ECE 330
POWER CIRCUITS AND ELECTROMECHANICS

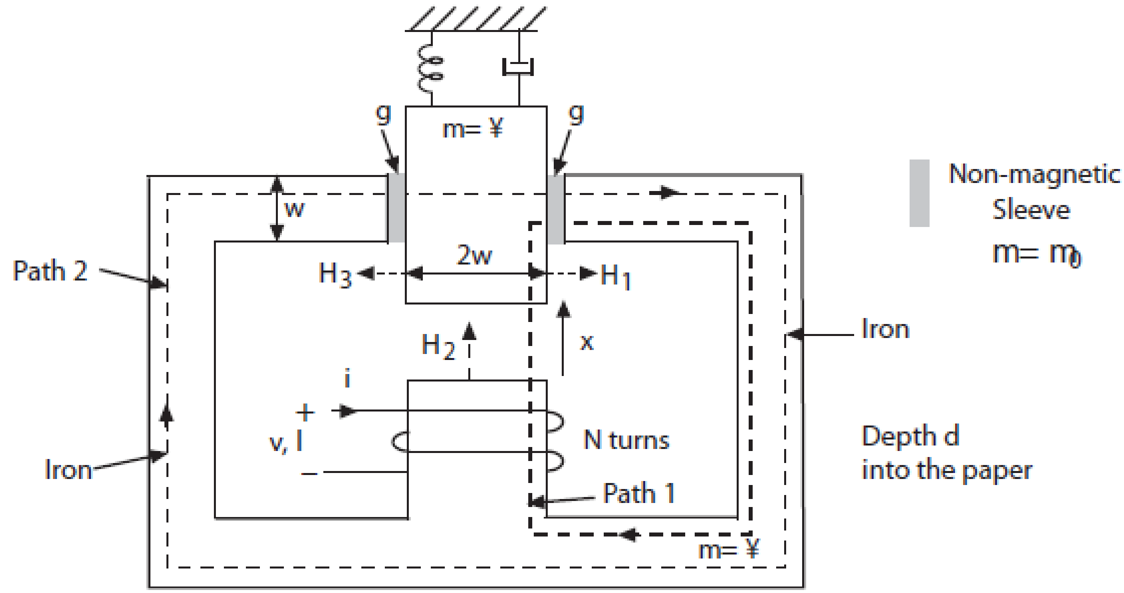
LECTURE 17

FORCES OF ELECTRIC ORIGIN – ENERGY APPROACH(1)

Acknowledgment-These handouts and lecture notes given in class are based on material from Prof. Peter Sauer's ECE 330 lecture notes. Some slides are taken from Ali Bazi's presentations

MECHANICAL EQUATIONS

- Newton's law states that *acceleration force in the positive x direction is equal to the algebraic sum of all the forces acting on the mass in the positive x direction*



MECHANICAL EQUATIONS

- In a linear or translational system with displacement x, velocity υ, mass M, and different forces (along x):

$$\frac{dx}{dt} = \upsilon$$

$$M\frac{d\upsilon}{dt} = f^{e} + f_{spring} + f_{damper} + f_{ext}$$

- In a rotational system with displacement θ,rotational speed ω, moment of inertia J, and different torques:

$$\frac{d\theta}{dt} = \omega, \qquad J\frac{d\omega}{dt} = T^{e} + T_{spring} + T_{damper} + T_{ext}$$

MECHANICAL EQUATIONS

- Forces and torques with superscript *e* are of electrical origin.

