ECE 330
POWER CIRCUITS AND ELECTROMECHANICS

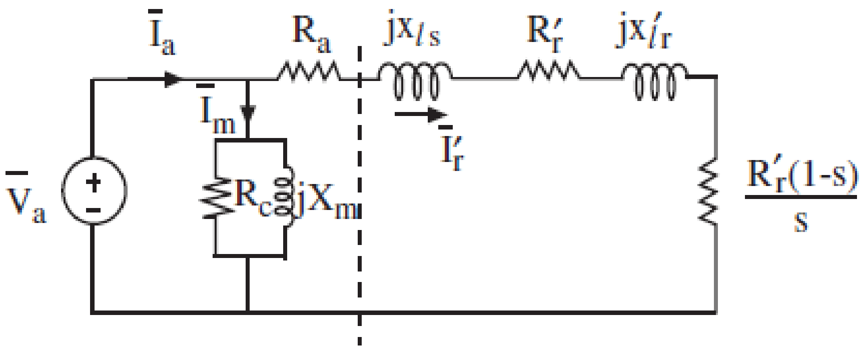
LECTURE 22
INDUCTION MACHINES (3)

Acknowledgment-These handouts and lecture notes given in class are based on material from Prof. Peter Sauer's ECE 330 lecture notes. Some slides are taken from Ali Bazi's presentations

EXPRESSION FOR T^e IN TERMS OF V_a

Using approximate per-phase equivalent circuit:

$$\bar{I}_r' = \frac{\bar{V}_a}{(R_a + \frac{R_r'}{s}) + j(X_{\ell s} + X_{\ell r}')}$$



The magnitude of the current $\bar{I}_r'$

$$I_r' = \frac{V_a}{\sqrt{(R_a + \frac{R_r'}{s})^2 + (X_{\ell s} + X_{\ell r}')^2}}$$

$$P_m = 3(I_r')^2 R_r' \frac{(1-s)}{s} = \frac{3V_a^2 R_r' \frac{(1-s)}{s}}{(R_a + \frac{R_r'}{s})^2 + (X_{\ell s} + X_{\ell r}')^2}$$

EXPRESSION FOR T^e IN TERMS OF V_a

For the two-pole machine $\omega_m = \omega_s(1-s)$

$$P_m = T^e \omega_m = T^e \omega_s (1-s)$$

$$T^e = \frac{P_m}{\omega_s(1-s)}$$

$$T^e = \frac{3V_a^2 \dfrac{R_r'}{s\omega_s}}{(R_s + \dfrac{R_r'}{s})^2 + (X_{\ell s} + X_{\ell r}')^2} = f(V_a, s)$$

