ECE 330
POWER CIRCUITS AND ELECTROMECHANICS

LECTURE 18

FORCES OF ELECTRIC ORIGIN – ENERGY APPROACH(2)

Acknowledgment-These handouts and lecture notes given in class are based on material from Prof. Peter Sauer's ECE 330 lecture notes. Some slides are taken from Ali Bazi's presentations

Disclaimer- These handouts only provide highlights and should not be used to replace the course textbook.

ENERGY AND CO-ENERGY

the force of electric origin f^e derived from either energy or co-energy expressions is identical. Depending on the system description, one or the other may be used

In electrically linear system the energy and co-energy are numerically equal.

ENERGY AND CO-ENERGY

Suppose $\lambda(i,x) = L(x)\, i$ which is electrically linear, then

$$i = \frac{\lambda(i,x)}{L(x)}$$

$$W_m(\lambda,x) = \int_0^{\lambda} i(\hat{\lambda},x)\Big|_{x=const.} d\hat{\lambda}$$

$$W_m'(i,x) = \int_0^{i} \lambda(\hat{i},x)\Big|_{x=const.} d\hat{i}$$

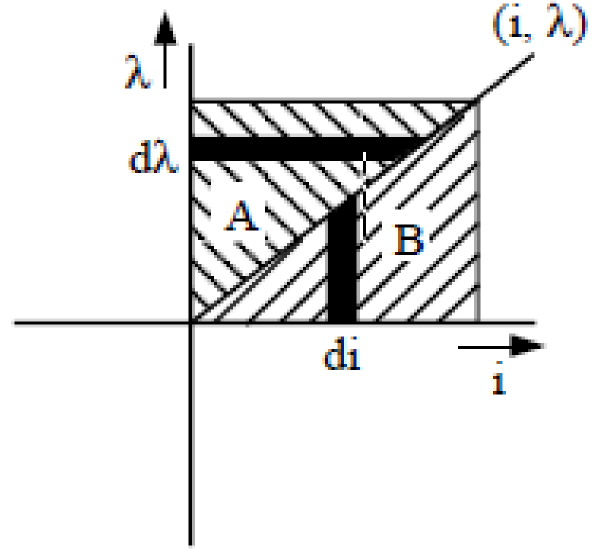
ENERGY AND CO-ENERGY

Since $\lambda(i,x) = L(x)i$, W_m and W'_m are equivalent

For a given value of x, the relationship between λ and i is linear

$$Area\ A = \int_o^\lambda i(\lambda, x)d\lambda \text{ is the ``energy''}$$

$$Area\ B = \int_o^i \lambda(i, x)di \text{ is the ``co-energy''}$$



for linear system $W_m = W'_m$

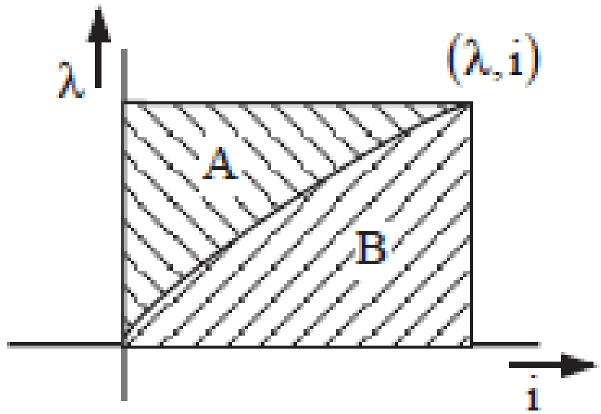
And note

$$\lambda i = W_m + W'_m$$

ENERGY AND CO-ENERGY

If $\lambda(i, x)$ is a nonlinear function as shown, then area

A $\neq$ B and $W_m \neq W_m'$ but still $\lambda i = W_m + W_m'$



MULTI-PORT SYSTEMS

For the case of multi-port system, we have the flux linkages as

$$\lambda_1 = L_{11}(x)i_1 + L_{12}(x)i_2$$
$$\lambda_2 = L_{21}(x)i_1 + L_{22}(x)i_2$$

Path from $0 \rightarrow i_1$, $0 \rightarrow i_2$, $x_0 \rightarrow x$

Must be path independent $(L_{12} = L_{21}$ *for linear system*$)$

Choose path x first while i_1 and i_2 are zero

i_1 next while $x =$ constant , $i_2 = 0$

i_2 next while $x =$ constant , $i_1 = i_1$

MULTI-PORT SYSTEMS

Computation of W'_m

$$W'_m(i_1, i_2, x) = \int_0^{i_1} \lambda_1 \Big|_{\substack{i_2=0 \\ x=const.}} di_1 + \int_0^{i_2} \lambda_2 \Big|_{\substack{i_1=i_1 \\ x=const.}} di_2 + \int_0^{x} f^e \Big|_{\substack{i_1=0 \\ i_2=0}} dx$$

$$W'_m(i_1, i_2, x) = L_{11}(x) i_1^2 + L_{21}(x) i_1 i_2 + L_{22}(x) i_2^2$$

$$f^e(i_1, i_2, x) = \frac{\partial W'_m(i_1, i_2, x)}{\partial x}$$

EXAMPLE

A certain multiple-port rotational system has the following flux-current relations, where θ is a rotational variable

$$\lambda_s = L_s i_s + M \cos\theta \, i_r$$
$$\lambda_r = M \cos\theta i_s + L_r i_r$$