- In a translational system, power is

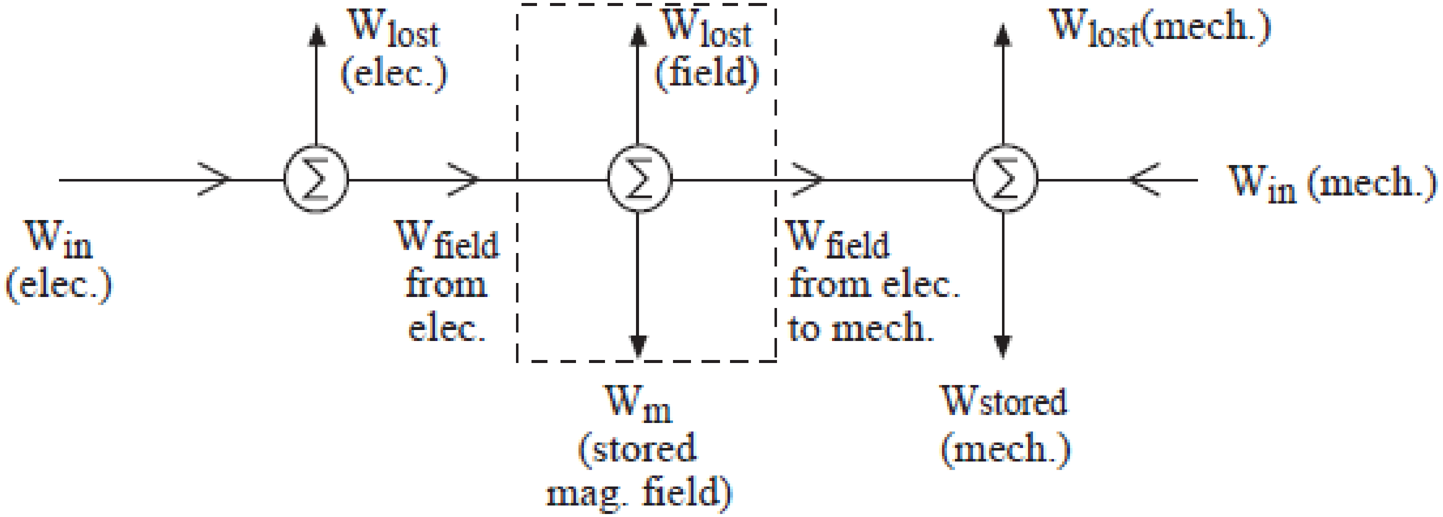
$$P = f \upsilon = f \frac{dx}{dt}$$

- In a rotational system, power is

$$P = T \omega = T \frac{d\theta}{dt}$$

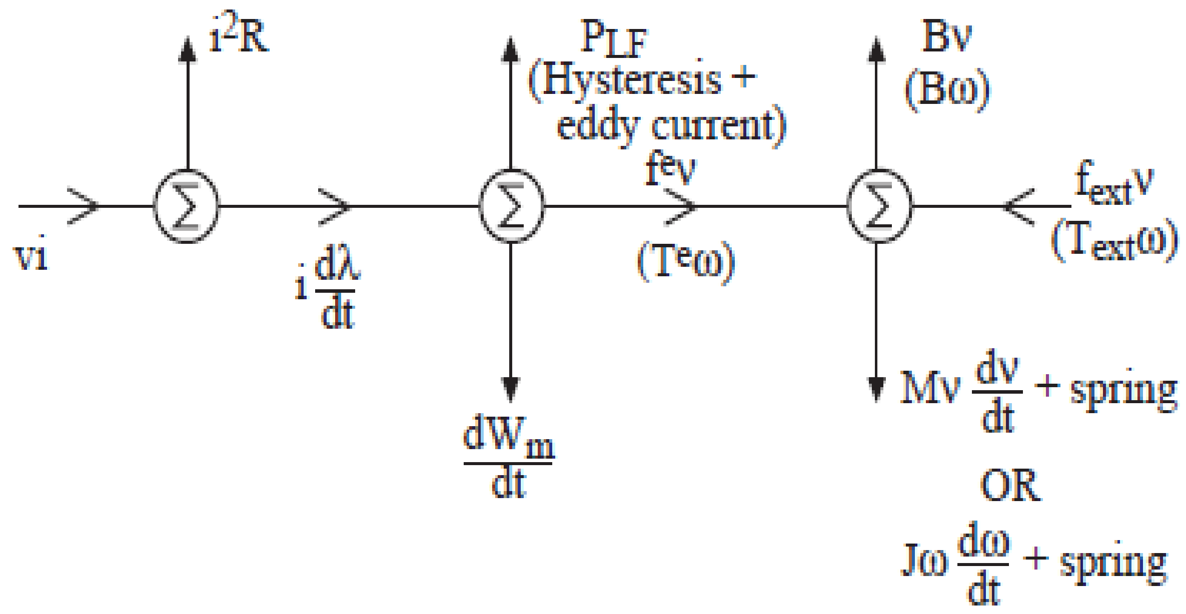
ENERGY CONVERSION

In an electro-mechanical energy conversion system, energy is transferred from the electrical side to the mechanical side as follows: (mechanical to electrical transfer is backwards):



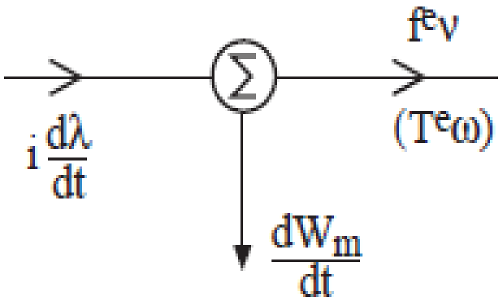
ENERGY CONVERSION

- By differentiating the energy with respect to time, power transfer can be shown to be:



ENERGY CONVERSION

Neglecting the field losses, we get the simple relation for the coupled system (shown dotted)



ENERGY CONVERSION

The force of electrical origin can be derived using the energy function W_{m} :

$$\frac{dW_m}{dt} = vi - f^e \frac{dx}{dt}$$

$$\frac{dW_m}{dt} = i\frac{d\lambda}{dt} - f^e \frac{dx}{dt}$$

