ECE 330
POWER CIRCUITS AND ELECTROMECHANICS

LECTURE 12
ELECTROMECHANICAL SYSTEMS (2)

Acknowledgment-These handouts and lecture notes given in class are based on material from Prof. Peter Sauer's ECE 330 lecture notes. Some slides are taken from Ali Bazi's presentations

ROTATIONAL SYSTEM

This is the basic configuration of many machines.

Both rotor and stator have $\mu = \infty$.

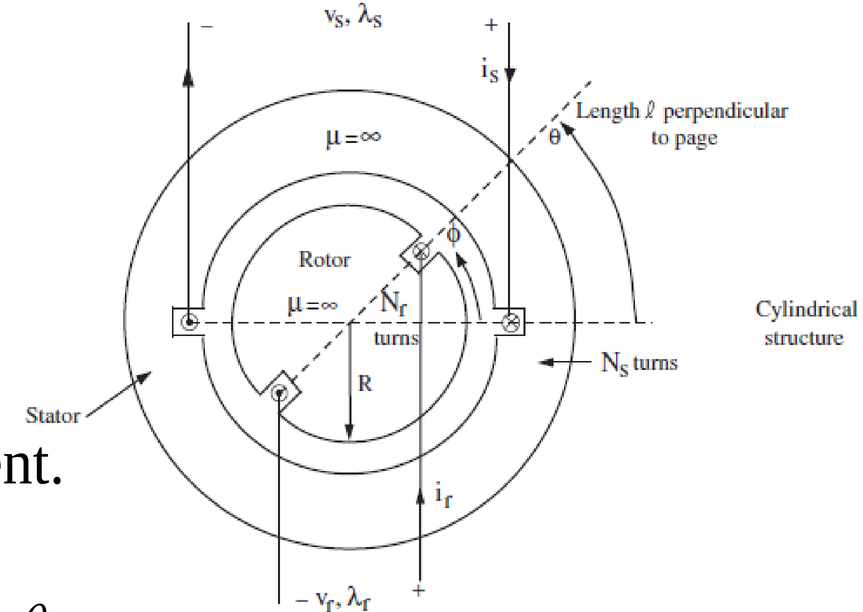
The radial air gap has length g.

The cylindrical rotor is of length ℓ.

At time t, θ is the angular displacement.

Find λ_s, λ_r as a function of i_s, i_r, and θ.

Also find v_s and v_r of the cylindrical rotor.