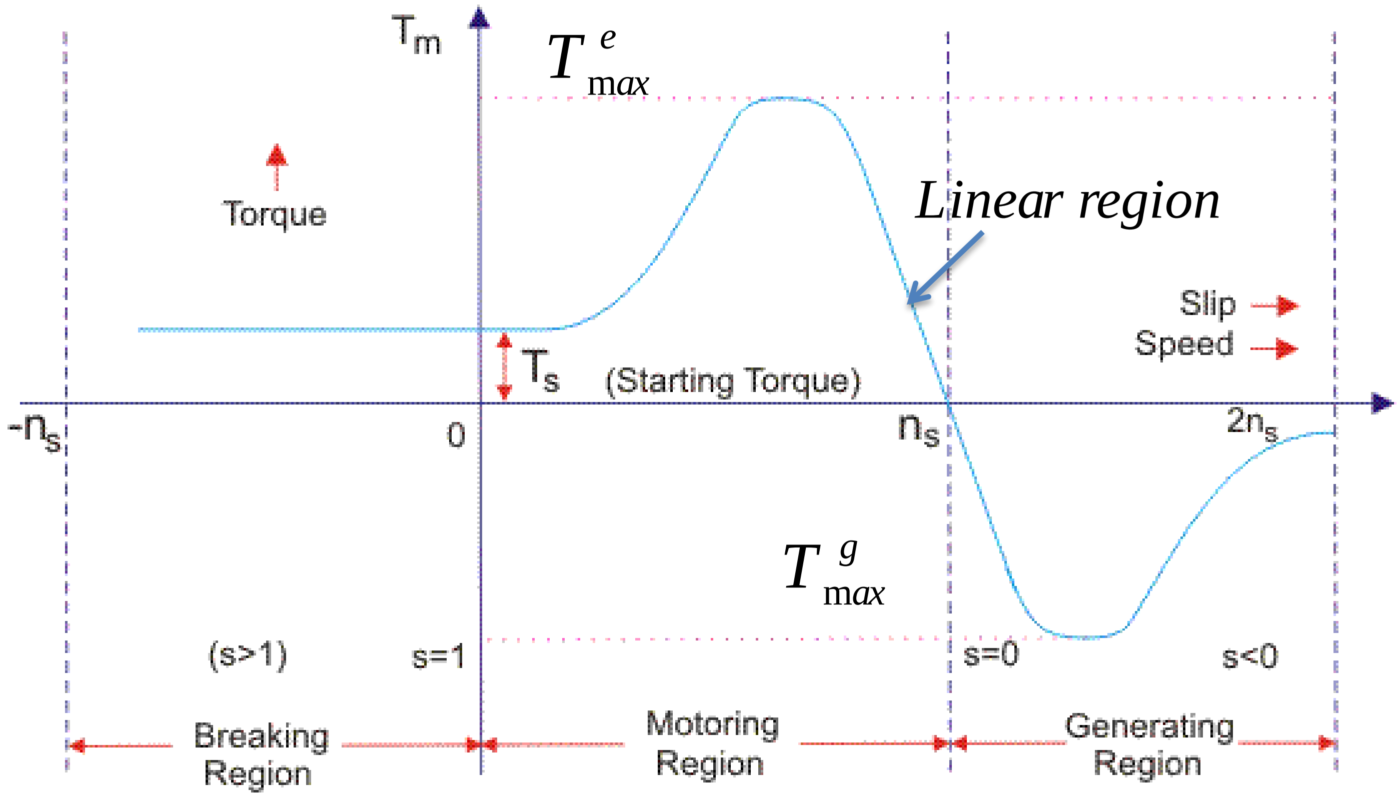
TORQUE-SPEED CURVE

T^e can be approximated as

$$T^e = \frac{p_{ag}}{\omega_s \frac{2}{P}} = \frac{3I_r'^2 \frac{R_r'}{s}}{\omega_s \frac{2}{P}}$$

Note: T^e is max. when p_{ag} is max.

