ECE 330
POWER CIRCUITS AND ELECTROMECHANICS

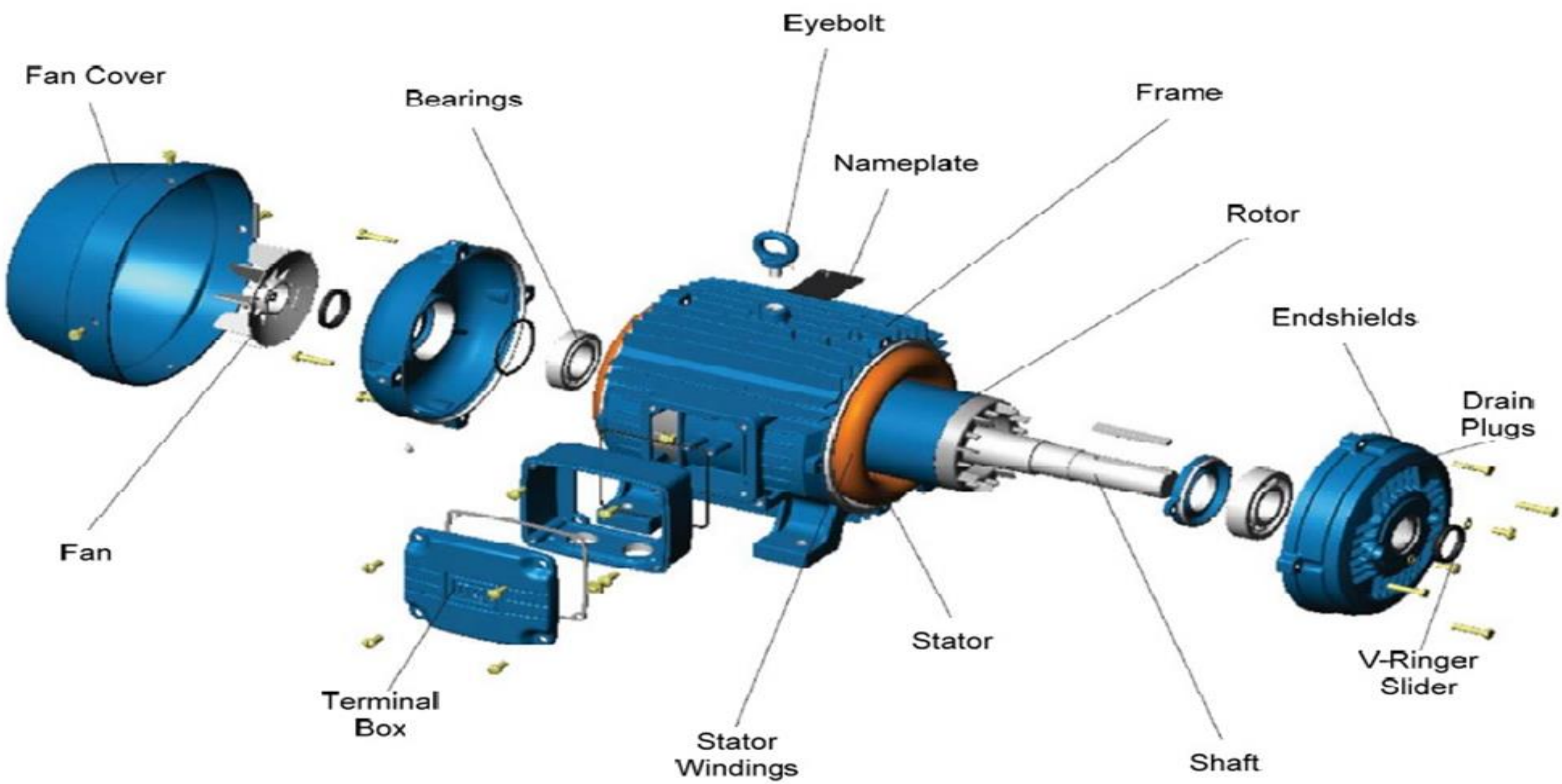
LECTURE 20
INDUCTION MACHINES (1)

Acknowledgment-These handouts and lecture notes given in class are based on material from Prof. Peter Sauer's ECE 330 lecture notes. Some slides are taken from Ali Bazi's presentations