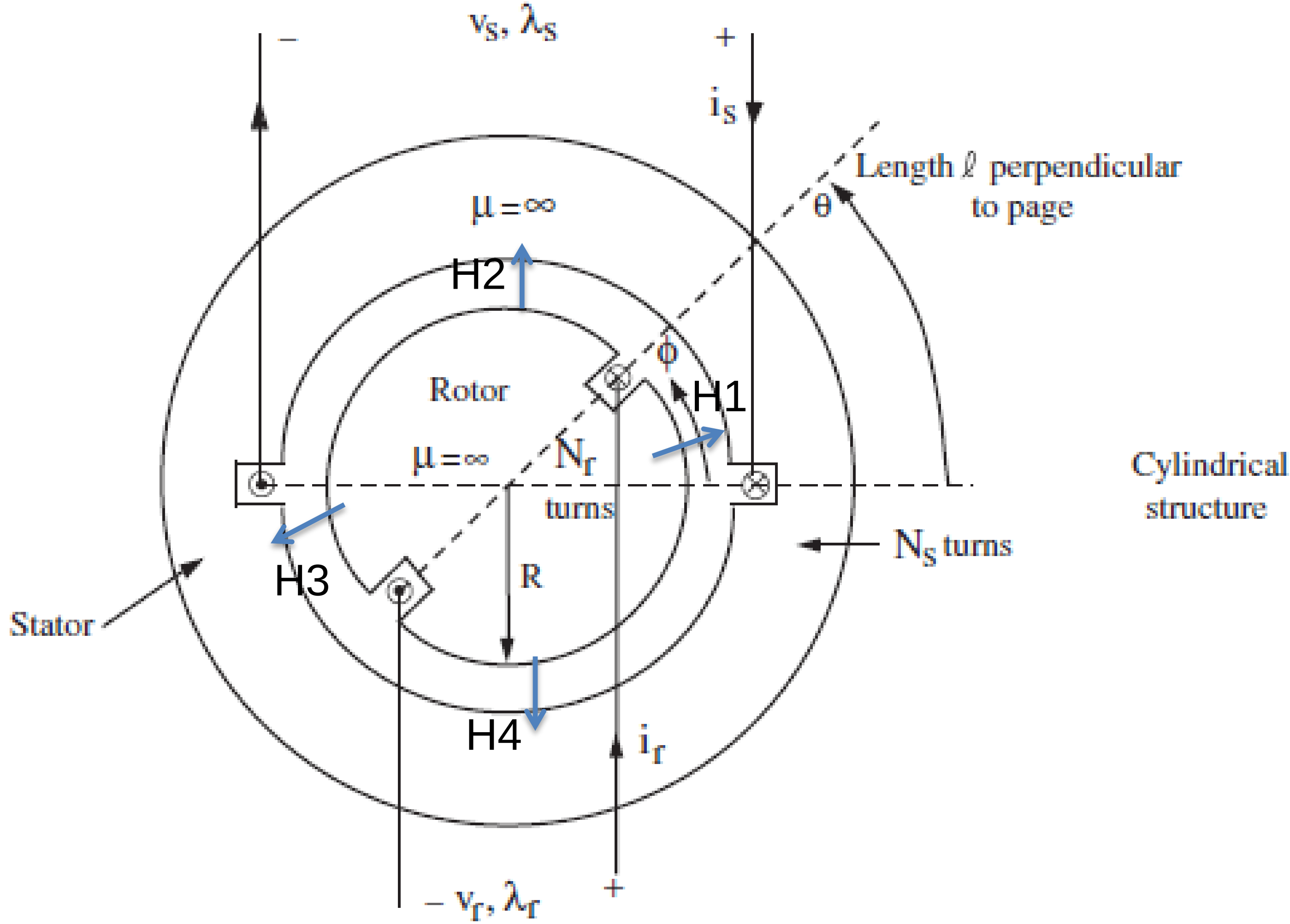


ECE ILLINOIS

ILLINOIS

ROTATIONAL SYSTEM

Assume H is constant in each region



ROTATIONAL MECHANICAL SYSTEM

Four regions with different value of H:

$$0 < \phi < \theta \quad , \quad \theta < \phi < \pi \quad , \quad \pi < \phi < \pi + \theta \quad , \quad \pi + \theta < \phi < 2\pi$$

$$H_1 \qquad\qquad H_2 \qquad\qquad H_3 \qquad\qquad H_4$$

Ampere's circuital law

Contour 1 $$H_1 - H_4 = \frac{N_s i_s}{g} \text{.............(1)}$$

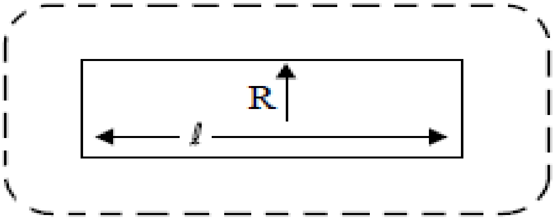
Contour 2 $$H_2 - H_1 = \frac{N_r i_r}{g} \text{...........(2)}$$

Contour 3 $$H_3 - H_2 = -\frac{N_s i_s}{g} \text{...........(3)}$$

Contour 4 $$H_4 - H_3 = -\frac{N_r i_r}{g} \text{...........(4)}$$

ROTATIONAL MECHANICAL SYSTEM

Gauss's law for magnetic fields



$$\oint_s B \bullet n da = 0$$

Multiplying by the circumferential area of each region

$$\mu_0 \ell R (H_1\theta + H_2(\pi - \theta) + H_3\theta + H_4(\pi - \theta)) = 0$$

$$\mu_0 \ell R[\theta(H_1 - H_2 + H_3 - H_4) + \pi(H_2 + H_4)] = 0$$

$$H_1 - H_2 + H_3 - H_4 = 0$$

$$H_2 + H_4 = 0 \Rightarrow H_2 = -H_4 \ldots\ldots\ldots\ldots(5)$$

ROTATIONAL MECHANICAL SYSTEM

$$H_1 + H_3 = 0 \Rightarrow H_1 = -H_3 \ldots\ldots\ldots\ldots(6)$$

Subtracting 2 from 1 and using 5

$$2H_1 = \frac{N_s i_s - N_r i_r}{g} \quad , \quad H_1 = \frac{N_s i_s - N_r i_r}{2g} = -H_3$$

