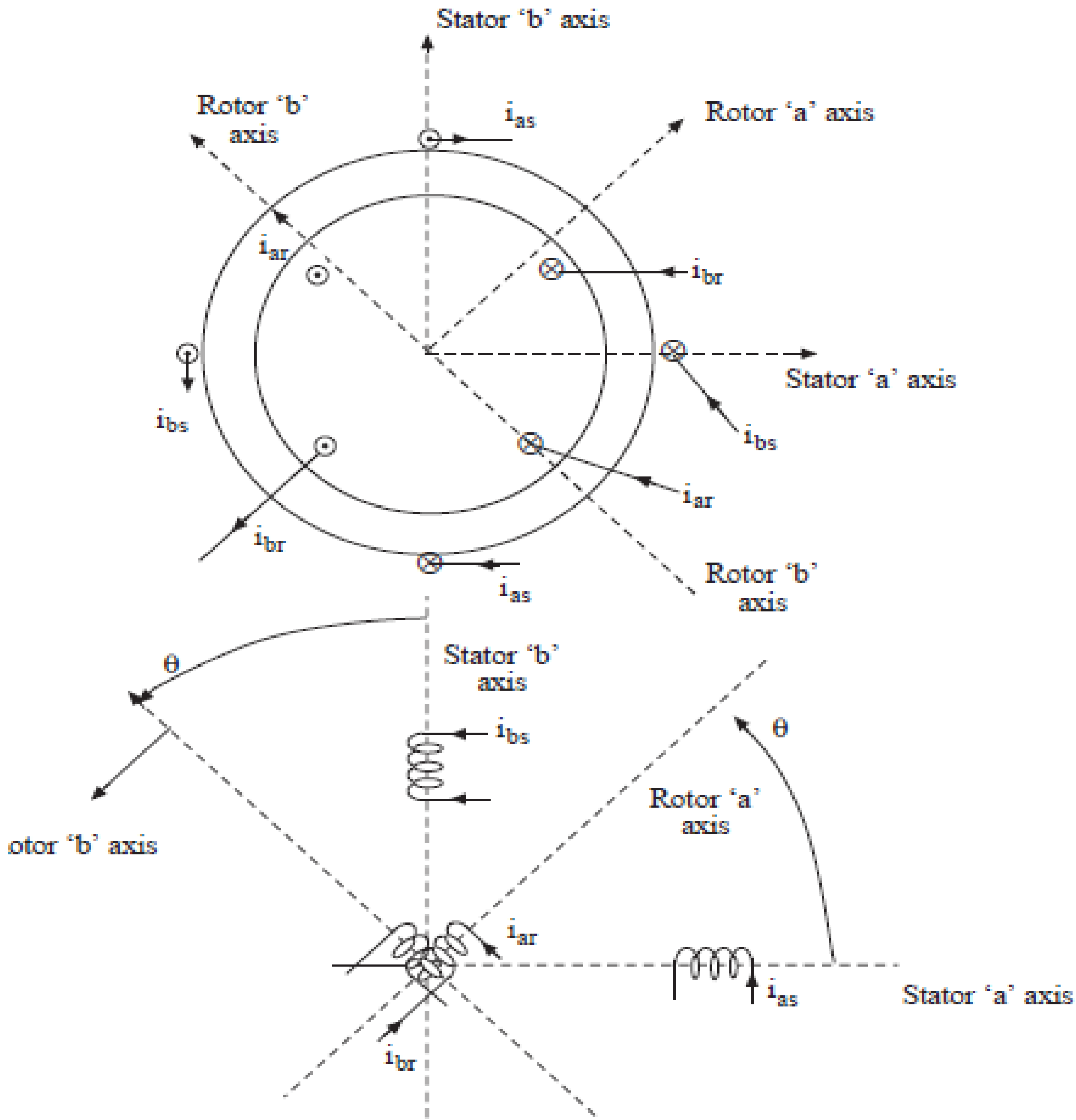
ECE 330
POWER CIRCUITS AND ELECTROMECHANICS

LECTURE 18
SYNCHRONOUS MACHINES (2)

Acknowledgment-These handouts and lecture notes given in class are based on material from Prof. Peter Sauer's ECE 330 lecture notes. Some slides are taken from Ali Bazi's presentations

TWO-PHASE ROTATING MACHINE



TWO-PHASE ROTATING MACHINE

The flux linkage equations can be written by considering each coil coupled with two other coils.