INDUCTION MACHINES

- Most widely used machines in the industry
- Both the stator and the rotor carry alternating currents
- Used in pumps, compressors, fans, etc.
- Motor inductance can be controlled to have excellent torque-speed capabilities.
- Analysis based on equivalent circuits derived from the energy point of view

INDUCTION MACHINES



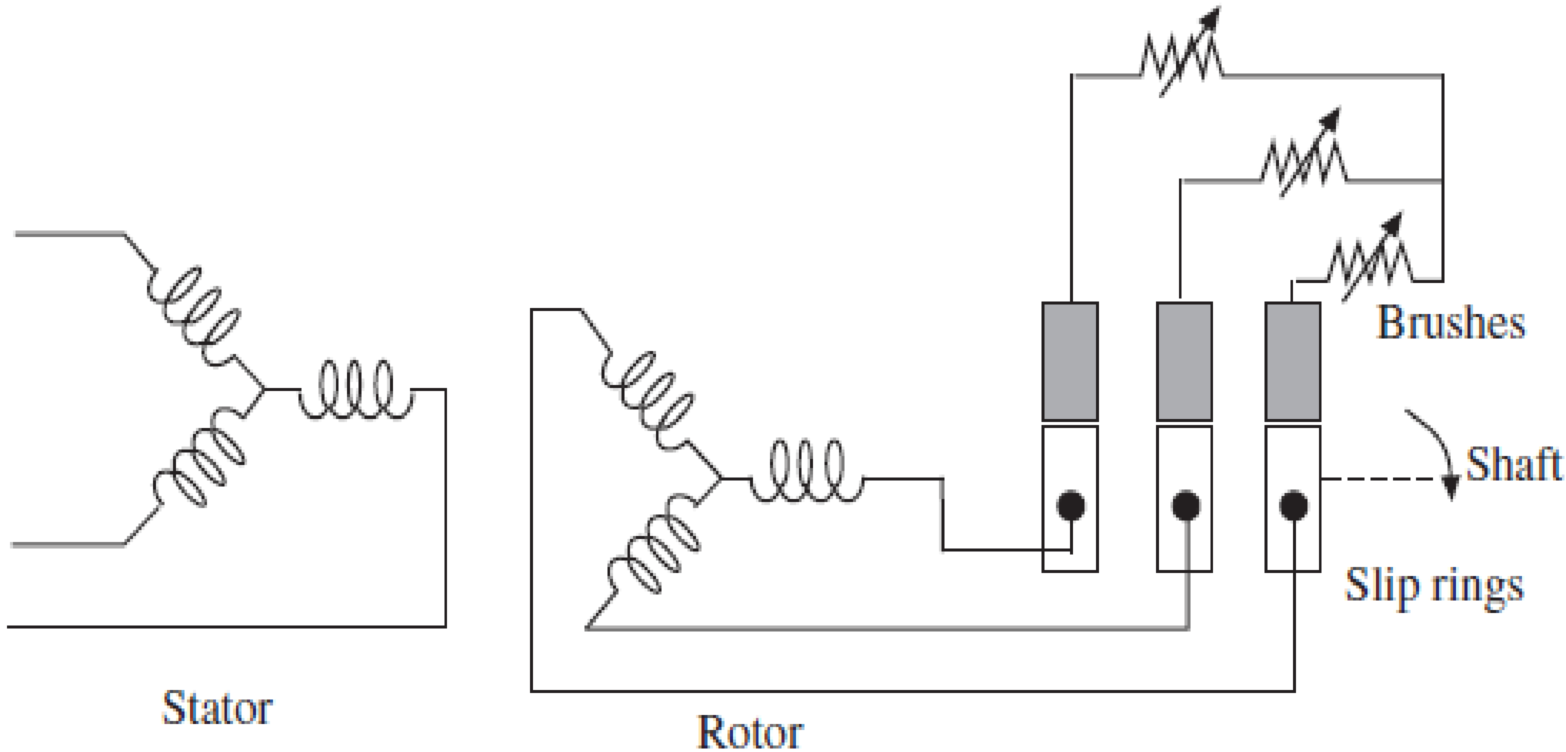
A typical 3-phase induction motor [Courtesy of Electromotors WEG SA, Brazil]

