

ECE 330

POWER CIRCUITS AND ELECTROMECHANICS

LECTURE 21

INDUCTION MACHINES (2)

Acknowledgment-These handouts and lecture notes given in class are based on material from Prof. Peter Sauer's ECE 330 lecture notes. Some slides are taken from Ali Bazi's presentations

Disclaimer- These handouts only provide highlights and should not be used to replace the course textbook.

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ECE ILLINOIS

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DERAVATION OF AN EQUIVALENT CIRCUIT

For two-phase machine

$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{ar} \\ \lambda_{br} \end{bmatrix} = \begin{bmatrix} L_s & 0 & M \cos \theta & -M \sin \theta \\ 0 & L_s & M \sin \theta & M \cos \theta \\ M \cos \theta & M \sin \theta & L_r & 0 \\ -M \sin \theta & M \cos \theta & 0 & L_r \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ \hat{i}_{ar} \\ \hat{i}_{br} \end{bmatrix}$$

Let the angle between the rotor and stator axes of phase a be

$$\theta = \omega_m t + \gamma$$

Electric torque requires $\omega_r = \omega_s - \omega_m$

Instead of $\omega_m = \omega_s$, $\omega_r = 0$ (synchronous)

DERAVATION OF AN EQUIVALENT CIRCUIT

Define slip

$$s = \frac{\omega_s - \omega_{act}}{\omega_s} \Rightarrow \omega_m = \omega_s - s \omega_s \Rightarrow \omega_m = (1-s) \omega_s$$

So $\omega_r = s \omega_s$ *slip frequency*

Assume stator and rotor currents as

$$i_{as} = I_{ms} \cos \omega_s t$$

$$i_{ar} = I_{mr} \cos (s \omega_s t + \beta)$$

$$i_{bs} = I_{ms} \sin \omega_s t$$

$$i_{br} = I_{mr} \sin (s \omega_s t + \beta)$$

$$\theta = \omega_m t + \gamma = (1-s) \omega_s t + \gamma$$

DERAVATION OF AN EQUIVALENT CIRCUIT

Stator circuit

$$v_{as} = R_s \hat{i}_{as} + \frac{d\lambda_{as}}{dt}$$

$$v_{as} = R_s I_{ms} \cos \omega_s t - \omega_s L_s I_{ms} \sin \omega_s t + \frac{d}{dt} [M \cos((1-s)\omega_s t + \gamma) I_{mr} \cos(s\omega_s t + \beta) - M \sin((1-s)\omega_s t + \gamma) I_{mr} \sin(s\omega_s t + \beta)]$$

$$v_{as} = R_s I_{ms} \cos \omega_s t - \omega_s L_s I_{ms} \sin \omega_s t + \frac{d}{dt} [M I_{mr} \cos((1-s)\omega_s t + \gamma + s\omega_s t + \beta)]$$

$$v_{as} = R_s I_{ms} \cos \omega_s t - \omega_s L_s I_{ms} \sin \omega_s t - \omega_s M I_{mr} \sin(\omega_s t + \gamma + \beta)$$

$$v_{as} = V_{ms} \cos(\omega_s t + \phi_{PF})$$

DERAVATION OF AN EQUIVALENT CIRCUIT

Solving for rotor circuit:

$$v_{ar} = R_r \hat{i}_{ar} + \frac{d \lambda_{ar}}{dt}$$
$$v_{ar} = R_r I_{mr} \cos(\omega_s t + \beta) - s \omega_s L_r I_{mr} \sin(s \omega_s t + \beta)$$
$$+ \frac{d}{dt} [M \cos((1-s)\omega_s t + \gamma) I_{ms} \cos \omega_s t + M I_{ms} \sin((1-s)\omega_s t + \gamma) I_{ms} \sin \omega_s t]$$

