ECE 330
POWER CIRCUITS AND ELECTROMECHANICS

LECTURE 17
SYNCHRONOUS MACHINES (1)

Acknowledgment-These handouts and lecture notes given in class are based on material from Prof. Peter Sauer's ECE 330 lecture notes. Some slides are taken from Ali Bazi's presentations

SYNCHRONOUS MACHINES

- The main element in terms of generation of power.
- Range all the way from a few MVA to 1100 MVA.
- Can be operated as either a generator or motor.
- The large majority of applications are as generators.
- The three-phase generators have an AC winding on the stator with a wye-connection.
- The rotor is excited by a DC field winding

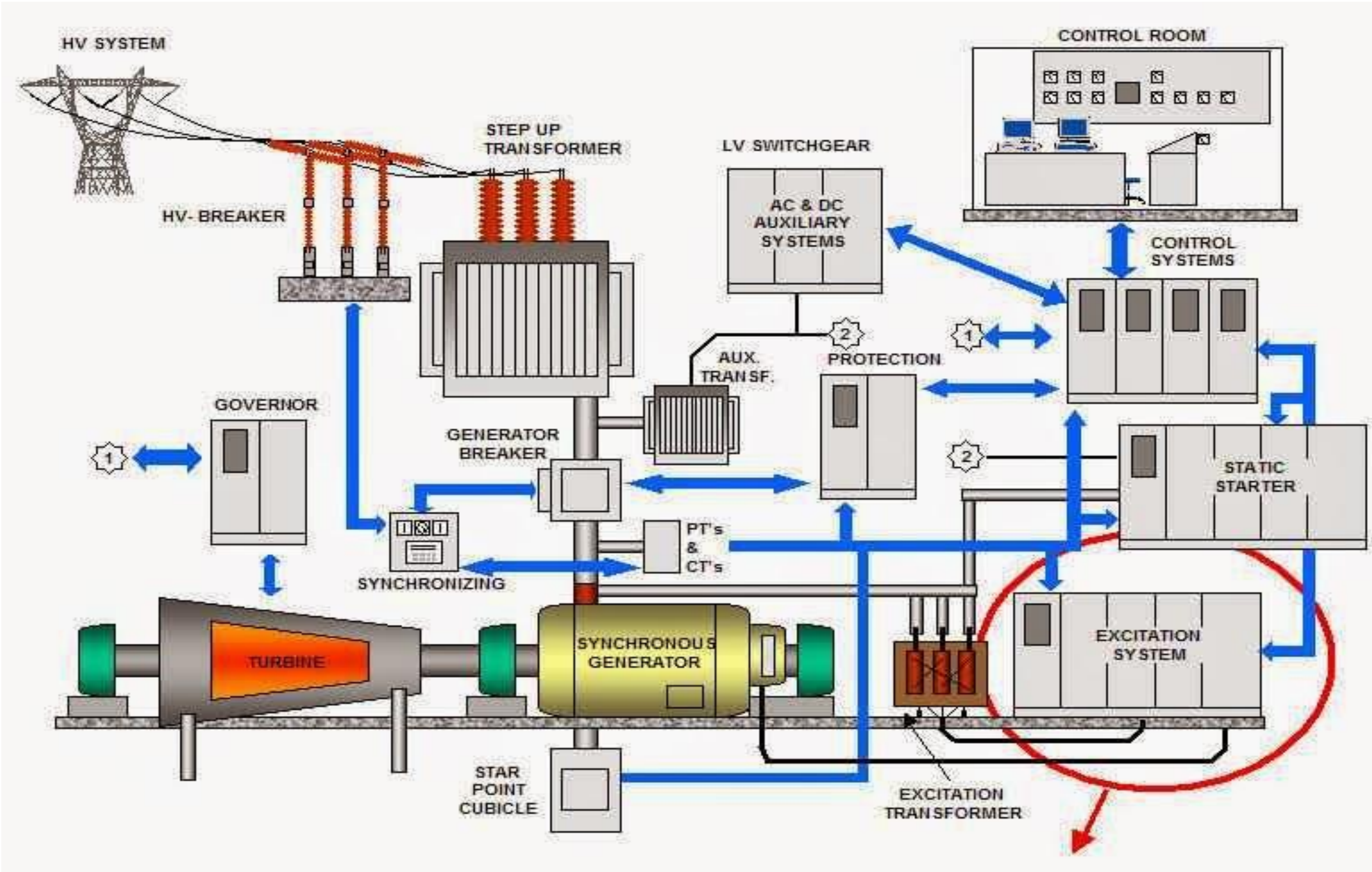