TORQUE-SPEED CURVE



Torque Slip Curve for Three Phase Induction Motor

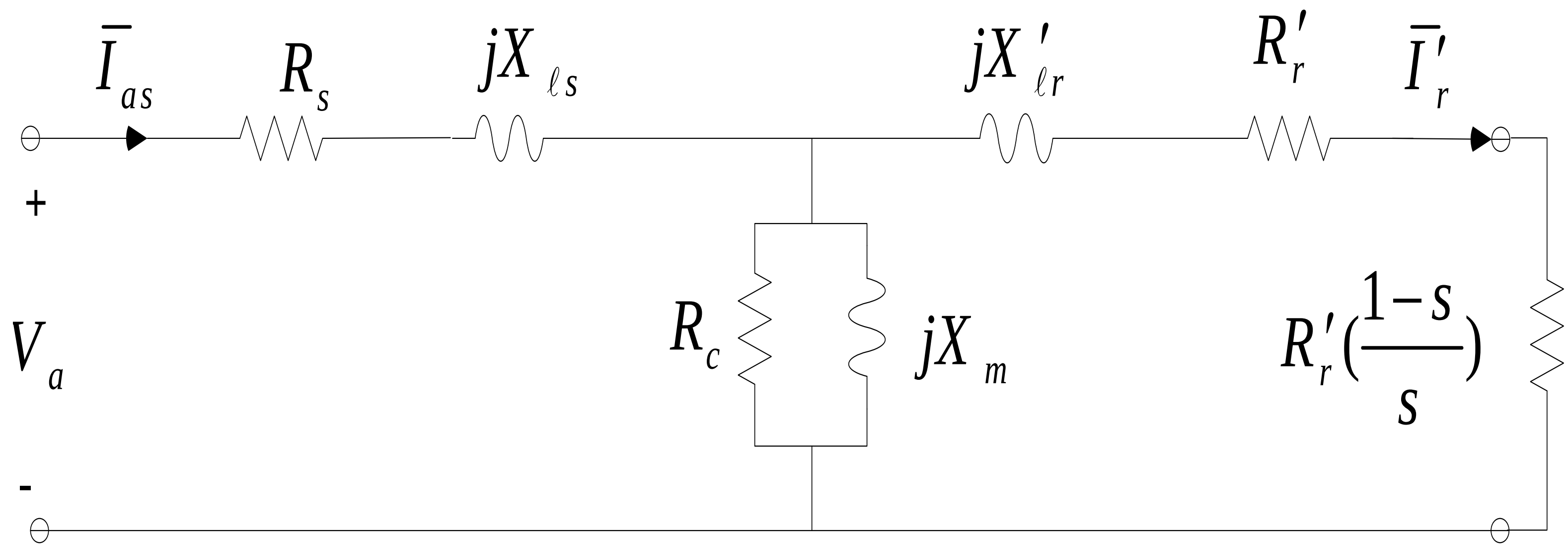
Source:electrical4u.com

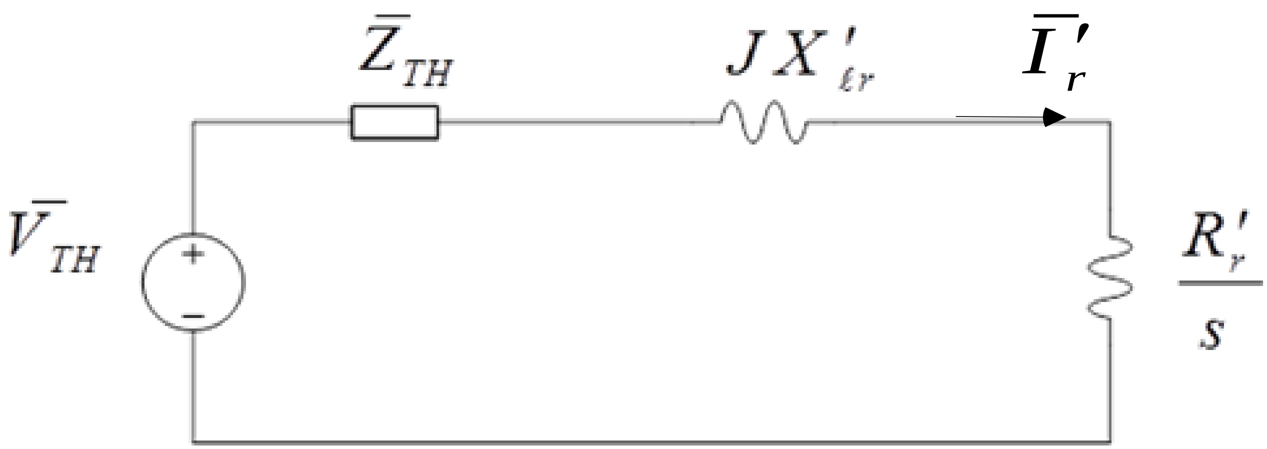
$$0 < s < 1 \Rightarrow Motor$$

$$s < 0 \Rightarrow Generator$$

$$1 < s < 2 \Rightarrow Breaking$$

USING THEVENIN THEORY





USING THEVENIN THEORY

$$p_{ag} = 3\left|I_r'\right|^2 \frac{R'}{s}$$

$$\left|\bar{I}_r'\right| = \left| \frac{V_{TH}}{Z_{TH} + \frac{R_r'}{s} + jX_{\ell r}'} \right|$$