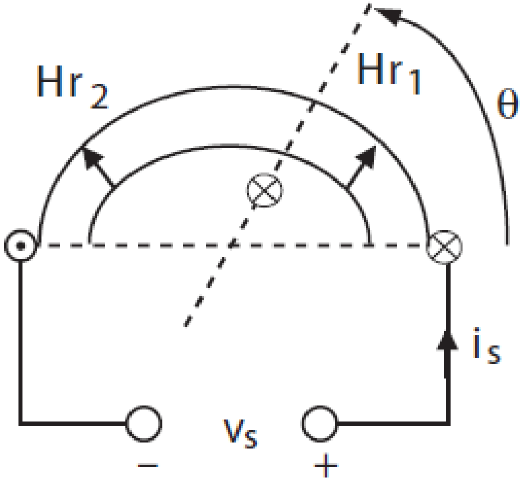
Subtracting 3 from 2 and using 6:

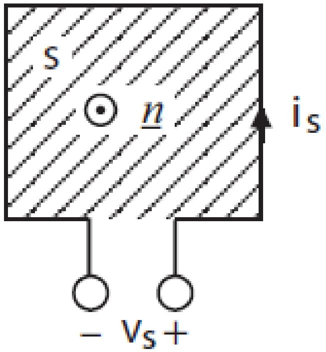
$$H_2 = \frac{N_s i_s + N_r i_r}{2g} = -H_4$$

ROTATIONAL MECHANICAL SYSTEM

STATOR FLUX LINKAGES

We first note that arc length of the coil with an angle θ is $R\theta$, where R is the radius





$$\phi_s = \int_s \underline{B} \cdot \underline{n}\, da = \int_0^{\theta} \mu_o H_1 \ell R d\phi + \int_{\theta}^{\pi} \mu_o H_2 \ell R d\phi$$

$$= (\mu_o H_1)(R\theta l) + (\mu_o H_2)(\pi - \theta) R \ell$$

ROTATIONAL MECHANICAL SYSTEM

the flux through one turn of stator coil is

$$\lambda_s = N_s \phi_s = N_s \mu_o H_1 R \theta \ell + N_s \mu_o H_2 R (\pi - \theta) \ell$$

$$\lambda_s = \frac{N_s^2 \mu_0 R l \pi}{2g} i_s + N_s N_r \frac{\mu_0 R l \pi}{2g} \left(1 - \frac{2\theta}{\pi}\right) i_r$$

$$\lambda_s = N_s^2 L_0 i_s + N_s N_r L_0 \left(1 - \frac{2\theta}{\pi}\right) i_r, \qquad L_0 = \frac{\mu_0 R l \pi}{2g}$$

$$\lambda_s = L_s i_s + L_m(\theta) i_r \quad ,$$ where

$$L_s = N_s^2 L_0, \qquad L_m(\theta) = N_s N_r L_0 \left(1 - \frac{2\theta}{\pi}\right)$$

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ROTATIONAL MECHANICAL SYSTEM

Rotor flux linkages:

$$\lambda_r = N_r \int_s B \bullet n\, da$$

$$= N_r \int_{\theta}^{\pi} \mu_0 H_2 \ell R d\phi + N_r \int_{\pi}^{\pi+\theta} \mu_0 H_3 \ell R d\phi$$

$$\lambda_r = N_r \mu_0 H_2 \ell R(\pi-\theta) + N_r \mu_0 H_3 \ell R \theta$$

$$\lambda_r = N_r \mu_0 \frac{N_s i_s + N_r i_r}{2g} R\ell(\pi-\theta) - N_r \mu_0 \frac{N_s i_s - N_r i_r}{2g} R\ell\theta$$

