ECE 330
POWER CIRCUITS AND ELECTROMECHANICS

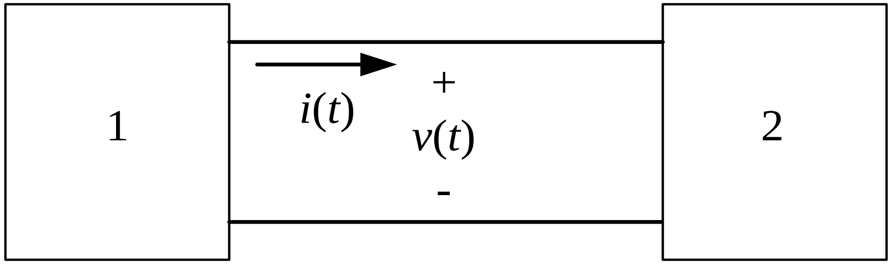
LECTURE 2
CONSERVATION OF COMPLEX POWER

Acknowledgment-These handouts and lecture notes given in class are based on material from Prof. Peter Sauer's ECE 330 lecture notes. Some slides are taken from Ali Bazi's presentations

Disclaimer- These handouts only provide highlights and should not be used to replace the course textbook.

SIMPLE NETWORK

- Complex power is conserved in a network.
- This can be proved in this simple network.
- Given the current direction,



$$\bar{S}_{in,1} = -\bar{V}\ \bar{I}^* \text{ and } \bar{S}_{out,1} = \bar{S}_{in,2} = \bar{V}\ \bar{I}^* \text{ and } \bar{S}_{out,2} = -\bar{V}\ \bar{I}^*$$

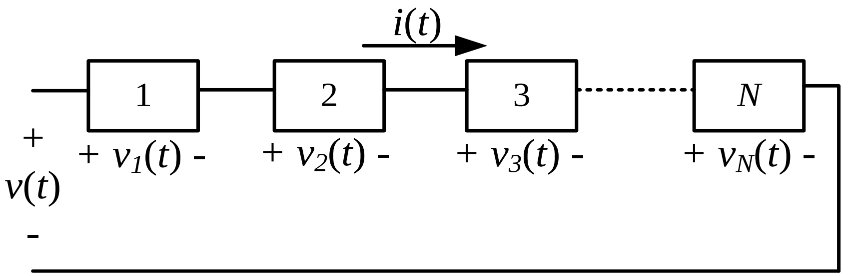
Thus, $\bar{S}_{in,1} = \bar{S}_{out,2}, \bar{S}_{in,1} + \bar{S}_{out,1} = 0, \text{ and } \bar{S}_{in,2} + \bar{S}_{out,2} = 0.$

- Electric charge is conserved. All the charge that passes any point of the circuit in a given time interval must pass any other point in the same time interval.

SERIES AND PARALLEL NETWORKS

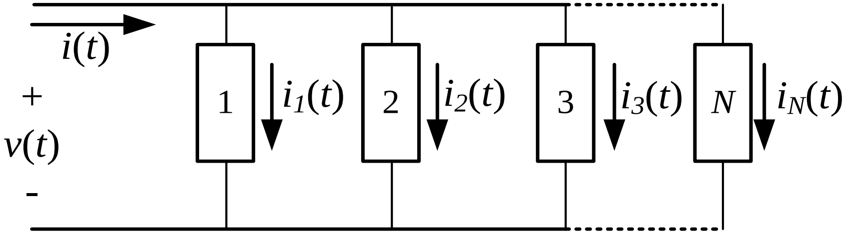
- Series network:

$$\bar{S}_{in} = \bar{V}_1 \bar{I}^* + \bar{V}_2 \bar{I}^* + \bar{V}_3 \bar{I}^* + \ldots + \bar{V}_N \bar{I}^*$$
$$= \bar{S}_1 + \bar{S}_2 + \bar{S}_3 + \ldots + \bar{S}_N$$



- Parallel network

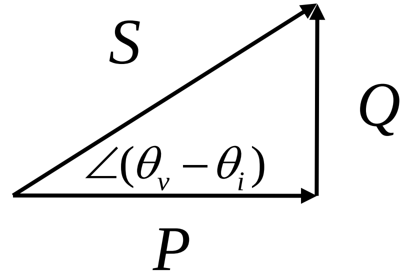
$$\bar{S}_{in} = \bar{V}\, \bar{I}_1^* + \bar{V}\, \bar{I}_2^* + \bar{V}\, \bar{I}_3^* + \ldots + \bar{V}\, \bar{I}_N^*$$
$$= \bar{S}_1 + \bar{S}_2 + \bar{S}_3 + \ldots + \bar{S}_N$$



The total complex power is the sum of the complex powers in individual elements (power is conserved).