$$P_{ag} = \frac{3\left|\bar{V}_{TH}\right|^2 \frac{R_r'}{s}}{(R_{TH} + \frac{R_r'}{s})^2 + (X_{TH} + X_{\ell r}')^2}$$

$$T^e = \frac{P_m}{\omega_m} = \frac{P_{ag}(1-s)}{(1-s)\omega_s} = \frac{P_{ag}}{\omega_s}$$

USING THEVENIN THEORY

$$T^e = \frac{3\left|\bar{V}_{TH}\right|^2 \frac{R_r'}{s}}{\omega_s \frac{2}{P}\left[\left(R_{TH} + \frac{R_r'}{s}\right)^2 + \left(X_{TH} + X_{\ell r}'\right)^2\right]}$$

$$T^e = \frac{3\left|\bar{V}_{TH}\right|^2 R_r' s}{\omega_s \frac{2}{P}\left[\left(R_{TH} s + R_r'\right)^2 + s^2\left(X_{TH} + X_{\ell r}'\right)^2\right]}$$

For small slip $\left(so \;\; R_{TH} s \ll R_r' \quad and \quad s^2\left(X_{TH} + X_{\ell r}'\right)^2 \ll R_r'^2\right)$

$$T^e = \frac{3\left|\bar{V}_{TH}\right|^2}{\omega_s \frac{2}{P} R_r'} s$$

EXPRESSION FOR MAXIMUM TORQUE

$$T^e = \frac{3V_a^{\ 2}\,\dfrac{R_r'}{s\,\omega_s}}{(R_s + \dfrac{R_s'}{s})^2 + (X_{\ell s} + X_{\ell r}')^2} = f\,(V_a, s)$$

For T^e to have maximum value , $\dfrac{dT^e}{ds} = 0$

$$\frac{R_r'}{s} = \sqrt{R_a^2 + (x_{\ell s} + x_{\ell r}')^2}$$

The slip at which maximum torque occurs

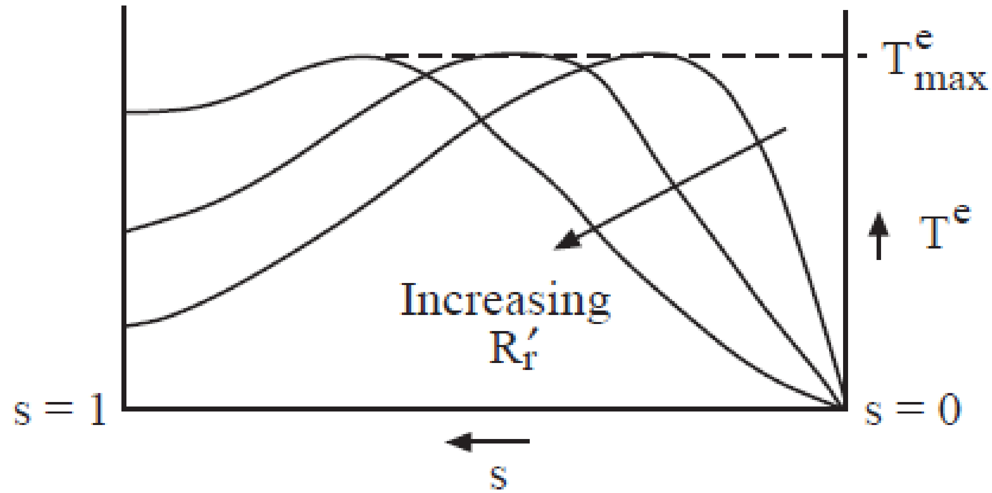
$$s_{mT} = \frac{R_r'}{\sqrt{R_a^2 + (x_{\ell s} + x_{\ell r}')^2}}$$

EXPRESSION FOR MAXIMUM TORQUE

In special case when $R_a = 0$

$$s_{mT} = \frac{R'_r}{(x_{\ell s} + x'_{\ell r})}$$

$$T^e_{\max} = \frac{3}{2} \frac{V_a^2}{\omega_s (x_{\ell s} + x'_{\ell r})}$$



$T^e_{\max}$ is independent of the rotor resistance, but s_{mT} is dependent on the value of R'_r .