SYNCHRONOUS MACHINES

- It is connected to the prime movers, such as steam or hydro-turbine.
- As motors, synchronous machines are less used except at low power levels such as *permanent magnet synchronous motors* (PMSM).
- In certain cases, synchronous machines at a high rating are operated to act as power factor correcting devices.

SYNCHRONOUS MACHINES

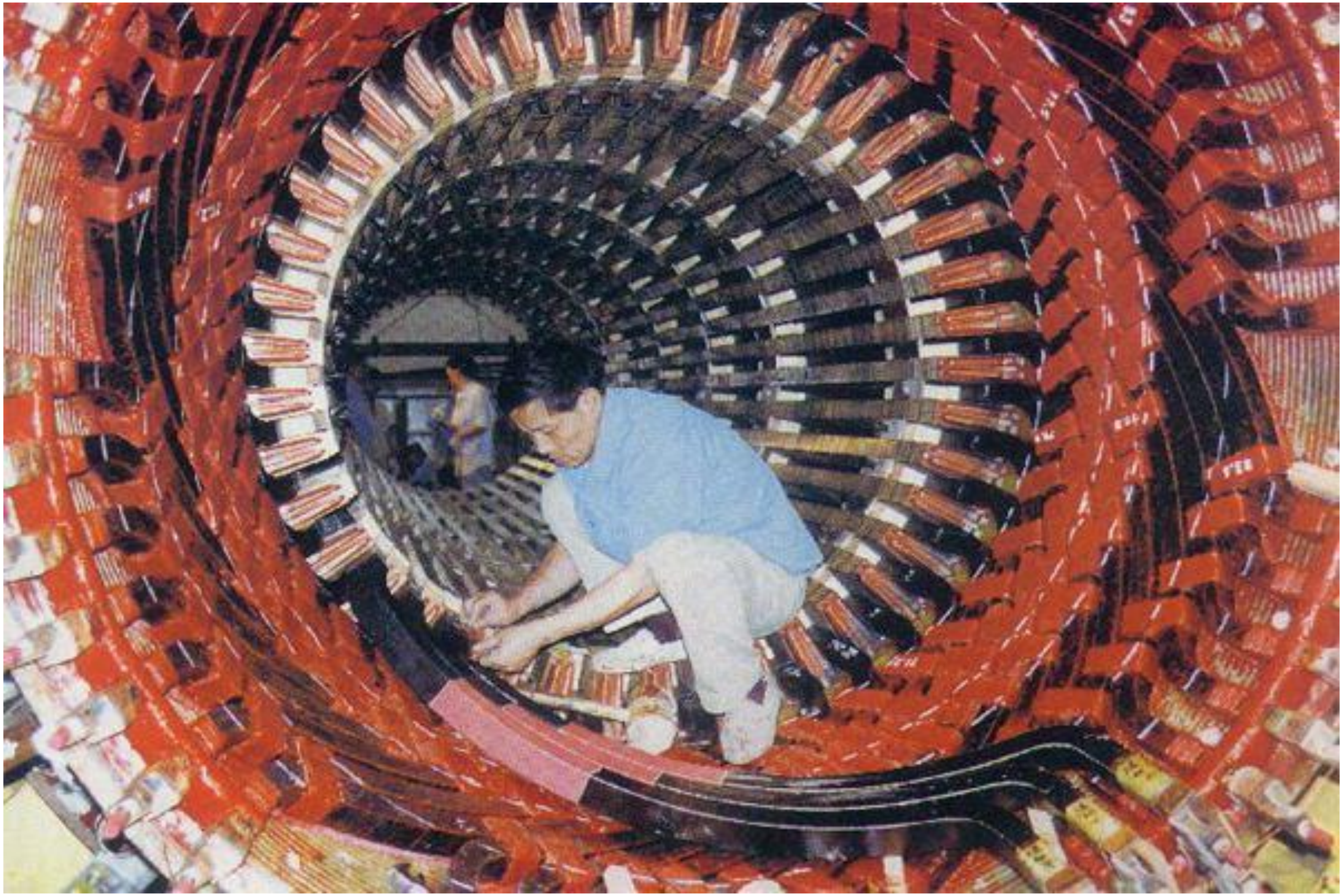
- We will discuss the fundamental concepts of deriving torque expressions and only the sinusoidal steady-state operation using the equivalent circuit.
- For a proper understanding of a three-phase machine, we will motivate it via the single- and two-phase machines.