Choosing λ and x as independent variables:

$$\frac{dW_m(\lambda,x)}{dt} = \frac{\partial W_m(\lambda,x)}{\partial \lambda}\frac{d\lambda}{dt} + \frac{\partial W_m(\lambda,x)}{\partial x}\frac{dx}{dt}$$

$$i(\lambda,x) = \frac{\partial W_m(\lambda,x)}{\partial \lambda}, \qquad f^e(\lambda,x) = -\frac{\partial W_m(\lambda,x)}{\partial x}$$

In a rotational system

$$\frac{dW_m}{dt} = i\,\frac{d\lambda}{dt} - T^e\,\frac{d\theta}{dt}$$

Choosing λ and θ as independent variables:

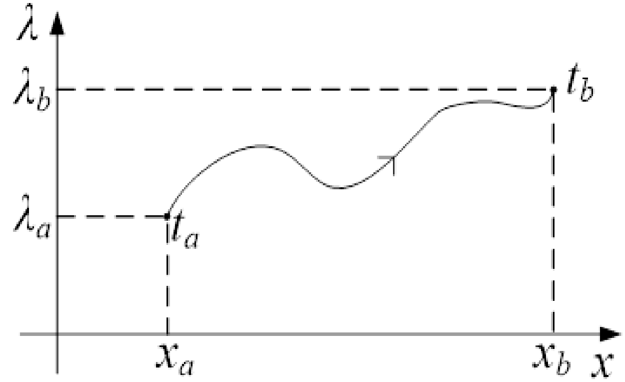
$$\frac{dW_m(\lambda,\theta)}{dt} = \frac{\partial W_m(\lambda,\theta)}{\partial \lambda}\frac{d\lambda}{dt} + \frac{\partial W_m(\lambda,\theta)}{\partial \theta}\frac{d\theta}{dt}$$

$$i(\lambda,\theta) = \frac{\partial W_m(\lambda,\theta)}{\partial \lambda} \quad , \quad f^e(\lambda,x) = -\frac{\partial W_m(\lambda,\theta)}{\partial x}$$