ROTATIONAL MECHANICAL SYSTEM

$$\lambda_r = N_r \mu_0 \frac{N_s i_s}{2g} R\ell\pi - N_r \mu_0 \frac{N_s i_s}{2g} R\ell\theta + N_r \mu_0 \frac{N_r i_r}{2g} R\ell\pi - \frac{N_r i_r}{2g} R\ell\theta$$

$$-N_r \mu_0 \frac{N_s i_s}{2g} R\ell\theta + N_r \mu_0 \frac{N_r i_r}{2g} R\ell\theta$$

$$\lambda_r = N_s N_r L_0 \left(1 - \frac{2\theta}{\pi}\right) i_s + N_r^2 L_0 i_r$$

$$= L_m(\theta) i_s + L_r i_r$$

Note: $L_r = N_r^2 L_0$

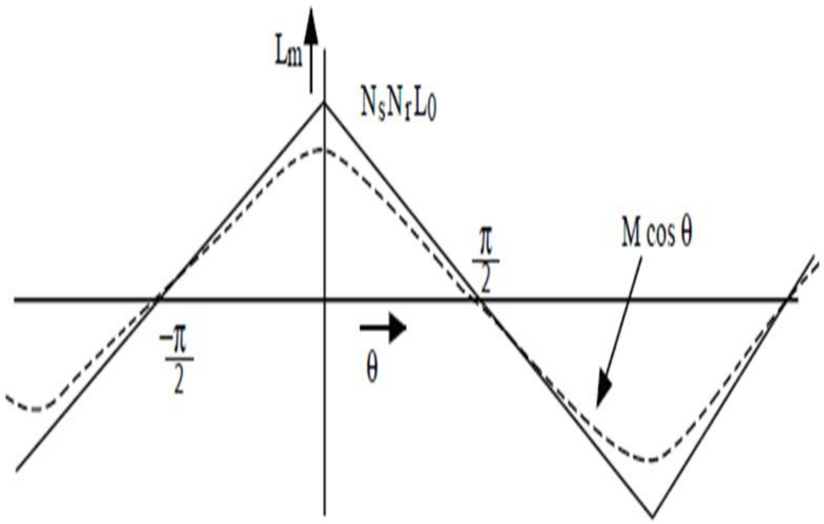
INDUCTANCE IN THE ROTATIONAL EXAMPLE

- The flux linkages are

$$\lambda_s = L_s i_s + L_m(\theta) i_r$$
$$\lambda_r = L_r i_r + L_m(\theta) i_s$$

The actual mutual inductance is a triangular wave, but is usually approximated as a sine wave.

$$L_m = M\cos(\theta).$$



ROTATIONAL MECHANICAL SYSTEM

If:

$$L_m = M \cos\theta$$

$$v_s = \frac{d\lambda_s}{dt} = L_s \frac{di_s}{dt} + M \cos\theta \frac{di_r}{dt} \underbrace{- i_r M \sin\theta \frac{d\theta}{dt}}_{Speed\ voltage}$$

$$v_r = \frac{d\lambda_r}{dt} = L_r \frac{di_r}{dt} + M \cos\theta \frac{di_s}{dt} \underbrace{- i_s M \sin\theta \frac{d\theta}{dt}}_{Speed\ voltage}$$

LINEAR SYSTEMS

- A linear system can be formulated based on the flux linkage and current equations.

$$\begin{bmatrix} i_s \\ i_r \end{bmatrix} = \begin{bmatrix} L_s & L_m(\theta) \\ L_m(\theta) & L_r \end{bmatrix}^{-1} \begin{bmatrix} \lambda_s \\ \lambda_r \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} i_s \\ i_r \end{bmatrix} = \frac{1}{L_s L_r - L_m^2(\theta)} \begin{bmatrix} L_r & -L_m(\theta) \\ -L_m(\theta) & L_s \end{bmatrix} \begin{bmatrix} \lambda_s \\ \lambda_r \end{bmatrix}$$