SYNCHRONOUS MACHINES



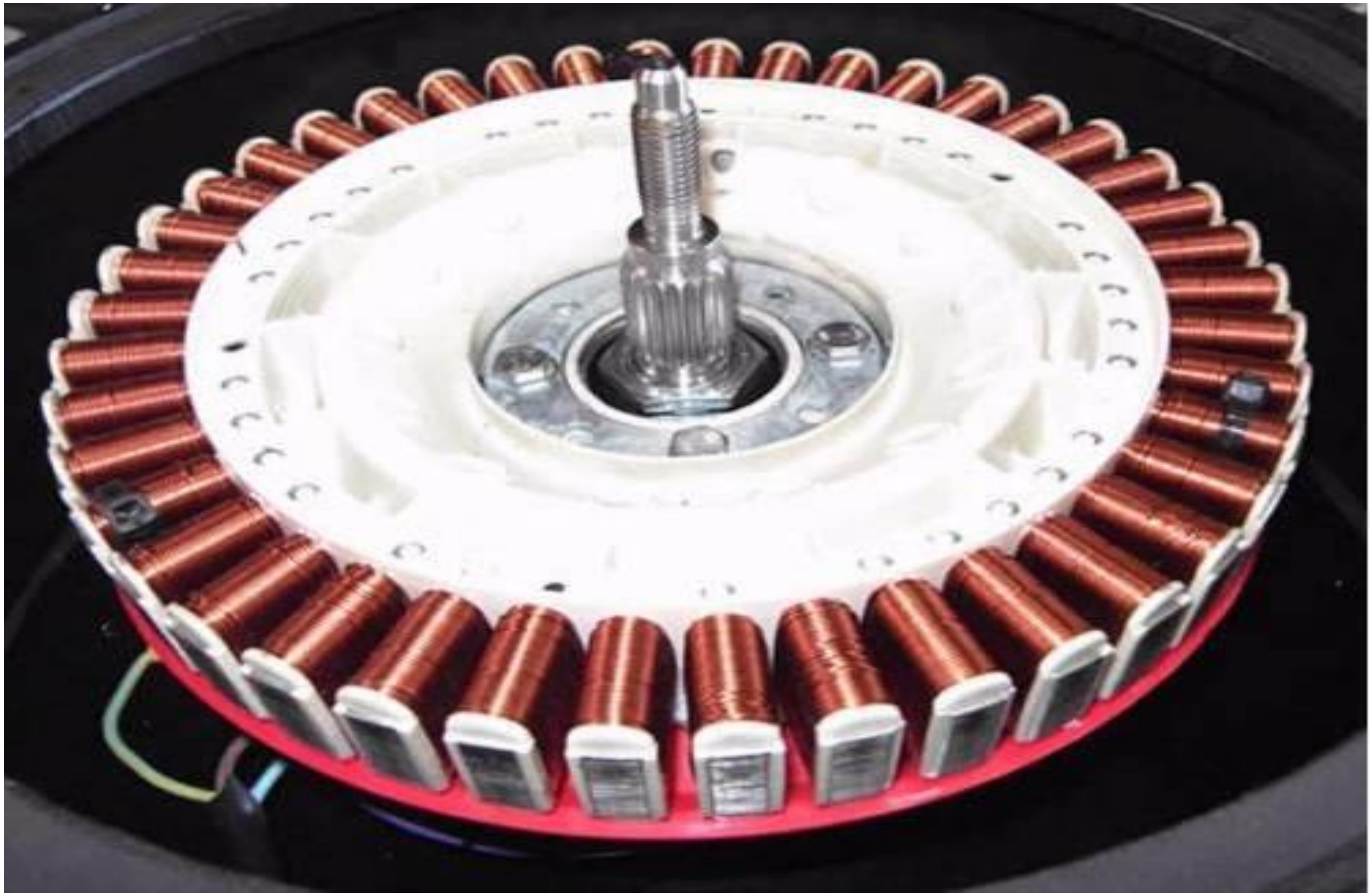
Source: emadrlc.blogspot.com

SYNCHRONOUS MACHINES



Source: pelectric.blogsky.com

SYNCHRONOUS MACHINES
SALIENT POLE



SYNCHRONOUS MACHINES
ROUND ROTOR

- Source: en.partzsch.de

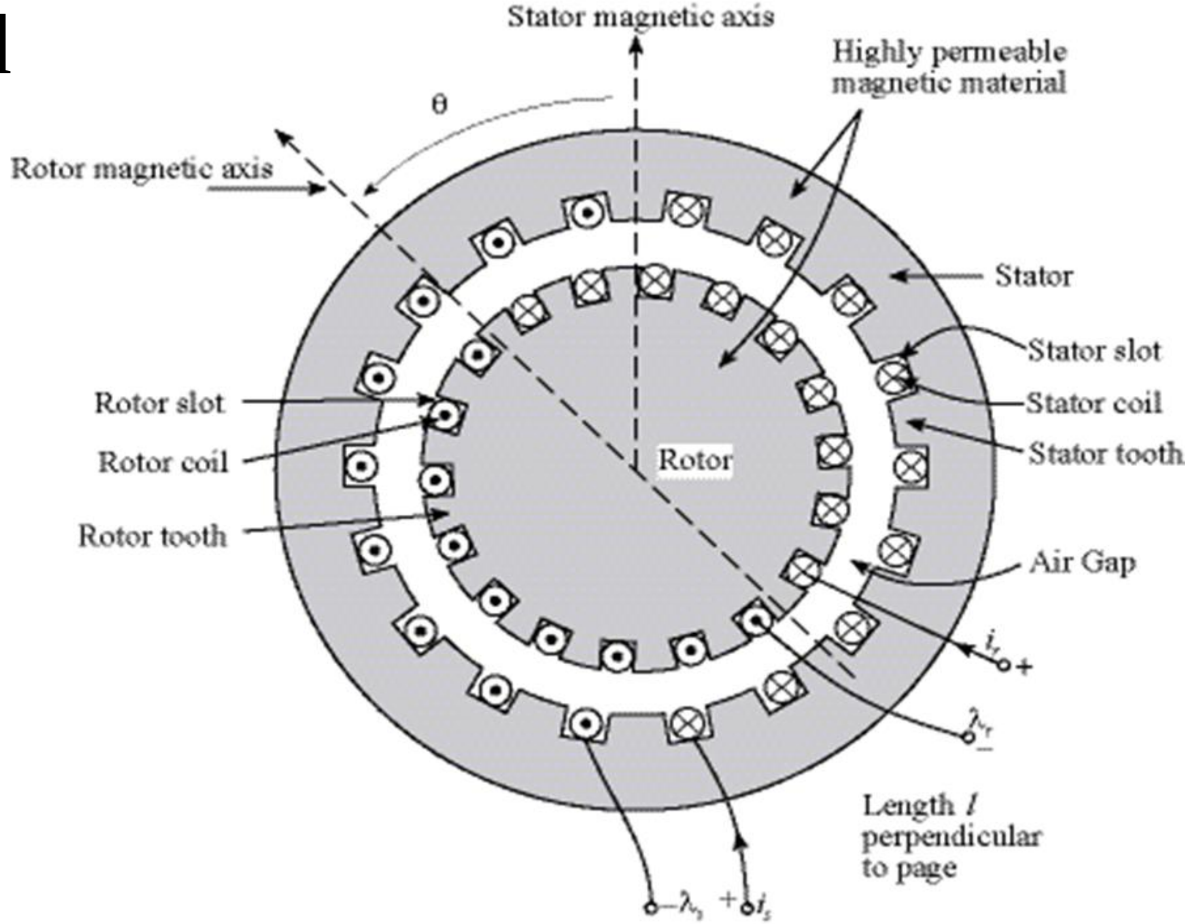
SINGLE-PHASE ROTATING MACHINE

The fundamental component of mutual inductance $N_s N_r L_0 \left(1-2\theta/\pi\right)$ will be $M\cos\theta$.

In practical machines, the number of turns are so positioned on stator and rotor that the higher harmonics are minimized and the mutual inductance is largely due to this fundamental component.