FORCES OF ELECTRICAL ORIGIN

When the system moves from one operating point to another

$$\int_{t_a}^{t_b} \frac{dW_m}{dt} dt = \int_{t_a}^{t_b} \left(i(\lambda, x) \frac{d\lambda}{dt} - f^e(\lambda, x) \frac{dx}{dt} \right) dt$$



Change variable

$$\int_{m_a}^{m_b} dW_m = \int_{\lambda_a}^{\lambda_b} i(\lambda, x) d\lambda - \int_{x_a}^{x_b} f^e(\lambda, x) dx$$

$$W_{mb} - W_{ma} = \int_{\lambda_a}^{\lambda_b} i(\lambda, x) d\lambda - \int_{x_a}^{x_b} f^e(\lambda, x) dx$$

FORCES OF ELECTRICAL ORIGIN

Integrate x keeping λ constant at λ_a, then integrate λ keeping x at x_b

$$W_{mb} - W_{ma} = \int_{\lambda_a}^{\lambda_b} i(\lambda, x_b) d\lambda - \int_{x_a}^{x_b} f^e(\lambda_a, x) dx$$

If $\lambda_a = 0$ and $W_{ma} = 0$ ($f^e = 0$), then:

$$W_{mb} = \int_0^{\lambda_b} i(\lambda, x) d\lambda$$

Letting λ_b any λ and x_b any x, then:

$$W_m(\lambda, x) = \int_0^{\lambda} i(\lambda, x) d\lambda$$

ENERGY AND CO-ENERGY

To compute $W_m(\lambda, x)$, we need to express $i = i(\lambda, x)$ from the flux linkage equation. This would require solving for $\boldsymbol{i}$ from $\lambda = \lambda(i, x)$. Particularly in multi-port systems, this could be quite complicated.

We can avoid this problem by defining a quantity called *co-energy* directly computable from $\lambda = \lambda(i, x)$ and then using it to compute f^e .

ENERGY AND CO-ENERGY

Choosing λ and x as independent variables, define:

$$W_m' \triangleq \lambda i - W_m$$

Using i and x as independent variables

$$\frac{dW_m'(i,x)}{dt} = \frac{\partial W_m'(i,x)}{\partial i}\frac{di}{dt} + \frac{\partial W_m'(i,x)}{\partial x}\frac{dx}{dt}$$

$$\frac{dW_m'(i,x)}{dt} = \lambda\frac{di}{dt} + i\frac{d\lambda}{dt} - \frac{dW_m}{dt}$$

$$\frac{dW_m'(i,x)}{dt} = \lambda\frac{di}{dt} + i\frac{d\lambda}{dt} - (i\frac{d\lambda}{dt} - f^e\frac{dx}{dt})$$

The left sides of these equations are the same, comparing terms:

$$\lambda(i,x) = \frac{\partial W_m'}{\partial i}$$

$$f^e(i,x) = -\frac{\partial W_m'}{\partial x}$$

Computation of W_m'

Method # 1

$$W_m'(i,x) \triangleq \lambda(i,x) - W_m(\lambda(i,x))$$

ENERGY AND CO-ENERGY

Method # 2

$$\frac{dW'_m}{dt} = \lambda \frac{di}{dt} + f^e \frac{dx}{dt}$$

$$\int_{t_a}^{t_b} \frac{dW'_m}{dt} dt = \int_{t_a}^{t_b} (\lambda \frac{d\lambda i}{dt} + f^e \frac{dx}{dt}) dt$$

Change of variables using i and x as independent variables

$$W'_{mb} - W'_{ma} = \int_{i_a}^{i_b} \lambda(i,x)\, di + \int_{x_a}^{x_b} f^e(i,x)\, dx$$

ENERGY AND CO-ENERGY

Choose a path

Integrate x keeping i constant at i_a, then integrate λ keeping x at x_b

$$W'_{mb} - W'_{ma} = \int_{i_a}^{i_b} \lambda(i, x_b)\,di + \int_{x_a}^{x_b} f^e(i_a, x)\,dx$$

