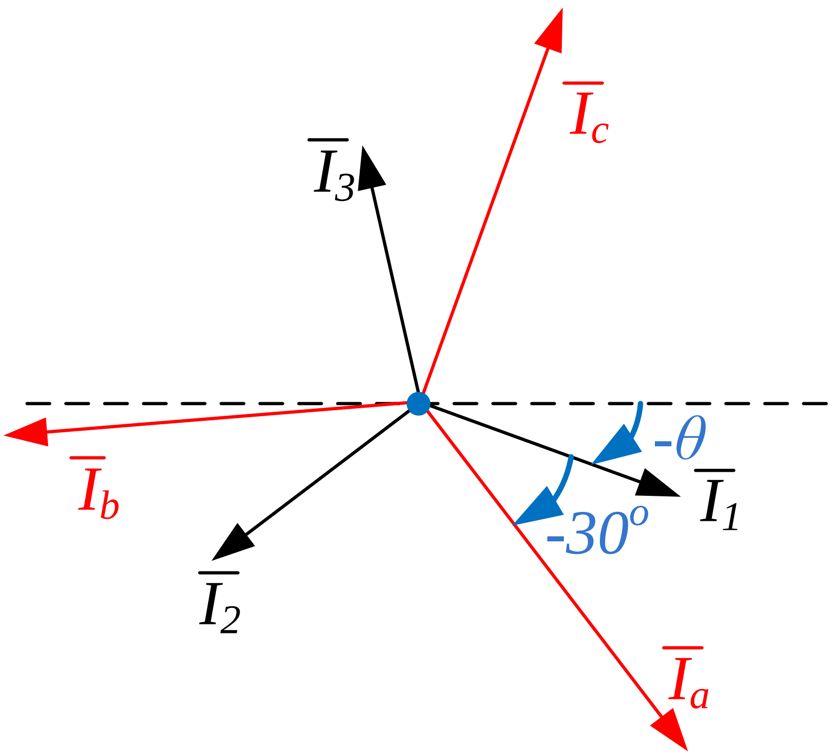
Y-Δ Connection

- Then,

$$\bar{I}_a = \sqrt{3} I_\varphi \angle -\theta - 30^o$$

$$\bar{I}_b = \sqrt{3} I_\varphi \angle -\theta - 150^o$$

$$\bar{I}_c = \sqrt{3} I_\varphi \angle -\theta + 90^o$$



- Therefore,

$$| I_l | = \sqrt{3} | I_\varphi |$$

$\angle I_l = \angle I_\varphi - 30^o$ where $\{l, \varphi\}$ are either $\{a,1\}$, $\{b,2\}$, or $\{c,3\}$.

Y-Δ Connection

- What about complex power?

$$\bar{S}_{3\varphi} = \bar{S}_a + \bar{S}_b + \bar{S}_c$$

$$= \bar{V}_{ab}\bar{I}_1^* + \bar{V}_{bc}\bar{I}_2^* + \bar{V}_{ca}\bar{I}_3^*$$

$$= V_\varphi \angle 0^o I_\varphi \angle \theta + V_\varphi \angle -120^o I_\varphi \angle \left(120^o + \theta\right) + V_\varphi \angle 120^o I_\varphi \angle \left(-120^o + \theta\right)$$

$$= 3V_\varphi I_\varphi \angle \theta$$

$$\Leftrightarrow \bar{S}_{3\varphi} = \sqrt{3} V_{ll} I_l \angle \theta$$

$$\Rightarrow P_{3\varphi} = \sqrt{3} V_{ll} I_l \cos(\theta)$$

and $Q_{3\varphi} = \sqrt{3} V_{ll} I_l \sin(\theta)$

- Note that these are the same expressions as in a Y-connected load.

Y-Δ Connection

- Example: three-phase 208 V source supplies a 900 VA load at 0.9 lagging P.F.

a) Find the load per phase.

$$|S_{3\varphi}| = 3 \times 208 \times |I_{\varphi}| = 900 \text{ VA}$$

$$\Rightarrow |I_{\varphi}| = 1.44\text{A}$$

$$\text{but } \angle I_{\varphi} = -\angle S_{3\varphi}$$

$$\text{then } \bar{I}_{\varphi} = 1.44\angle -25.84^{o}$$

$$\bar{Z}_{\varphi} = \frac{\bar{V}_{\varphi}}{\bar{I}_{\varphi}} = \frac{208\angle 0}{1.44\angle -25.84^{o}} = 138.9\angle 25.84^{o}\,\Omega$$

Y-Δ Connection

b) Find the line and phase current magnitudes.

$$|I_\varphi| = 1.44\text{A}$$

$$|I_l| = \sqrt{3}\,|I_\varphi| = 2.5\text{A}$$

c) Find the real and reactive power per phase.

$$\bar{S}_{3\varphi} = 900\angle 25.84^o\,\text{VA}$$

$$\bar{S}_{1\varphi} = 900/3 = 300\angle 25.84^o\,\text{VA}$$

$$\Rightarrow P_{1\varphi} = 300\cos(25.84^o) = 270\text{W}.$$

$$Q_{1\varphi} = 300\sin(25.84^o) = 130.76\text{VAR}.$$

$Or,$

$$P_{1\varphi} = |V_\varphi||I_\varphi|\cos(25.84^o) = 270\text{W}.$$

$$Q_{1\varphi} = |V_\varphi||I_\varphi|\sin(25.84^o) = 130.76\text{VAR}.$$

Y-Δ Connection

d) Compute the total complex power.

$$\bar{S}_{3\varphi} = 3\times\left(270 + j130.76\right) = 810 + j392.28 = 900\angle 25.84^{o} \text{ VA (check)}$$

- Other important examples are 2.12 and 2.13 in the textbook, in addition to problem 2.22.
- The answers to 2.22 are:

$$\bar{S}_{3\varphi} = 95300\angle 28^{o} \text{ VA}$$

P.F.=0.88 lagging

$$Q_{3\varphi} = 45200\text{VAR}$$