PHYSICAL FEATURES AND FREQUENCY RELATIONSHIPS

- It has a three-phase winding on both the stator and the rotor, wound for P poles
- The three-phase winding on the rotor is short circuited either through external means or on the rotor itself
  - ➢ Wound rotor
  - ➢ *Squirrel cage rotor*
- Two basic components: the *stator* and the *rotor*
- *Analogous to a transformer with rotating secondary.*

WOUND ROTOR

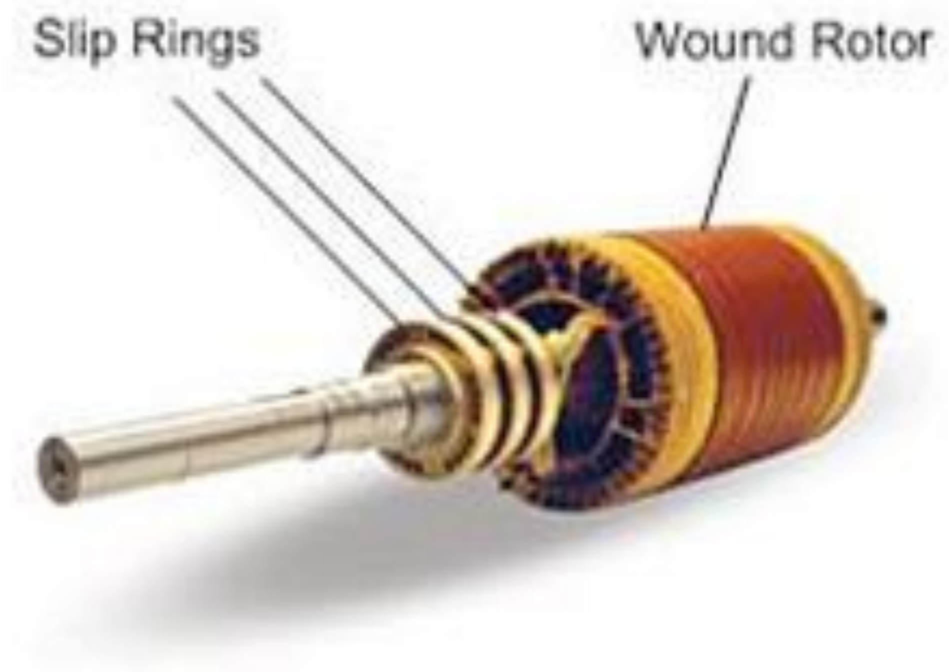
The rotor windings that rotate with the rotor are connected to slip rings.



The fixed brushes provide a short-circuited path through an external resistance

WOUND ROTOR

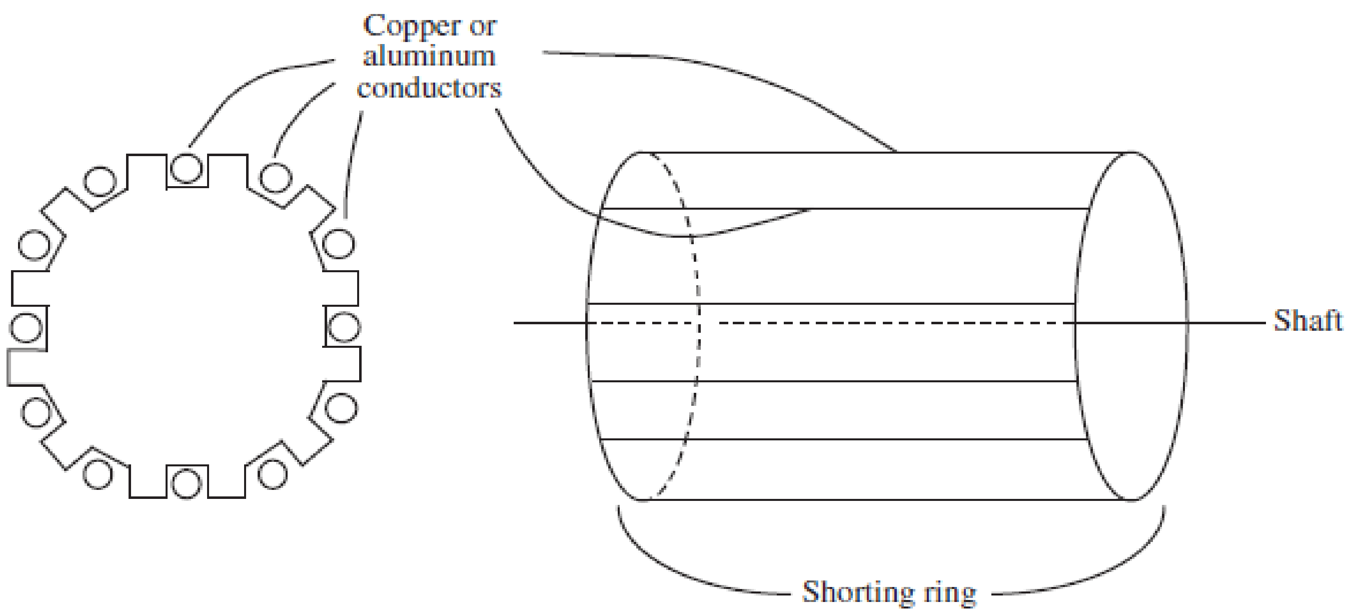
External resistance is inserted to achieve a desired torque-speed characteristic. A higher starting torque can be achieved together with limited control over speed.