SINGLE-PHASE ROTATING MACHINE

The winding, instead of being concentrated, is distributed



SINGLE-PHASE ROTATING MACHINE

- We have already seen flux linkage derivation of single-phase machines:

$$\lambda_s = N_s^2 L_0 i_s + N_s N_r L_0 \left(1 - \frac{2\theta}{\pi}\right) i_r = L_s i_s + L_{sr}(\theta) i_r$$

$$\lambda_r = N_r^2 L_0 i_r + N_s N_r L_0 \left(1 - \frac{2\theta}{\pi}\right) i_s = L_r i_r + L_{rs}(\theta) i_s$$

$$L_{sr}(\theta) = L_{rs}(\theta) = M \cos(\theta)$$

SINGLE-PHASE ROTATING MACHINE

- The co-energy and torque are:

$$W_m{}' = \int_0^{i_s} \lambda_s(i_s', 0, \theta)\, di_s' + \int_0^{i_r} \lambda_r(i_s, i_r', \theta)\, di_r'$$

$$= \frac{1}{2} L_s i_s^2 + \frac{1}{2} L_r i_r^2 + L_{sr}(\theta) i_s i_r$$

$$T^e = \frac{\partial W_m{}'}{\partial \theta} = \frac{\partial L_{sr}(\theta)}{\partial \theta} i_s i_r = -i_s i_r M \sin(\theta)$$

SINGLE-PHASE ROTATING MACHINE

- The electrical differential equations are:

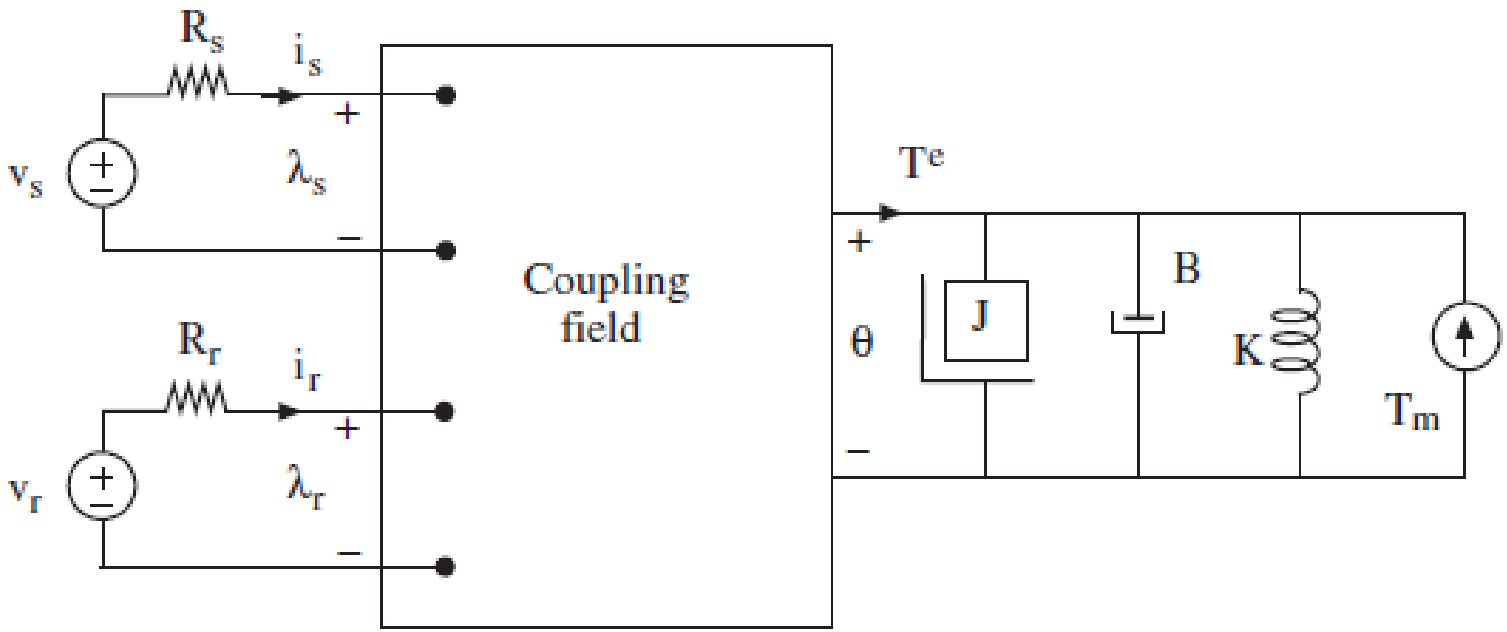
$$v_s = i_s R_s + \frac{d\lambda_s}{dt}$$