Compute the co-energy W'_m and the torque of electrical origin

$$W'_m(i_1, \mathrm{i}_2, x) = \frac{1}{2} L_s i_s^2 + M \cos\theta i_r + \frac{1}{2} L_r i_r^2$$

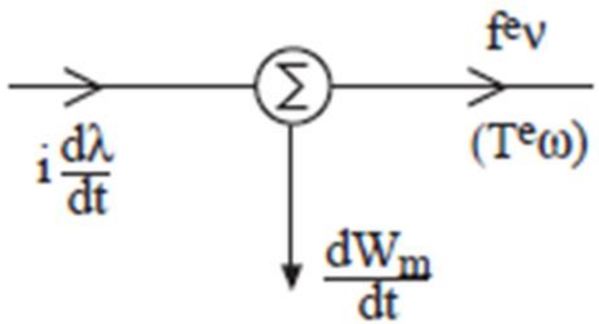
$$T^e(i_s, i_r, \theta) = \frac{\partial W'_m(i_s, i_r, \theta)}{\partial \theta} = -M \cos\theta i_s i_r$$

ENERGY CONVERSION BETWEEN TWO POINTS

In the $\lambda - i$ plane, to go from a to b

$$W_m(\lambda_b, x_b) - W_m(\lambda_a, x_a) = \int_{\lambda_a}^{\lambda_b} i\,d\lambda - \int_{x_a}^{x_b} f^e dx$$

$$= \int_{\lambda_a}^{\lambda_b} i\,d\lambda + \left[-\int_{x_a}^{x_b} f^e dx \right]$$



$$\Delta W_{m_{a\to b}} = EFE|_{a\to b} + EFM|_{a\to b}$$

Where EFE stands for “energy from electrical” and EFM stands for “energy from mechanical.” To evaluate EFE and EFM, we need to specify a particular path.

ENERGY CONVERSION OVER A CYCLE

Energy balance $$dW_m = id\lambda - f^e dx$$

Over complete cycle

$$dW_m = 0 \ (system \ \ returns \ \ to \ \ original \ \ state)$$

$$0 = \oint i \, d\lambda - \oint f^e dx$$

$$= \oint i \, d\lambda + (-\oint f^e dx)$$

$$id\lambda = Energy \ \ from \ electrical \ \ (EFE)$$

$$-f^e dx \ = Energy \ \ from \ \ mechanical \ \ (EFM)$$

ENERGY CONVERSION OVER A CYCLE

Over complete cycle

$$\oint EFE + \oint EFM = 0 \quad \text{or} \quad EFE|_{cycle} + EFM|_{cycle} = 0$$

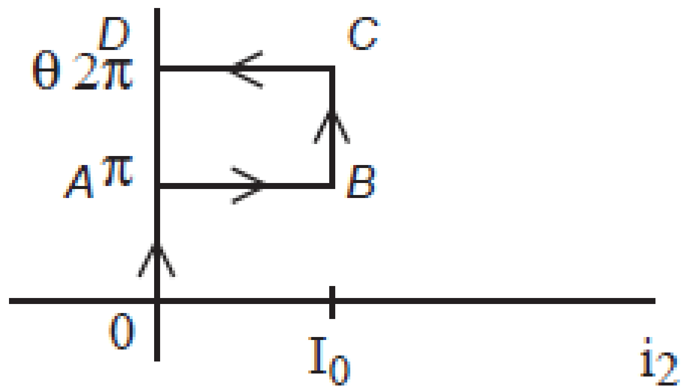
Because there is no net change in stored energy, one can compute either $EFE|_{cycle}$ or $EFM|_{cycle}$. If $EFE|_{cycle} > 0$, then the system is operating as a *motor* and $EFM|_{cycle} < 0$. If $EFM|_{cycle} > 0$, then the system is operating as a *generator* and $EFE|_{cycle} < 0$.

EXAMPLE

An electric machine has the relation as shown in the figure below. The relations are

$$\lambda_1 = L_{11} i_1 + M \cos\theta \, i_2$$

$$\lambda_2 = M \cos\theta \, i_1 + L_{22} i_2$$



The machine is operated over the cycle ABCD while $i_1 = I_0$. Find the energy converted from electrical to mechanical form for each cycle. Is this a motor or a generator?

EXAMPLE

Torque of electric origin $W_m' = \frac{1}{2}L_{11}i_1^2 + M\cos\theta\, i_1 i_2 + \frac{1}{2}L_{22}i_2^2$

$$T^e = \frac{\partial W_m'}{\partial\theta} = -Mi_1 i_2 \sin\theta$$

Since it is the $\theta - i$ plane, we compute $\int_{\theta_1}^{\theta_2} T^e d\theta$

$$EFM|_{cycle} = -\int_0^{2\pi} T^e d\theta = -\left[\int_0^{\pi} T^e d\theta + \int_{\pi}^{2\pi} T^e d\theta\right]$$

$$= \int_0^A -T^e d\theta + \int_A^B -T^e d\theta + \int_B^C -T^e d\theta + \int_C^D -T^e d\theta$$

$$EFM_{cycle} = \int_B^C -T^e d\theta = \int_{\pi}^{2\pi} MI_0^2 \sin\theta d\theta = -2M I_0^2$$

since $EFM|_{cycle} < 0$, it is motor