Source: pinterest.com

SQUIRREL CAGE ROTOR

- Rotor windings shorted on the rotor itself
- A fixed-torque speed characteristic
- The conductors through the slots are simply shorted through a ring at the ends



OPERATION OF AN INDUCTION MACHINE

Three-phase currents are supplied to the stator windings, resulting in a rotating field.

Each phase may be wound for p poles so that the rotating field revolves at the synchronous speed N_s given by $f = \frac{pN_s}{120}$ where f is the frequency in Hz and N_s is in revolutions per minute (rpm)

OPERATION OF AN INDUCTION MACHINE

- Currents are induced in the rotor windings, which interact with the rotating field to produce torque.
- The speed increases to a point where the torque developed between rotor and stator is just enough to balance the opposing mechanical load torque
- So long as the opposing torque (due to bearing friction,.. etc., under no-load conditions) exists, the rotor speed can never equal the speed of the stator rotating field speed.

OPERATION OF AN INDUCTION MACHINE

The rotor currents have a frequency ω_r .

Mechanical speed in mechanical rads per second satisfies the relation by $\omega_s - \omega_r = \left(\frac{p}{2}\right)\omega_m$

The frequency of induced currents in the rotor is

$$\omega_r = \omega_s - \left(\frac{p}{2}\right)\omega_m \ \ electrical \ \ rads / sec$$

We define a new dimensionless quantity called *slip* as

$$s = \frac{N_s - N_{act}}{N_s}$$

OPERATION OF AN INDUCTION MACHINE

N_s : The synchronous speed in rpm,

N_{act} : the actual speed of the rotor

Multiplying the right side by $\frac{2\pi p}{120}$ we get

$$s = \frac{2\pi\left(\frac{pN_s}{120}\right) - \frac{2\pi p}{120} N_{act}}{\frac{2\pi p N_s}{120}}$$

$$s = \frac{2\pi f - \left(\frac{p}{2}\right)\frac{2\pi N_{act}}{60}}{2\pi f} = \frac{\omega_s - \left(\frac{p}{2}\right)\omega_m}{\omega_s}$$

OPERATION OF AN INDUCTION MACHINE

So,

$$\omega_m = \frac{\omega_s(1-s)}{\frac{p}{2}} \; mechanical \; rads/sec \Rightarrow \omega_m = \omega_s(1-s)\frac{2}{p}$$

Since

$$\omega_r = \omega_s - \left(\frac{p}{2}\right)\omega_m \; electrical \; rads/sec$$

So, $\omega_r = s\,\omega_s$

OPERATION OF AN INDUCTION MACHINE

From this $f_r = s\,f$

Where f_r is the frequency of current in the rotor

Slip is dimensionless, and it gives an idea of the speed of the machine with respect to the synchronous speed.

$s = 0$ corresponds to synchronous speed

$s = 1$ corresponds to standstill or starting conditions

s is a very small number between 0.01 to 0.05.