Windings on rotor and stator are identical but displaced mechanically by 90 deg. in positive direction.

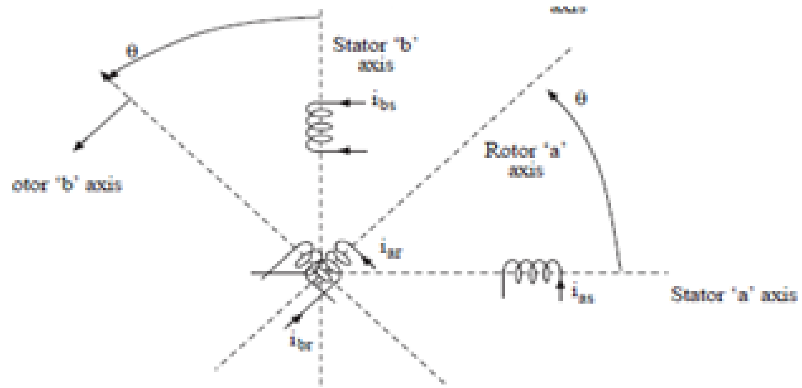
The two stator coils have no mutual coupling, similarly rotor coils do not have mutual coupling.

TWO-PHASE ROTATING MACHINE

The two-phase machine

flux linkages are:

For the stator:



$$\lambda_{as} = L_s i_{as} + M\ i_{ar} \cos(\theta) + M\ i_{br} \cos\left(\theta + \frac{\pi}{2}\right)$$

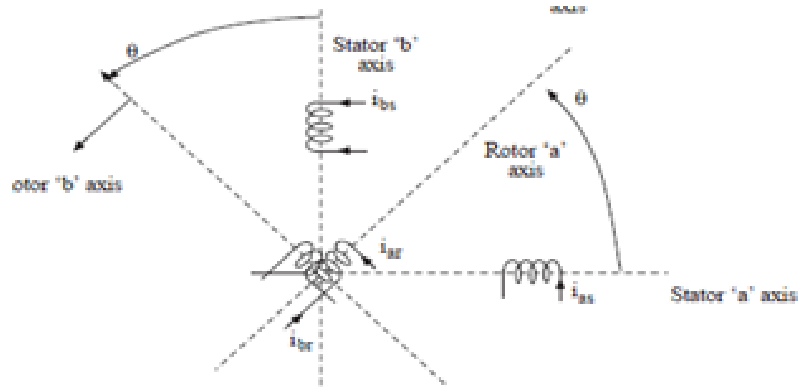
$$= L_s i_{as} + M\ i_{ar} \cos(\theta) - M\ i_{br} \sin(\theta)$$

$$\lambda_{bs} = L_s i_{bs} + M\ i_{ar} \cos\left(-(\frac{\pi}{2} - \theta)\right) + M\ i_{br} \cos(\theta)$$

$$= L_s i_{bs} + M\ i_{ar} \sin(\theta) + M\ i_{br} \cos(\theta)$$

TWO-PHASE ROTATING MACHINE

For the rotor



$$\lambda_{ar} = L_r i_{ar} + M\ i_{as} \cos(-\theta) + M\ i_{bs} \cos\left(\frac{\pi}{2} - \theta\right)$$

$$\lambda_{ar} = L_r i_{ar} + M\ i_{as} \cos(\theta) + M\ i_{bs} \sin(\theta)$$

$$\lambda_{br} = L_r i_{br} + M\ i_{as} \cos\left(-(\frac{\pi}{2} + \theta)\right) + M\ i_{bs} \cos(-\theta)$$

$$\lambda_{br} = L_r i_{br} - M\ i_{as} \sin(\theta) + M\ i_{bs} \cos(\theta)$$

TWO-PHASE ROTATING MACHINE

The flux linkages in matrix form:

$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{ar} \\ \lambda_{br} \end{bmatrix} = \begin{bmatrix} L_s & 0 & M\cos(\theta) & -M\sin(\theta) \\ 0 & L_s & M\sin(\theta) & M\cos(\theta) \\ M\cos(\theta) & M\sin(\theta) & L_r & 0 \\ -M\sin(\theta) & M\cos(\theta) & 0 & L_r \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{ar} \\ i_{br} \end{bmatrix}$$

Note: the inductance matrix is symmetric

TWO-PHASE ROTATING MACHINE

We can derive co-energy as:

$$W_m' = \frac{1}{2}L_s i_{as}^2 + \frac{1}{2}L_s i_{bs}^2 + \frac{1}{2}L_r i_{ar}^2 + \frac{1}{2}L_r i_{br}^2$$
$$+ M\cos(\theta) i_{as} i_{ar} + M\sin(\theta) i_{bs} i_{ar}$$
$$- M\sin(\theta) i_{as} i_{br} + M\cos(\theta) i_{bs} i_{br}$$

The torque can be expressed as:

$$T^e = \frac{\partial W_m'}{\partial \theta} = M\left[-\sin(\theta) i_{as} i_{ar} + \cos(\theta) i_{bs} i_{ar} - \cos(\theta) i_{as} i_{br} - \sin(\theta) i_{bs} i_{br}\right]$$

AVERAGE POWER CONDITIONS

The mechanical output power for sinusoidal excitation is then:

$$i_{as} = I_s \cos(\omega_s t), \quad i_{bs} = I_s \sin(\omega_s t)$$

$$i_{ar} = I_r \cos(\omega_r t), \quad i_{br} = I_r \sin(\omega_r t)$$

$$\theta = \omega_m t + \gamma$$

$$P_m = T^e \frac{d\theta}{dt} = T^e \omega_m = -\omega_m I_s I_r M \sin\left[(\omega_m - \omega_s + \omega_r)t + \gamma\right]$$

TWO-PHASE ROTATING MACHINE

There is average power transfer if the coefficient of t is zero, i.e., $\omega_m = \omega_s - \omega_r$. This is the frequency condition.