The maximum value of T^e is the same but it occurs at different values of s depending on R'_r

MULTI-POLE INDUCTION MACHINES

For a p pole machine, we can repeat the analysis with mechanical angle θ substituted by $\left(\frac{p}{2}\right)\theta$

$$\omega_r = \omega_s - \left(\frac{p}{2}\right)\omega_m \quad , \quad s = \frac{\omega_s - (p/2)\omega_m}{\omega_s} \quad , \quad \omega_m = \frac{\omega_s(1-s)}{p/2}$$

Mechanical power P_m is given by $P_m = T^e \omega_m = T^e \frac{\omega_s(1-s)}{(p/2)}$

$$T^e = \left(\frac{p}{2}\right) 3V_a^2 \frac{R_r'}{s\,\omega_s} \Bigg/ \left[\left(R_a + \frac{R_r'}{s}\right)^2 + (x_{\ell s} + x_{\ell r}')^2\right] = f(V_a, s)$$

$$T_{\max}^e = \left(\frac{p}{2}\right)\frac{3}{2}\frac{V_a^2}{\omega_s(x_{\ell s} + x_{\ell r}')} \quad , \quad T_{\max}^e = \left(\frac{p}{2}\right)\frac{3}{2}\frac{V_{th}^2}{\omega_s(x_{th} + x_{\ell r}')}$$

PROBLEM 7.4

A three-phase, four pole, 60 Hz, 30 hp, 220 V, wye connected induction motor draws a current of 77 A from the line source at a P.F. of 0.88 *lagging*. $s = 0.05$, $P_{sc\ell} = 1033W$, $P_c = 485W$

Find:

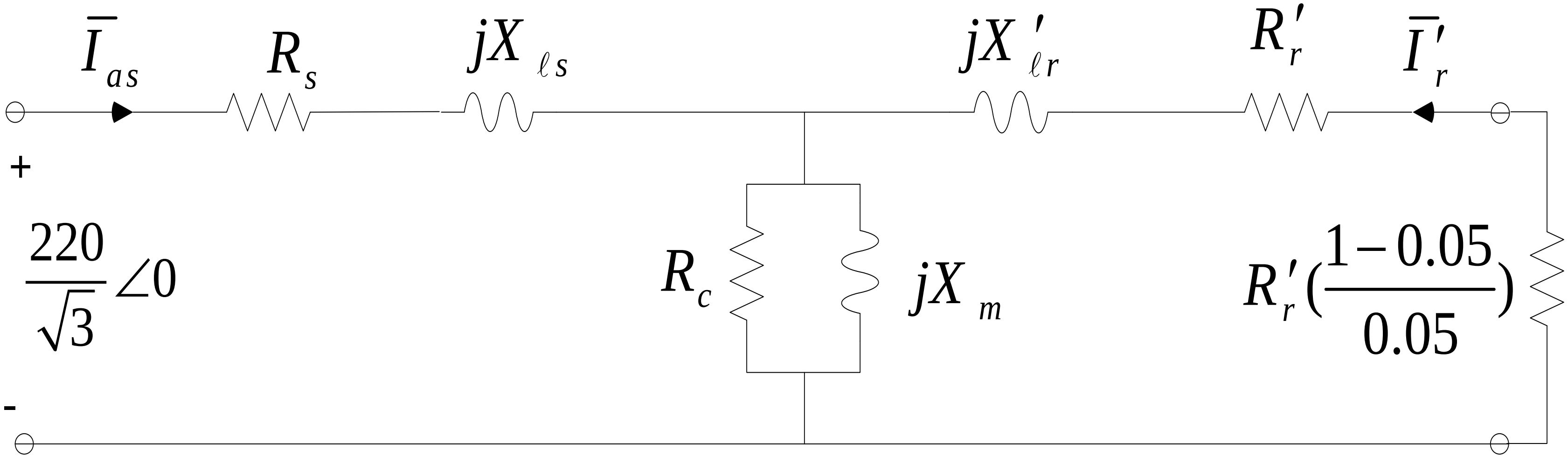
(a) the power across air gap P_{ag} .