DERAVATION OF AN EQUIVALENT CIRCUIT

$$v_{ar} = R_r I_{mr} \cos(s \omega_s t + \beta) - s \omega_s L_r I_{mr} \sin(s \omega_s t + \beta) \\ + \frac{d}{dt} [M I_{ms} \cos(s \omega_s t - \gamma)]$$

$$v_{ar} = R_r I_{mr} \cos(s \omega_s t + \beta) - s \omega_s L_r I_{mr} \sin(s \omega_s t + \beta) \\ - M I_{ms} \sin(s \omega_s t - \gamma)$$

$$v_{ar} = V_{mr} \cos(s \omega_s t + \theta_{v_{ar}})$$

PER-PHASE EQUIVALENT CIRCUIT

Phasors:

$$v_{as} = V_{ms} \cos(\omega_s t + \phi_{PF})$$

$$v_{as} = \frac{V_{ms}}{\sqrt{2}} \angle \phi_{PF} = R_s \frac{I_{ms}}{\sqrt{2}} \angle 0^\circ + j \omega_s L_s \frac{I_{ms}}{\sqrt{2}} \angle 0^\circ + j \omega_s M \frac{I_{mr}}{\sqrt{2}} \angle \gamma + \beta$$

$$v_{ar} = V_{mr} \cos(s \omega_s t + \theta_{v_{ar}})$$

$$v_{ar} = \frac{V_{mr}}{\sqrt{2}} \angle \theta_{vr} = R_r \frac{I_{mr}}{\sqrt{2}} \angle \beta + js \omega_s L_r \frac{I_{mr}}{\sqrt{2}} \angle \beta + js \omega_s M \frac{I_{ms}}{\sqrt{2}} \angle -\gamma$$

Rotate all vectors by γ

PER-PHASE EQUIVALENT CIRCUIT

$$V_{ar} = \frac{V_{mr}}{s\sqrt{2}} \angle \theta_{vr} + \gamma = \frac{R_r}{s} \frac{I_{mr}}{\sqrt{2}} \angle \gamma + \beta + js\omega_s L_r \frac{I_{mr}}{\sqrt{2}} \angle \gamma + \beta + js\omega_s M \frac{I_{ms}}{\sqrt{2}} \angle 0$$

Recall: $L_s = L_{\ell s} + \frac{N_s}{N_r} M$ $L_r = L_{\ell r} + \frac{N_r}{N_s} M$

Define: $I'_{mr} = I_{mr} \left(\frac{N_r}{N_s} \right)$, $V'_{mr} = V_{mr} \left(\frac{N_s}{N_r} \right)$

$$R'_r = R_r \left(\frac{N_s}{N_r} \right)^2 , \quad L'_{\ell r} = L_{\ell r} \left(\frac{N_s}{N_r} \right)^2$$

' Means referred to stator

PER-PHASE EQUIVALENT CIRCUIT

$$\frac{V_{ms}}{\sqrt{2}} \angle \phi_{PF} = R_s \frac{I_{ms}}{\sqrt{2}} \angle 0^\circ + j \omega_s \left(L_{\ell s} + \frac{N_s}{N_r} M \right) \frac{I_{ms}}{\sqrt{2}} \angle 0^\circ + j \omega_s M \frac{I_{mr}}{\sqrt{2}} \angle \gamma + \beta$$

$$= R_s \frac{I_{ms}}{\sqrt{2}} \angle 0^\circ + j \omega_s L_{\ell s} \frac{I_{ms}}{\sqrt{2}} \angle 0^\circ + j \omega_s \frac{N_s}{N_r} M \frac{I_{ms}}{\sqrt{2}} \angle 0^\circ + j \omega_s M \frac{I_{mr}}{\sqrt{2}} \angle \gamma + \beta$$

$$\frac{V'_{mr}}{s \sqrt{2}} \angle \theta_{vr} + \gamma \frac{N_r}{N_s} = \frac{R'_r}{s} \left(\frac{N_r}{N_s} \right) \frac{I'_{mr}}{\sqrt{2}} \left(\frac{N_s}{N_r} \right) \angle \gamma + \beta$$