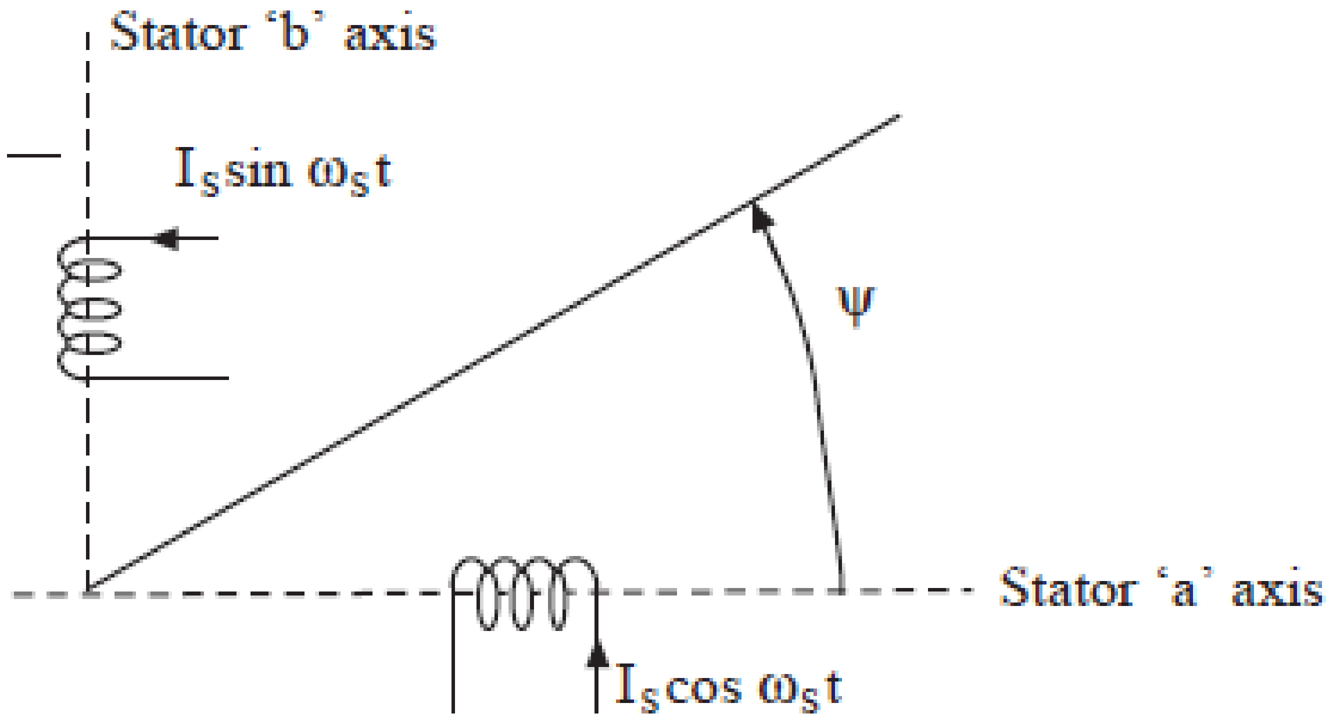
Notice that according to the power equation, there is no pulsating torque as in the single-phase machine!.

If we place the coils 90^o apart and the currents are 90^o out of phase then:

$$P_m = -\omega_m I_s I_r M \sin(\gamma)$$

CONCEPT OF ROTATING FIELD

Consider the two-phase machine with only the stator excited by current sources



CONCEPT OF ROTATING FIELD

The current in each winding produces a flux density that is maximum along its magnetic axis at some time instant and varies sinusoidally between the positive and negative directions of the axis.

CONCEPT OF ROTATING FIELD

(a) At $t = 0$, the flux is maximum along the positive stator a axis direction.

(b) At $t = \frac{\pi}{2\omega_s}$ seconds, it is maximum along the positive stator b axis direction.

(c) At $t = \frac{\pi}{\omega_s}$ seconds, it is maximum along the negative stator a axis direction.