(b) T^e torque of electric origin.

(c) P_m mechanical power delivered to shaft.

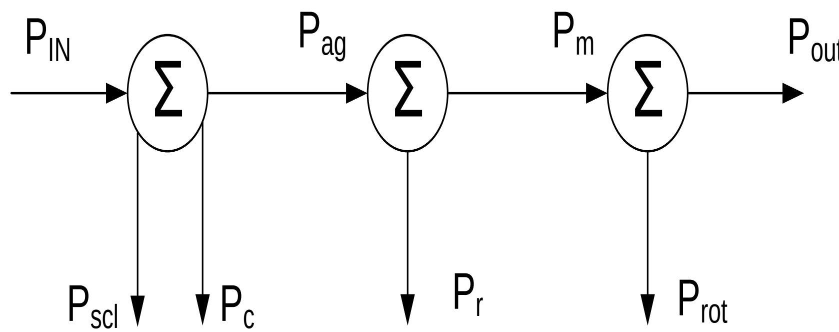
(d) actual horsepower developed and the overall efficiency if rotational losses are 540 W.

PROBLEM 7.4



$$\text{(a):}\ P_{IN} = 3\times\frac{220}{\sqrt{3}} X\ 77\times 0.88 = 25821\,W$$

$$P_{ag} = P_{IN} - P_{sc\ell} - P_c = 24303\ W$$



$$\text{(b):}\ T^e = \frac{p_{ag}}{\omega_s \frac{2}{P}} = \frac{24303}{2\pi 60 X\ \frac{2}{4}} = 129\ N-m$$

PROBLEM 7.4

(c): $$P_m = (1-s)P_{ag} = 0.95X\ 24303 = 23088\ \ W$$

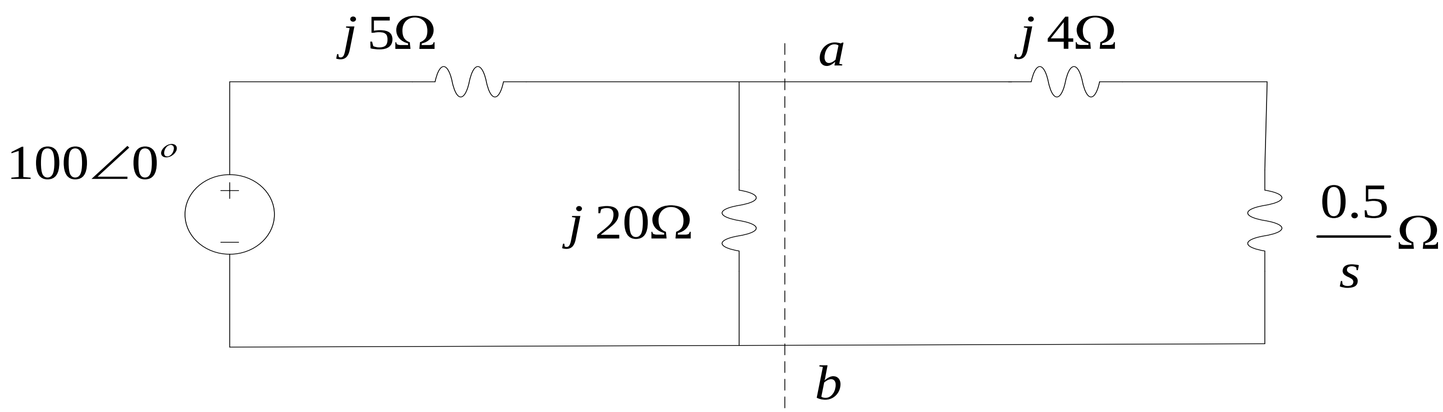
$$P_{out} = P_{shaft} = P_m - P_{rot} = 23088 - 540 = 22548\ \ W$$