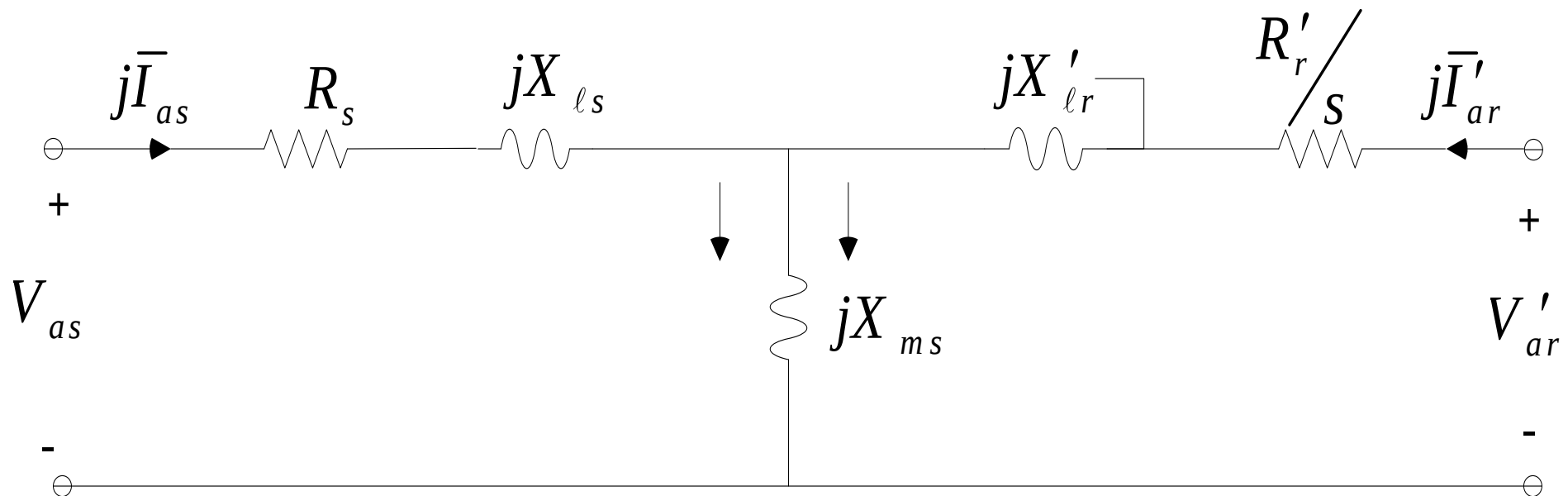
$$+ j \omega_s L'_r \left(\frac{N_r}{N_s} \right)^2 \frac{I'_{mr}}{\sqrt{2}} \left(\frac{N_s}{N_r} \right) \angle \gamma + \beta + j \omega_s M \frac{I_{ms}}{\sqrt{2}} \angle 0$$

PER-PHASE EQUIVALENT CIRCUIT

Multiply by $\frac{N_s}{N_r}$

$$\frac{V'_{mr}}{s\sqrt{2}} \angle \theta_{vr} + \gamma = \frac{R'_r}{s} \cdot \frac{I'_{mr}}{\sqrt{2}} \angle \gamma + \beta + j\omega_s (L'_{lr} + \frac{N_s}{N_r} M) \frac{I'_{mr}}{\sqrt{2}} \angle \gamma + \beta$$

$$+ j\omega_s \frac{N_s}{N_r} M \cdot \frac{I_{ms}}{\sqrt{2}} \angle 0$$



PER-PHASE EQUIVALENT CIRCUIT

Types of Induction Motors:

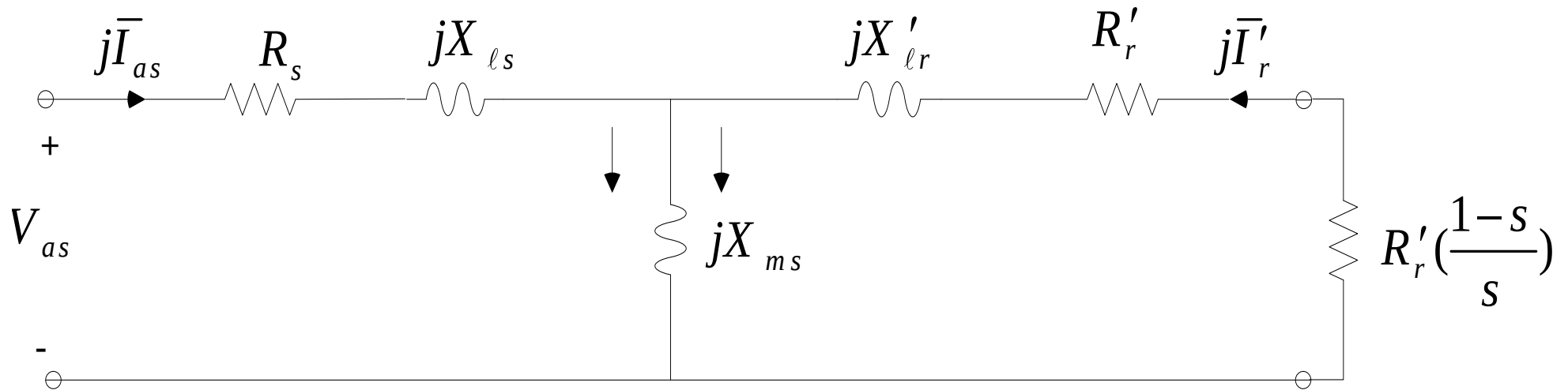
Wound rotor – Could connect something at V'_{ar}