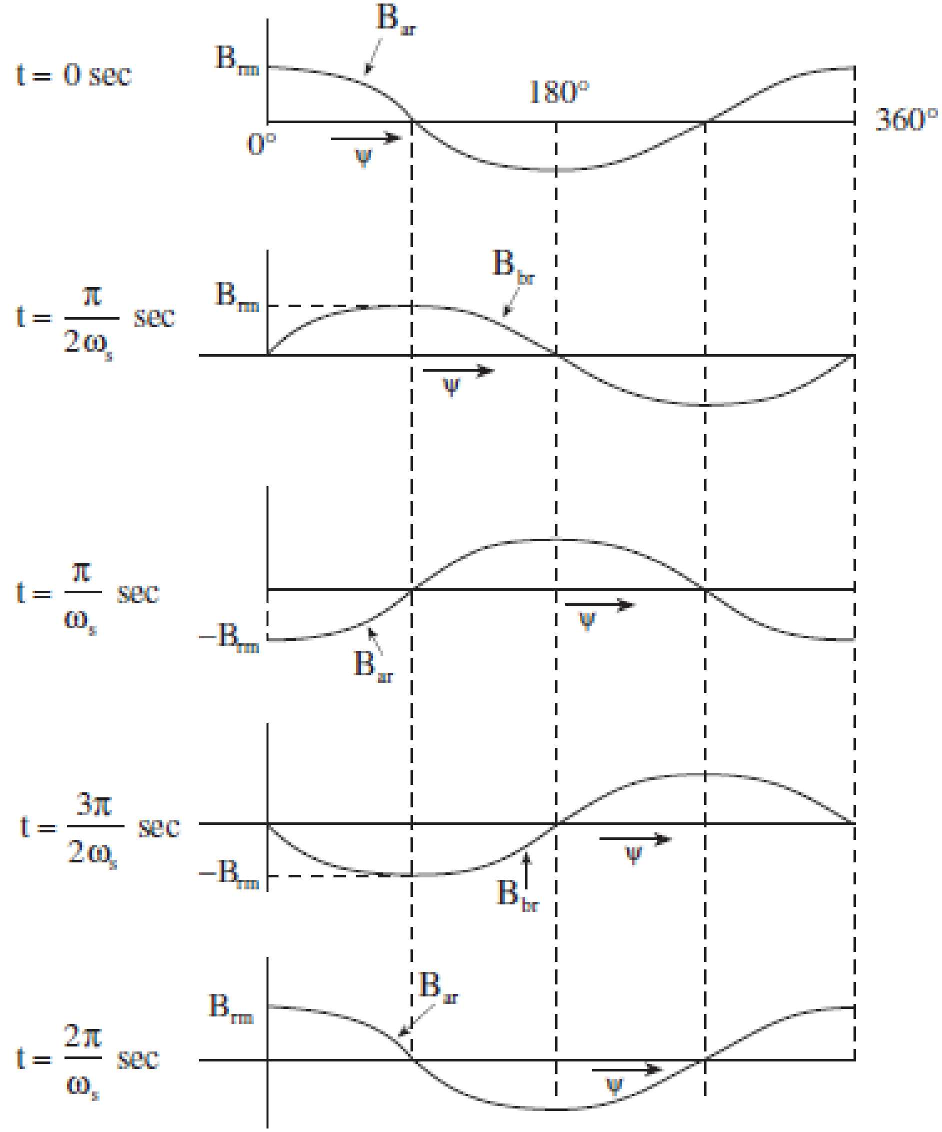
(d) At $t = \frac{3\pi}{2\omega_s}$ seconds, it is maximum along the negative b stator axis direction.

ECE ILLINOIS

ILLINOIS

CONCEPT OF ROTATING FIELD

Spatial distribution of flux B_r at different time instants



CONCEPT OF ROTATING FIELD

At an arbitrary time instant t, the magnetic field in the air gap at angle ψ is the sum of $B_{rm}\sin\omega_s t\sin\psi$ and $B_{rm}\cos\omega_s t\cos\psi$, i.e.,

$$B_r(t,\psi) = B_{rm}[\cos\omega_s t\cos\psi + \sin\omega_s t\sin\psi]$$

$$= B_{rm}(\cos(\omega_s t - \psi))$$

This is the magnetic field rotating at speed ω_s

CONCEPT OF ROTATING FIELD

- Rotor currents excited with frequency ω_r also create a rotating field similarly. This rotating field rotates at an angular velocity of ω_r .
- The two fields are fixed in space relative to each other.
- The rotor is running at a constant angular velocity. To do
- this a constant mechanical speed given by $\omega_m = \omega_s - \omega_r$
- is required.