Use $i_a = 0$ (so $f^e = 0$) and assume $W'_{ma} = 0$

Let i_b be any i and x_b *be* any x

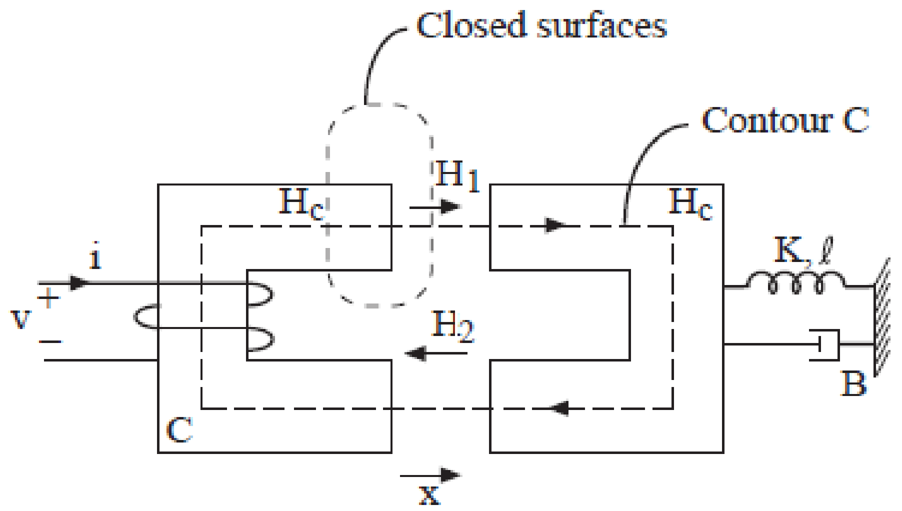
$$W'_m(i, x) = \int_0^i \lambda(i, x)\,di \qquad (x = const.)$$

EXAMPLE

ℓ_c: The mean length of the magnetic path

A : Cross-sectional area, the magnetic circuit has finite μ.

Find the force of electric origin f^e.



EXAMPLE

Ampere's circuital law gives:

$$H_c \ell_c + H_1 x + H_2 x = Ni$$

Applying Gauss's law around the moving part:

$$\mu_0 H_1 A = \mu_0 H_2 A$$

$$H_1 = H_2$$

Applying Gauss's law to the closed surface around the upper fixed surface

$$\mu H_c A = \mu_0 H_1 A$$

$$H_c = \frac{\mu_0}{\mu} H_1$$

EXAMPLE

$$H_c \ell_c + 2H_1 x = Ni$$

Solve for H_1:

$$H_1 = \frac{Ni}{\left(\frac{\mu_0}{\mu}\ell_c + 2x\right)}$$

The flux linkage of the coil:

$$\lambda = N\phi$$

$$= N\mu_0 H_1 A$$

$$= \frac{N\mu_0 Ni}{\left(\frac{\mu_0}{\mu}\ell_c + 2x\right)} A \qquad = \frac{N^2 i}{\left(\frac{\ell_c}{\mu A} + \frac{2x}{\mu_0 A}\right)}$$

EXAMPLE

The co-energy W_m' is $W_m' = \int_0^i \lambda(i, x)\,di = \dfrac{N^2 i^2}{2\left(\dfrac{\ell_c}{\mu A} + \dfrac{2x}{\mu_0 A}\right)}$

The force of electric origin: $f^e = \dfrac{\partial W_m'}{\partial x}$

$$f^e = \frac{-N^2 i^2}{2\left(\dfrac{\ell_c}{\mu A} + \dfrac{2x}{\mu_0 A}\right)^2}\left(\frac{2}{\mu_0 A}\right) = \frac{-N^2}{\mu_0 A}\frac{i^2}{\left(\dfrac{\ell_c}{\mu A} + \dfrac{2x}{\mu_0 A}\right)^2}$$

Note that $\ell_c \mu A$ is the reluctance of the iron path, and $\dfrac{2x}{\mu_0 A}$ is the reluctance of the two air gaps in series.