EXAMPLE 7.1

Consider a three-phase, 60 Hz, six-pole induction machine run at 1125 rpm. Find the slip and the rotor current frequency f_r .

$$f = \frac{pN_s}{120}$$

$$N_s = \frac{60 \times 120}{6} = 1200 \ rpm$$

$$s = \frac{1200 - 1125}{1200} = 0.0625$$

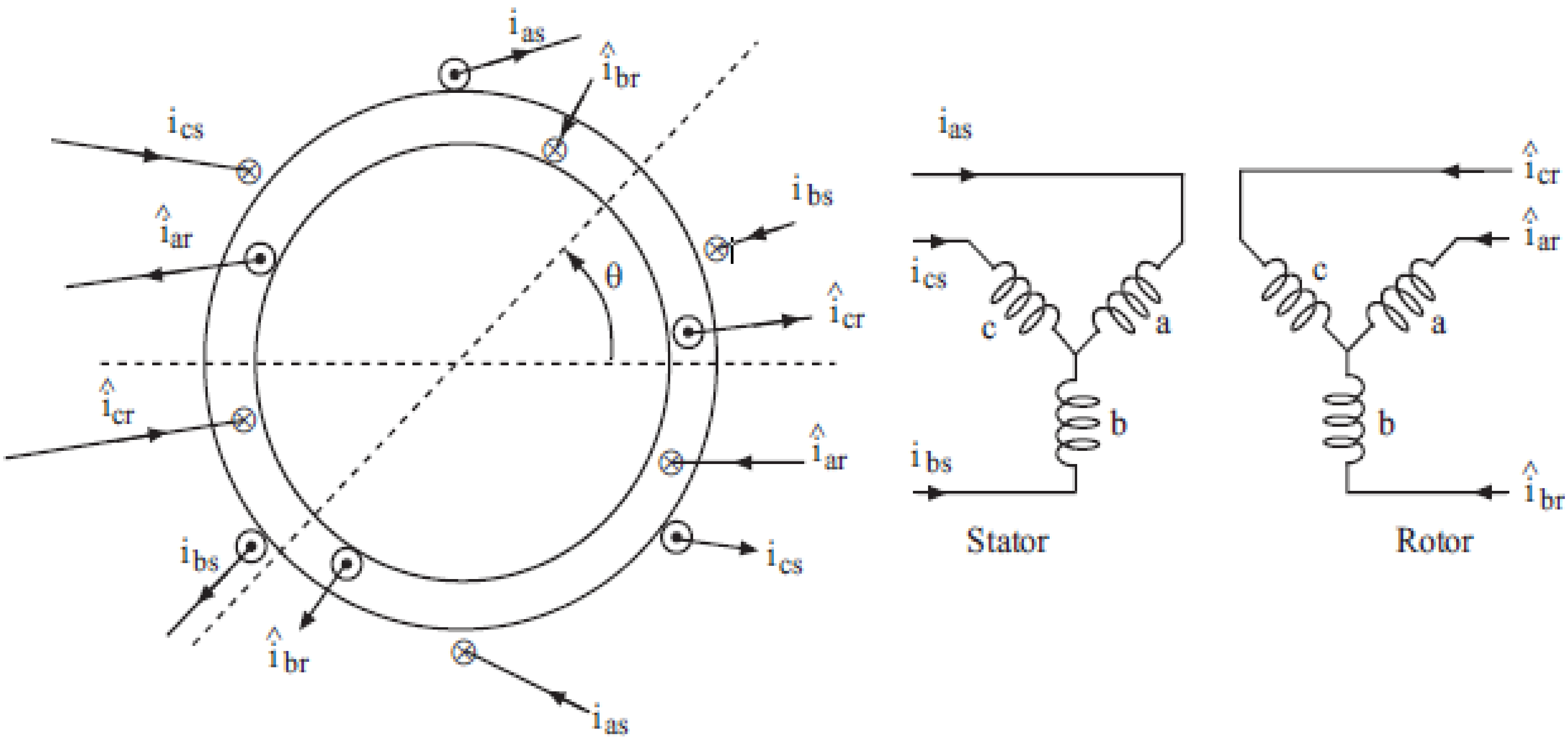
$$f_r = sf = (0.0625)60 = 3.75 \ Hz$$

ANALYSIS OF A TWO-POLE MACHINE

Just as in the case of the synchronous machine, we first do the analysis for a two-pole machine and then generalize to a P pole machine. The stator has three-phase windings. Unlike in the synchronous machine, there are three windings on the rotor,