$$v_r = i_r R_r + \frac{d\lambda_r}{dt}$$

SINGLE-PHASE ROTATING MACHINE

- The mechanical differential equation is:

$$J\frac{d^2\theta}{dt^2}+K\theta+B\frac{d\theta}{dt}=T^e+T_m$$



SINGLE-PHASE ROTATING MACHINE

Under sinusoidal excitation, the power becomes:

$$i_s = I_s \cos(\omega_s t)$$

$$i_r = I_r \cos(\omega_s t)$$

$$P_m = T^e \frac{d\theta}{dt} = T^e \omega_m = -\omega_m I_s I_r M \cos(\omega_s t)\cos(\omega_r t)\sin(\theta)$$

$$\theta = \omega_m t + \gamma \qquad (\gamma \ is \ some \ arbitary \ const.)$$

$$\Rightarrow P_m = -\omega_m I_s I_r M \cos(\omega_s t)\cos(\omega_r t)\sin(\omega_m t + \gamma)$$

SINGLE-PHASE ROTATING MACHINE

- Power can also be expressed as:

$$P_m = -\omega_m I_s I_r M \begin{bmatrix} \sin(\omega_1 t + \gamma) + \sin(\omega_2 t + \gamma) \\ + \sin(\omega_3 t + \gamma) + \sin(\omega_4 t + \gamma) \end{bmatrix} \Big/ 4$$

where:

$$\omega_1 = \omega_m + \omega_s - \omega_r, \omega_2 = \omega_m - \omega_s + \omega_r$$

$$\omega_3 = \omega_m + \omega_s + \omega_r, \omega_4 = \omega_m - \omega_s - \omega_r$$

SINGLE-PHASE ROTATING MACHINE

Since a sinusoidal function can have no average value

P_m can have an average value only if $\omega_i = 0$ for

$i = 1, 2, 3$, or 4, i.e., $\omega_m = \pm\omega_s \pm \omega_r$

- If $\omega_2 = 0$, $\omega_m = \omega_s - \omega_r$

$$< P_{m(av)} >= \frac{-\omega_m I_s I_r M \sin(\gamma)}{4}$$

A necessary condition for average power is that one of the ω_i 's is zero, and a sufficient condition is that $\sin\gamma \neq 0$