Squirrel cage $V'_{ar} = 0$

Let $V'_{ar} = 0$

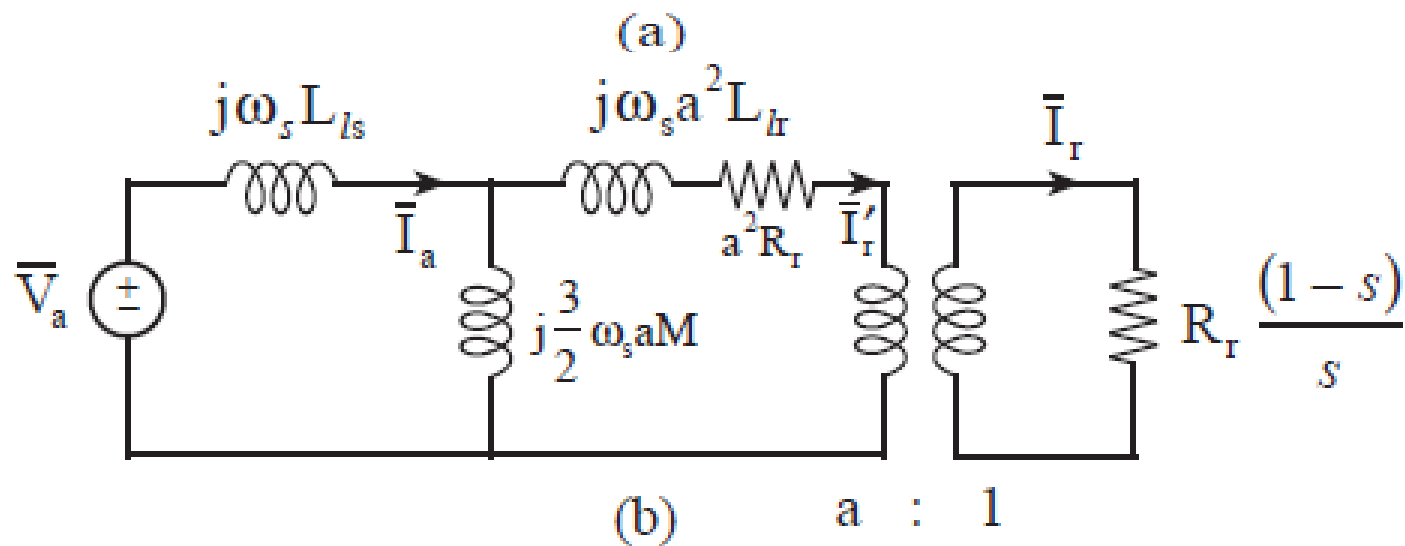
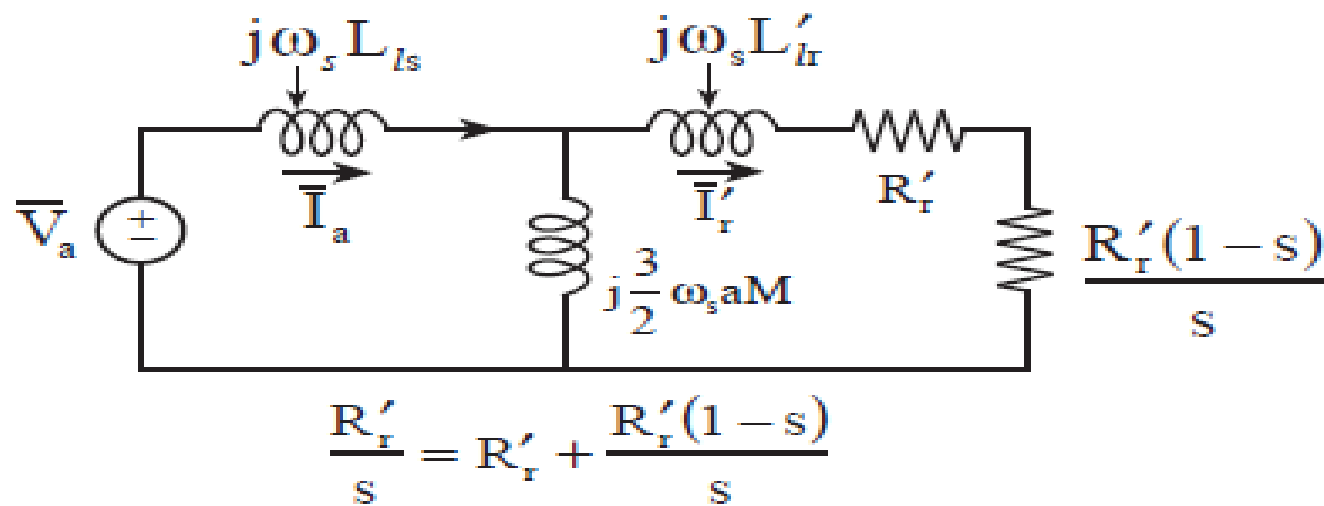
Split $\frac{R'_r}{s}$ into $R'_r + R'_r \left(\frac{1-s}{s} \right)$