- Label s for stator windings
- Label r for rotor windings

DERAVATION OF AN EQUIVALENT CIRCUIT



DERAVATION OF AN EQUIVALENT CIRCUIT

$\lambda - i$ *relation is*

$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \\ \lambda_{ar} \\ \lambda_{br} \\ \lambda_{cr} \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} \boldsymbol{i}_{as} \\ \boldsymbol{i}_{bs} \\ \boldsymbol{i}_{cs} \\ \hat{\boldsymbol{i}}_{ar} \\ \hat{\boldsymbol{i}}_{br} \\ \hat{\boldsymbol{i}}_{cr} \end{bmatrix}$$

DERAVATION OF AN EQUIVALENT CIRCUIT

where the matrix A is

$$
\begin{bmatrix}
L_s & \frac{-L_{ms}}{2} & \frac{-L_{ms}}{2} & M\cos\theta & M\cos(\theta+120^\circ) & M\cos(\theta-120^\circ) \\
\frac{-L_{ms}}{2} & L_s & \frac{-L_{ms}}{2} & M\cos(\theta-120^\circ) & M\cos\theta & M\cos(\theta+120^\circ) \\
\frac{-L_{ms}}{2} & \frac{-L_{ms}}{2} & L_s & M\cos(\theta+120^\circ) & M\cos(\theta-120^\circ) & M\cos\theta \\
 & & & L_r & \frac{-L_{mr}}{2} & \frac{-L_{mr}}{2} \\
 & M_r^T & & \frac{-L_{mr}}{2} & L_r & \frac{-L_{mr}}{2} \\
 & & & \frac{-L_{mr}}{2} & \frac{-L_{mr}}{2} & L_r
\end{bmatrix}
$$

DERAVATION OF AN EQUIVALENT CIRCUIT

$L_s = L_{ms} + L_{\ell s}$ is the self-inductance of the stator coil

$L_r = L_{mr} + L_{\ell r}$ is the self-inductance of the rotor coil

The leakage inductance $L_{\ell s}$, $L_{\ell r}$

The magnetizing inductance

$$L_{ms} = \left(\frac{N_s}{2}\right)^2 \frac{\pi \mu_0 r \ell}{g} \quad , \quad L_{mr} = \left(\frac{N_r}{2}\right)^2 \frac{\pi \mu_0 r \ell}{g} ,$$

Mutual inductance $M = \frac{N_s N_r}{4} \frac{\pi \mu_0 r \ell}{g}$

$$L_s = L_{\ell s} + \frac{N_s}{N_r} M \quad , \quad L_r = L_{\ell r} + \frac{N_r}{N_s} M$$

DERAVATION OF AN EQUIVALENT CIRCUIT

Machine parameters:

g is the air gap length, r is the radius of the air gap, ℓ is the axial length of the core, N_s and N_r are the effective turns per phase on the stator and rotor, respectively.

Note that for both the stator and rotor windings, the current directions are *into* the windings. The rotor currents are denoted as $\hat{i}_{ar}$, $\hat{i}_{br}$, and $\hat{i}_{cr}$ into the coils

DERAVATION OF AN EQUIVALENT CIRCUIT

For two-phase machine

$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{ar} \\ \lambda_{br} \end{bmatrix} = \begin{bmatrix} L_s & 0 & M\cos\theta & -M\sin\theta \\ 0 & L_s & M\sin\theta & M\cos\theta \\ M\cos\theta & M\sin\theta & L_r & 0 \\ -M\sin\theta & M\cos\theta & 0 & L_r \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ \hat{i}_{ar} \\ \hat{i}_{br} \end{bmatrix}$$

Let the angle between the rotor and stator axes of phase a be

$$\theta = \omega_m t + \gamma$$

Electric torque requires $\omega_r = \omega_s - \omega_m$

Instead of $\omega_m = \omega_s$, $\omega_r = 0$ (synchronous)