TWO-PHASE ROTATING MACHINE

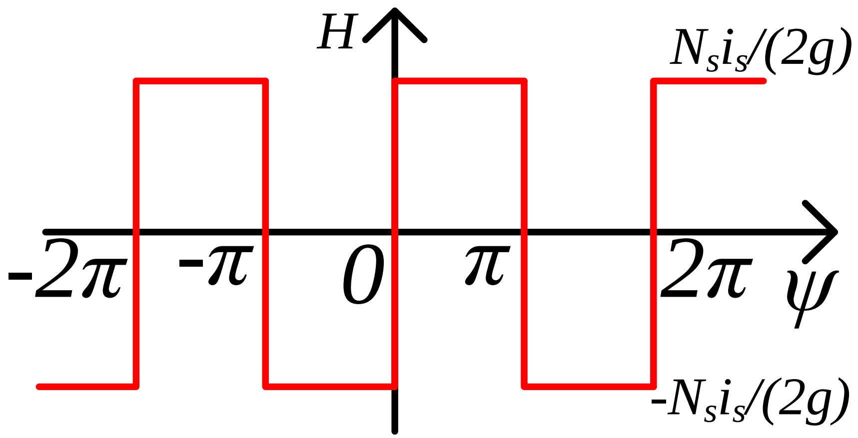
There is average power when there is pulsating torque due to other ω_i' s.

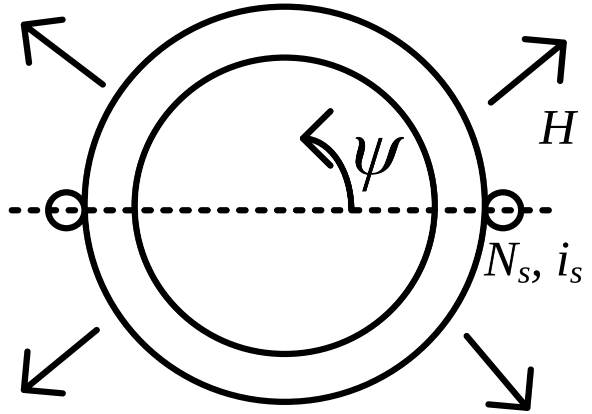
To eliminate this, we can have a two-phase machine.

In the two-phase machine, there is an additional winding on both the stator and the rotor. The two-phase machine creates a rotating magnetic field.

TWO-PHASE ROTATING MACHINE

- We look at the stator magnetic field of one- and two-phase machines.