PER-PHASE EQUIVALENT CIRCUIT

- Now we look at voltages with $\theta = \omega_m t + \gamma$

$$v_{as} = i_{as} R_s + \frac{d\lambda_{as}}{dt}$$

$$\lambda_s = L_{as} i_{as} + M \cos\theta\, i_{ar} - M \sin\theta\, i_{br}$$

$$v_{as} = i_{as} R_s + \frac{d}{dt}\left[L_{as} i_{as} + M \cos(\omega_m t + \gamma) i_{ar} - M \sin(\omega_m t + \gamma) i_{br}\right]$$

$$i_{as} = I_s \cos\omega_s t$$

$$i_{ar} = I_r \cos\omega_r t$$

$$i_{br} = I_r \cos\omega_r t$$

ECE ILLINOIS

ILLINOIS

PER-PHASE EQUIVALENT CIRCUIT

$$v_{as} = R_s I_s \cos\omega_s t - \omega_s L_s I_s \sin\omega_s t$$

$$+\frac{d}{dt}\left[M \cos\left(\omega_m t + \gamma\right) I_r \cos\omega_r t\right) - M \sin(\omega_m t + \gamma) I_r \sin\omega_r t)\right]$$

$$v_{as} = R_s I_s \cos\omega_s t - \omega_s L_s I_s \sin\omega_s t + \frac{d}{dt}\left[M I_r \cos\left((\omega_m + \omega_r)t + \gamma\right)\right]$$

$$= R_s I_s \cos\omega_s t - \omega_s L_s I_s \sin\omega_s t - (\omega_m + \omega_r) M I_r \sin((\omega_m + \omega_r)t + \gamma)$$

Require $\omega_s = \omega_m + \omega_r$

$$v_{as} = R_s I_s \cos\omega_s t - \omega_s L_s I_s \sin\omega_s t - \omega_s M I_r \sin(\omega_s t + \gamma)$$

$$V_s \cos(\omega_s t + \alpha) = R_s I_s \cos\omega_s t - \omega_s L_s I_s \sin\omega_s t - \omega_s M I_r \sin(\omega_s t + \gamma)$$

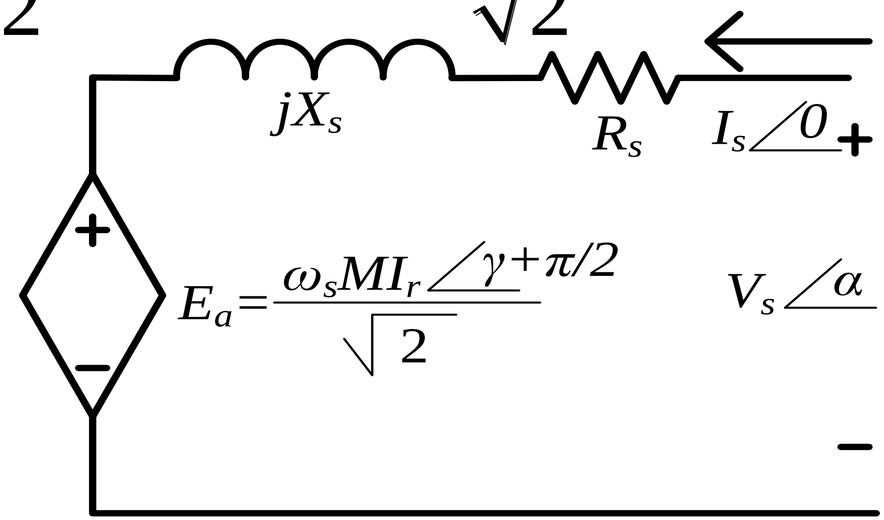
PER-PHASE EQUIVALENT CIRCUIT

or, $$\frac{V_s}{\sqrt{2}}\angle\alpha = R_s I_s \angle 0 + j\,\omega_s L_s \frac{I_s}{\sqrt{2}}\angle 0 + j\,\omega_s M \frac{I_r}{\sqrt{2}}\angle\gamma$$

If I_r= dc, $\omega_r = 0$, and $\omega_s = \omega_m$, then we have a

synchronous machine.

$$\frac{V_s}{\sqrt{2}}\angle\alpha = R_s I_s \angle 0 + j\omega_s L_s \frac{I_s}{\sqrt{2}}\angle 0 + j\omega_s M \frac{I_r}{\sqrt{2}}\angle\gamma$$



Note: α is the power factor angle