PER-PHASE EQUIVALENT CIRCUIT



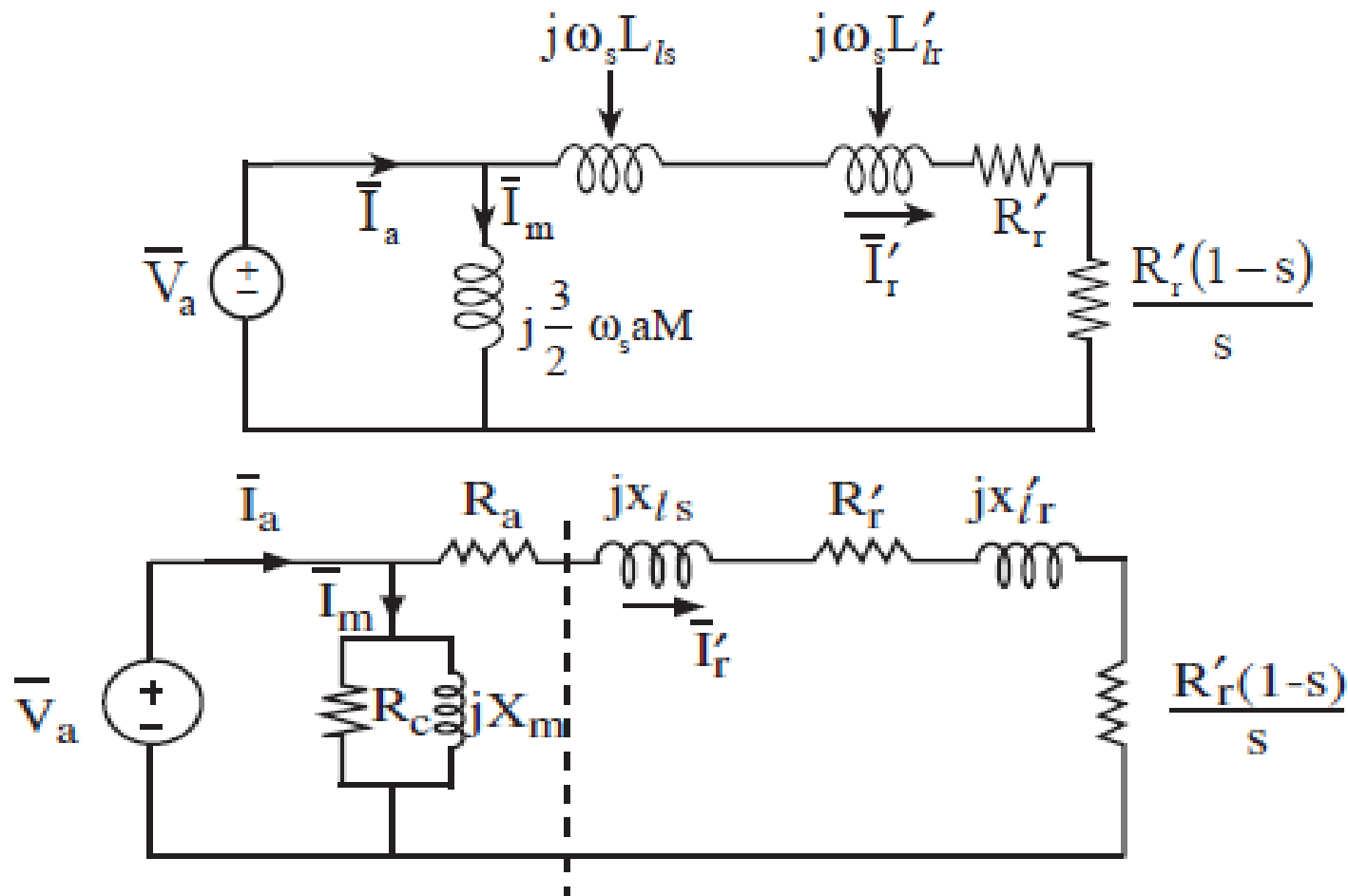
We could add R_c for eddy current and core losses in parallel with jX_{ms} (like transformer).

PER-PHASE EQUIVALENT CIRCUIT FOR TWO POLES THREE-PHASE INDUCTION MACHINE



PER-PHASE EQUIVALENT CIRCUIT FOR THREE-PHASE INDUCTION MACHINE

Approximate equivalent circuit:



POWER RELATIONSHIP

Power relationships:

The total input power is given by $P_{IN} = \text{Real}(3\bar{V}_a \bar{I}_a^*)$

$$= P_{ag} + P_{sc\ell} + P_c$$

Where

$$P_c = \text{core loss} = 3V_a^2 / R_c$$

$$P_{sc\ell} = \text{stator copper loss} = 3(I_r')^2 R_a$$

$$P_{ag} = \text{power across the air gap} = 3I_r'^2 R_r' / s$$

POWER RELATIONSHIP

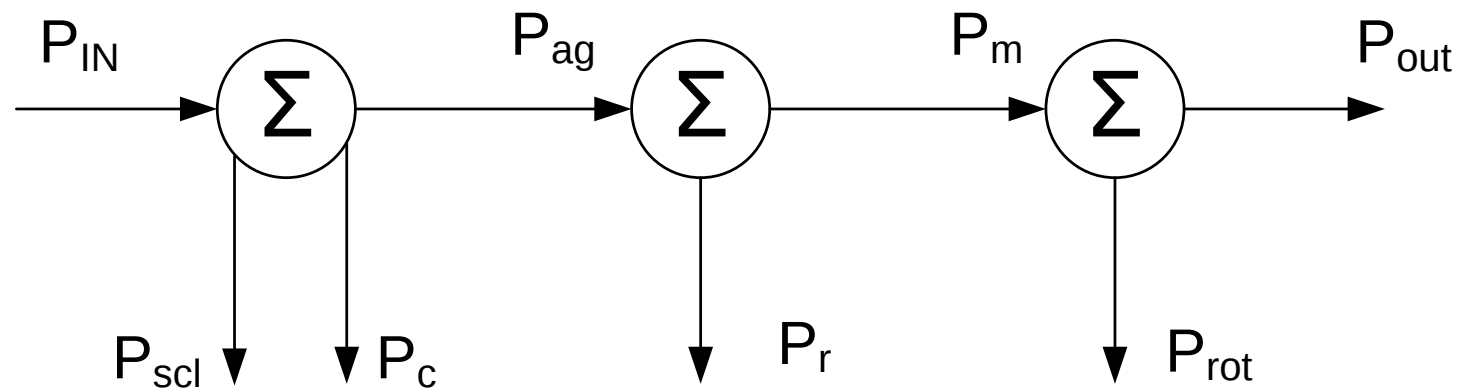
The power transformed across the air gap is the sum of the mechanical power P_m developed and the rotor loss

$$P_r = 3I_r'^2 R_r' = s P_{ag}$$

$$P_m = 3I_r'^2 R_r' \left(\frac{1-s}{s} \right) = P_{ag} (1-s)$$

$$P_{ag} = P_m + P_r$$

$$P_{shaft} = P_m - P_{rot}$$



POWER AND TORQUE RELATIONSHIP

Rotor copper loss in terms of P_{ag}

$$P_r = 3 \left| \bar{I}'_r \right|^2 R'_r = s P_{ag}$$

$$\text{For } P \neq 2 \quad \omega_m = (1-s) \omega_s \frac{2}{P}$$

$$T^e = \frac{P_m}{\omega_m} = \frac{P_{ag} (1-s)}{(1-s) \omega_s \frac{2}{P}} = \frac{P_{ag}}{\frac{2}{P} \omega_s}$$

EXAMPLE

Four pole induction motor, 60 Hz, 120 V, has

$$R_s = 0, \quad X_{\ell s} = 0, \quad X_{ms} = 40 \, \Omega$$

$$X'_{\ell r} = 1 \, \Omega, \quad R'_r = 1.3 \, \Omega$$

$$N_{act} = 1720 \, \text{rpm}. \quad \text{Neglect } P_c \text{ and } P_{rot}$$

Find s , f_r , $|I'_{ar}|$, P_m , ω_m , T^e , P_{ag} , P_{IN} , $\bar{I}_{as}$, $P.F.$