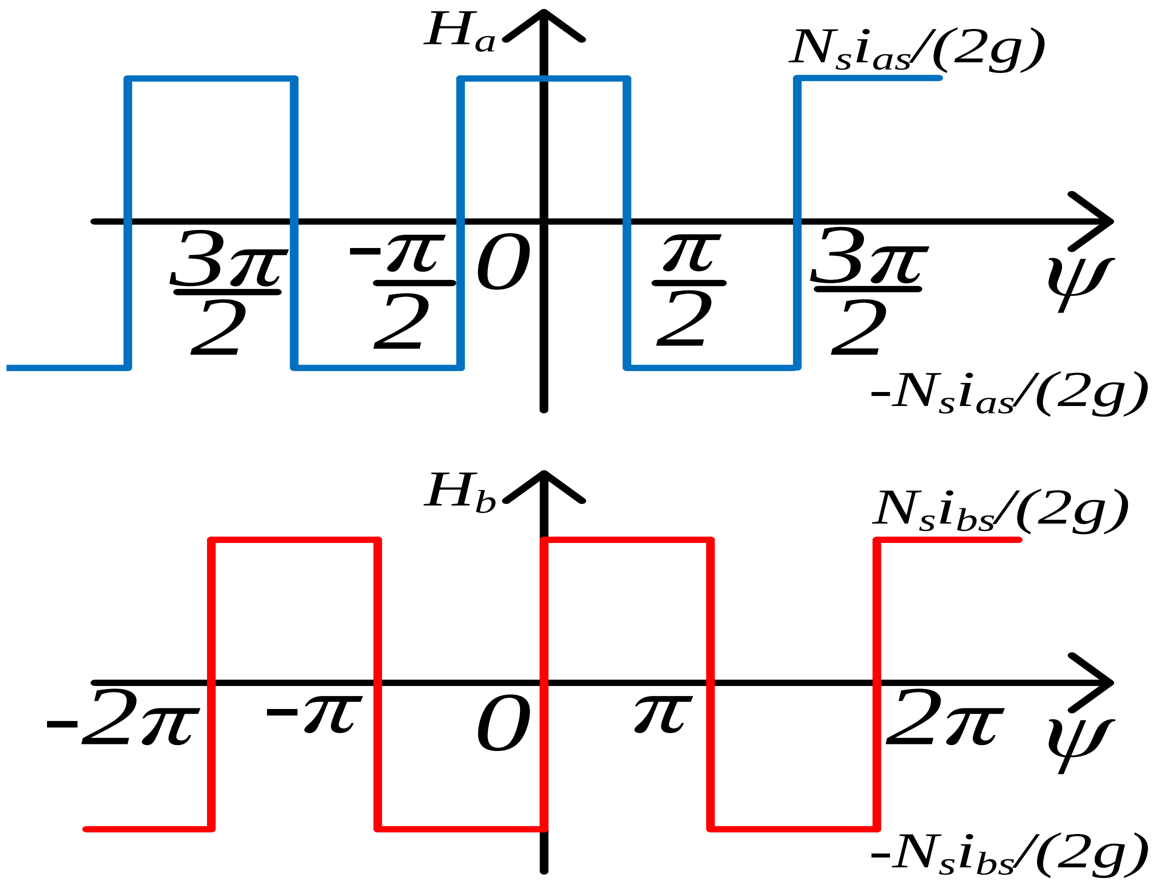


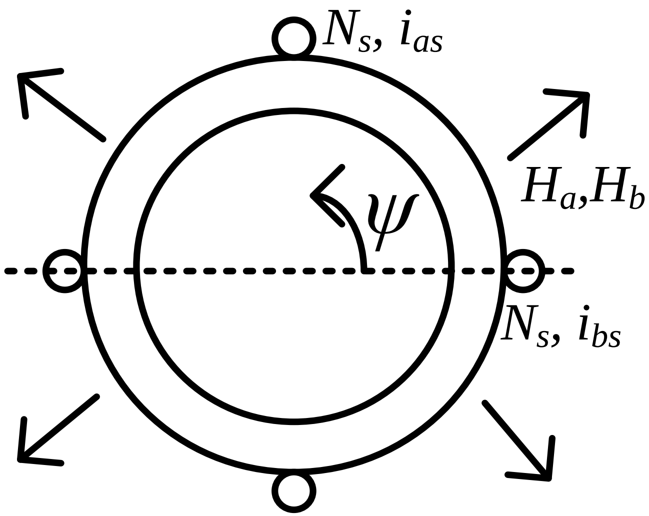
$$H \approx \frac{N_s i_s}{2g}\sin(\psi)$$

- With sinusoidal excitation, no rotating field as it is
- always maximum at $\psi = 90^o$: $H = \frac{N_s I_s}{2g}\sin(\psi)\cos(\omega_s t)$

TWO-PHASE ROTATING MACHINE

- Two-phase machine: Rotating magnetic field!





TWO-PHASE ROTATING MACHINE

$$H_s = H_{as} + H_{bs}$$

$$H_s = \frac{N_s i_{as}}{2g}\cos(\psi) + \frac{N_s i_{bs}}{2g}\sin(\psi)$$

$$Assume: i_{as} = I_s \cos\omega_s t, \; i_{bs} = I_s \sin\omega_s t$$

$$H = \frac{N_s I_s}{2g}\left[\cos(\psi)\cos(\omega_s t) + \sin(\psi)\sin(\omega_s t)\right]$$

TWO-PHASE ROTATING MACHINE

$$H = \frac{N_s I_s}{2g} \cos(\omega_s t - \psi)$$

Revolving magnetic field

at $t = 0, \;\; peak \;\; is \;\; at \;\; \psi = 0$

$t = t, \;\; peak \;\; is \;\; at \;\; \psi = \omega_s t$

Revolves *continue clockwise*