(d): $$Actual\ \ hp = \frac{P_{out}}{746} = \frac{22548}{746} = 302\ \ hp$$

$$\mu = \frac{P_{out}}{P_{IN}} X\ 100 = \frac{22548}{25821} X\ 100 = 87\%$$

PROBLEM 7.12

The per-phase equivalent circuit of a three-phase, four-pole, 60 Hz induction motor is given in the figure below.

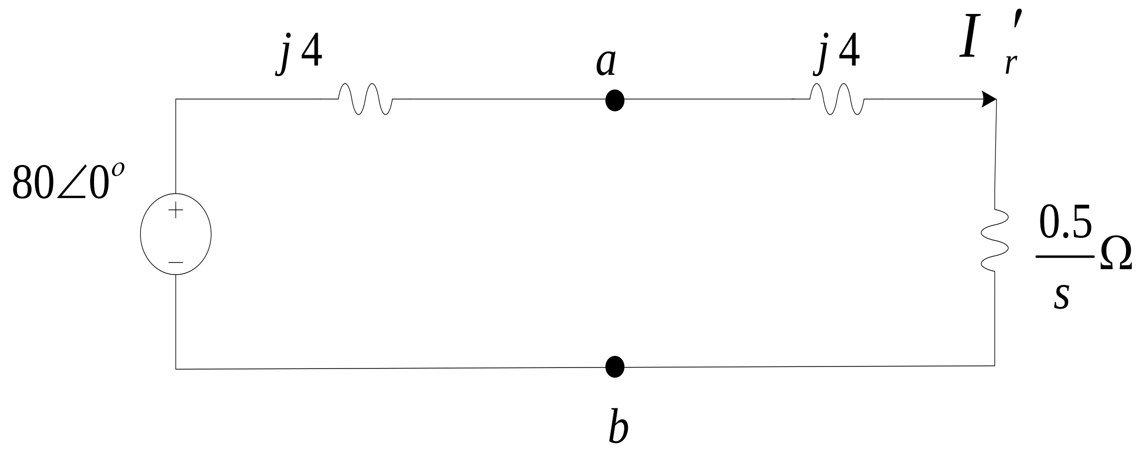


(a): Find T^e using the Thevenin equivalent to the left of the dotted line.

(b): Find $T^e_{\max}$, s_{mT}, and $T^e_{starting}$.

PROBLEM 7.12

$$\bar{V}_{\substack{TH\\ab}} = j20\left(\frac{100\angle 0^o}{j25}\right) = 80\angle 0^o$$



$$\bar{Z}_{TH} = \frac{j5Xj20}{j25} = j4$$

$$I_r' = \frac{80\angle 0^o}{\frac{0.5}{s} + j8} = \frac{80(s)}{0.5 + j8(s)}$$

PROBLEM 7.12

$$T^e = \frac{3\left|I_r'\right|^2 \dfrac{0.5}{s}}{2\pi 60 X \dfrac{2}{4}} = \frac{3X\ 6400(s)^2\left(\dfrac{0.5}{s}\right)}{2\pi 30 X\ (0.25 + 64(s)^2)} = \frac{51(s)}{0.25 + 64(s^2)}$$

$$s_{\max T} = \frac{0.5}{8} = 0.0625$$

(b) $T^e_{\max} = 6.4\ N - m$

$$T^e_{start} = T^e\Big|_{s=1} = 0.8\ N - m$$