POWER TRIANGLE

- The complex power can be represented in a power triangle where $\bar{S} = P + jQ = S\angle(\theta_V - \theta_i) = S\angle\theta$

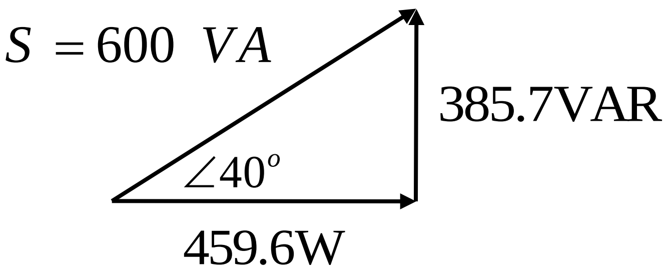


- Example:

$$\bar{V} = 120\angle 25^o \text{V}, \bar{I} = 5\angle -15^o \text{A}$$

$$\bar{S} = \bar{V}\ \bar{I}^* = 600\angle 40^o \text{VA}$$

$$\bar{S} = 459.6 + j385.7\text{VA}$$



POWER TRIANGLE

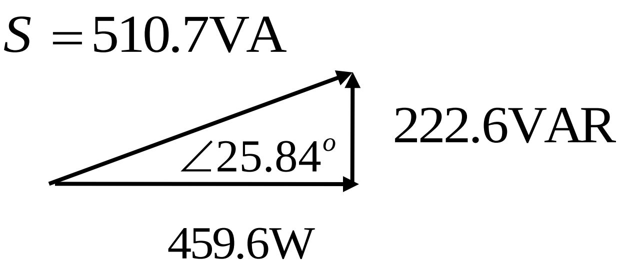
- The P.F. in this example is cos (40°). How can we make it unity?
- The power triangle is very useful in such an application.
- How can we make the P.F.= 0.9 lagging?

$$\theta = \cos^{-1}(0.9) = 25.84^o,\ S = \frac{459.6}{0.9} = 510.7\text{VA}$$

$$Q_{new} = 510.7\sin(25.84^o) = 222.6\text{VAR}$$

$$Q_{added} = Q_{new} - Q = -163.1\text{VAR}$$

$$\text{Check: } \cos(\tan^{-1}(Q_{new}/P)) = 0.899$$



LOAD

- The load can be represented using any of these combinations:

$$S\left(VA\right)\ and\ \ P.F.$$

$$S\left(VA\right)\ and\ \ P\left(W\right)$$

$$S\left(VA\right)\ and\ \ Q\left(VAR\right)$$

$$P\left(W\right)\ and\ \ Q\left(VAR\right)$$

$$\bar{V}\left(V\right)\ and\ \ \bar{I}\left(A\right)$$

$$\bar{V}\left(V\right)\ and\ \ \bar{Z}\left(\Omega\right)$$

$$\bar{I}\left(A\right)\ and\ \ \bar{Z}\left(\Omega\right)$$

LOAD

- Since there is need for a reference in phasor analysis, we take $\bar{V} = V\angle 0^\circ$. The power factor $PF = \cos\theta$ and $\theta = \pm\ \cos^{-1}$(PF)(+ for lag and - for lead).

The current is given by $\bar{I} = I\angle -\theta$.

If $\bar{V}$ and $\bar{I}$ are specified it is equivalent to specifying V , I , and the PF.

LOAD

Another way is to specify V, PF, and P. Then

$$\theta = \pm\cos^{-1}(PF)(+ \; for \; lag \; and \; - \; for \; lead)$$

$$P = VI\cos\theta$$

$$I = \frac{P}{V\cos\theta}$$

$$Q = VI\sin\theta$$

$$\overline{S} = P + jQ$$