EXAMPLE

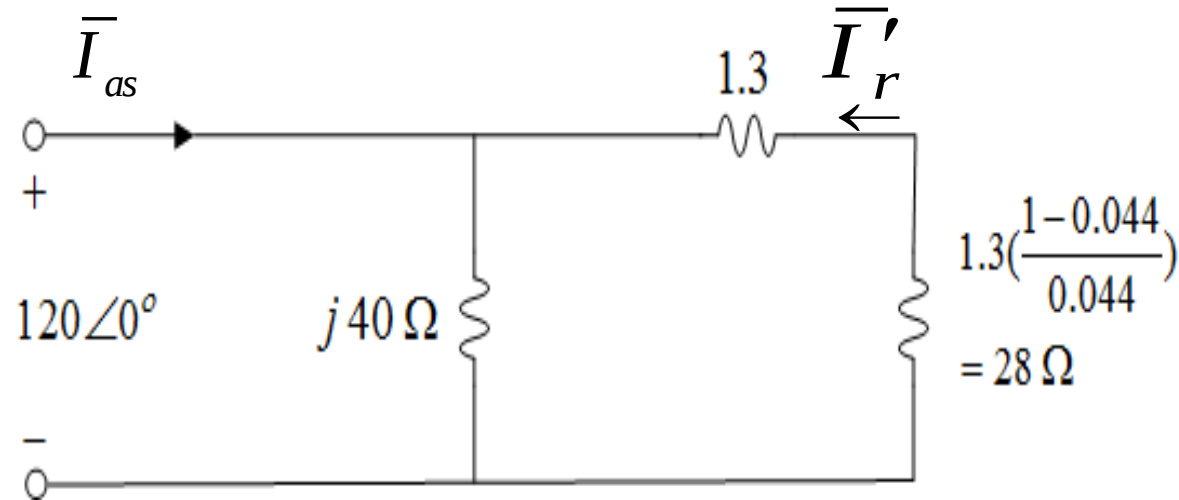
$$N_s = \frac{120f_s}{P} = \frac{120 \times 60}{4} = 1800$$

$$s = \frac{1800 - 1720}{1800} = 0.044$$

$$f_r = s f_s = 0.044 \times 60 = 2.7 \text{ Hz}$$

$$\bar{I}'_r = \frac{-120 \angle 0}{29.3 + j1} = -4.1 \angle 0$$

$$P_m = 3 \times 4.1^2 \times 28 = 1412 \text{ W}$$



EXAMPLE

$$\omega_{m_{mech}} = 1720 \frac{REV}{MIN} \left(\frac{2\pi R}{60 \text{sec}} \right) = 180 \text{ R / sec}$$

$$T^e = \frac{1420}{180} = 7.8 \text{ N - m}$$

$$P_{ag} = \frac{1412}{1 - 0.044} = 1477 \text{ W}$$

$$\text{Check : } T^e = \frac{1477}{2\pi 60 \left(\frac{2}{4} \right)} = 7.8 \text{ N - m}$$

EXAMPLE

$$P_{IN} = P_{ag} + 3 \left| \bar{I}_{as} \right|^2 R_s = 1477 \quad (R_s = 0)$$

$$\bar{I}_{as} = \frac{120 \angle 0}{j 40} - (-4.1 \angle 0) = 4.1 - j 3 = 5.1 \angle -36^\circ$$

$$P.F. = \cos(-36^\circ) = 0.81 \quad \text{lagging}$$

EFFICIENCY

The efficiency is

$$\eta = \frac{P_{out}}{P_{IN}} = \frac{P_m}{P_{IN}} \times 100 = \frac{P_{IN} - (P_{sc\ell} + P_c + P_r)}{P_{IN}} \times 100$$

If we neglect the stator copper loss $P_{sc\ell}$ and the core losses P_c

$$\eta = \frac{\text{mechanical power developed}}{\text{input power}}$$

$$\eta = \frac{P_m}{P_{ag}} \times 100 = (1 - s) \times 100$$

Given the actual speed, it gives a quick idea about the efficiency of the motor.

EFFICIENCY

If rotational losses P_{rot} are considered, they should be subtracted from P_m to get P_{shaft}

$$P_{shaft} = P_m - P_{rot}$$

The overall efficiency

$$\eta = \frac{P_{shaft}}{P_{IN}} \times 100$$

