ECE330: Power Circuits & Electromechanics

Lecture 10. Ideal transformers

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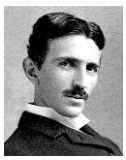
Schedule

- ~~Mon 9/23: Coupled coils~~
- Wed 9/25: Ideal transformers
- **Friday 9/27: Review + Quiz 4**
- Mon 9/30: Review
- Wed 10/2: Review
- **Thursday 10/3: Mid-term Exam 1**
- Friday 10/4: No class

War of Currents (circa 1880-1900)

Edison

Westinghouse



Tesla

1880 Edison receives patent for lightbulb, founds the Edison Illuminating Company. (Today: ConEd)

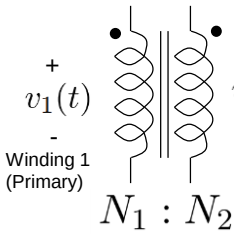
1888 Tesla joins Westinghouse to commercialize AC.

1888 Edison: "It will never be free from danger." In memo concerning the Westinghouse AC system.

1893 Westinghouse wins contract for Chicago World Fair.

1908 Edison: "Tell your father I was wrong." To George Stanley, the son of William Stanley who had invented an AC transformer for Westinghouse.

The Transformer: AC's secret weapon



Winding 1 (Primary)

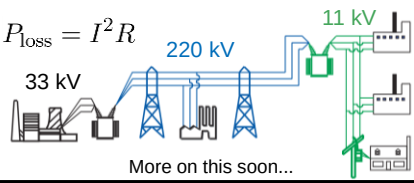
Winding 2 (Secondary)

Turns ratio

$N_2 > N_1$ Step-up
$N_2 < N_1$ Step-down
$N_2 = N_1$ Isolation

Transformers in the AC power system

$P_{\text{loss}} = I^2 R$



More on this soon...

Genius is one percent inspiration and ninety-nine percent perspiration.

---Thomas Edison (c. 1903)

Just a little theory and calculation would have saved him ninety percent of his labor.

---Nikola Tesla (1931)

Today

- Ideal transformer model
- Apparent impedance
- Practical transformer model

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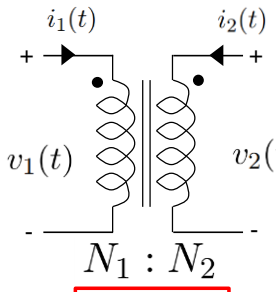
Transformer (ideal)



Definition (Ideal transformer)

$$\frac{v_1(t)}{v_2(t)} = \frac{N_1}{N_2} = a$$

(Transformation ratio)

$$P_{\text{in}} = P_{\text{out}} = 0$$

(Power conservation)

Implies current transformation

$$0 = v_1(t)i_1(t) + v_2(t)i_2(t)$$
$$= v_1(t)\left[i_1(t) + \frac{N_2}{N_1}i_2(t)\right]$$
$$= i_1(t) + \frac{N_2}{N_1}i_2(t)$$

Minus sign!!

$$\frac{i_1(t)}{i_2(t)} = -\frac{N_2}{N_1} = -\frac{1}{a}$$

Turns ratio $a = N_1/N_2$

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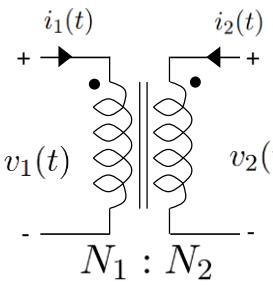
Transformer (ideal)



Minus sign!!

$$i_2(t) = -\frac{N_1}{N_2}i_1(t)$$

Can reverse minus sign at risk of confusion

higher voltage
→ lower current

$$v_2(t) = \frac{N_2}{N_1}v_1(t)$$

more turns
→ higher voltage

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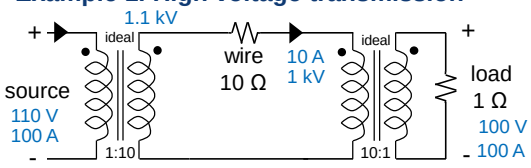
Example 1. High voltage transmission



$v_{\text{load}}(t) = \sqrt{2}(100)\cos\omega t$ [V]

$i_{\text{wire}}(t) =?$ [A]

~~A) $\sqrt{2}\cos\omega t$~~

B) $\sqrt{2}(10)\cos\omega t$

~~C) $\sqrt{2}(100)\cos\omega t$~~

~~D) $\sqrt{2}(1000)\cos\omega t$~~

~~E) $\sqrt{2}(10000)\cos\omega t$~~

$v_{\text{source}}(t) =?$ [V]

~~A) $\sqrt{2}(100)\cos\omega t$~~

B) $\sqrt{2}(110)\cos\omega t$

~~C) $\sqrt{2}(200)\cos\omega t$~~

~~D) $\sqrt{2}(1100)\cos\omega t$~~

~~E) $\sqrt{2}(2000)\cos\omega t$~~

$i_{\text{source}}(t) =?$ [A]

~~A) $\sqrt{2}\cos\omega t$~~

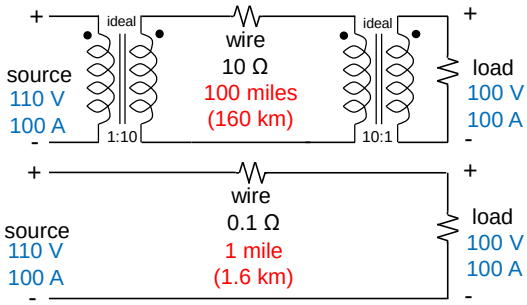
~~B) $\sqrt{2}(10)\cos\omega t$~~

C) $\sqrt{2}(100)\cos\omega t$

~~D) $\sqrt{2}(1000)\cos\omega t$~~

~~E) $\sqrt{2}(10000)\cos\omega t$~~

Why Edison lost the war (1890s)



10x step-up yields 100x distance
(for the same efficiency)

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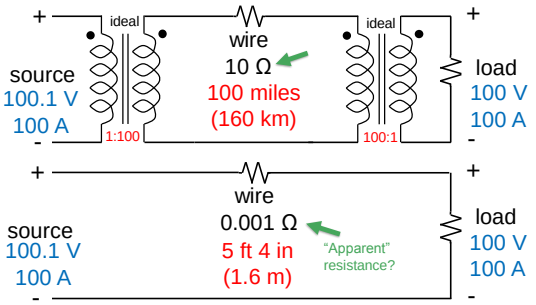
Why Edison lost the war (1960s)



100x step-up yields 10,000x distance
(for the same efficiency)

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Today

- ~~Ideal transformer model~~
- Apparent impedance
- Practical transformer model

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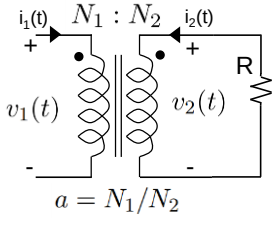
Ideal transformer with resistance



$a = N_1/N_2$

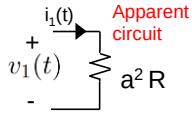
Transformation equations

$$\frac{v_1(t)}{v_2(t)} = a \qquad \frac{i_1(t)}{i_2(t)} = -\frac{1}{a}$$

$$\frac{v_2(t)}{i_2(t)} = -R$$

$$\frac{v_1(t)}{i_1(t)} = \frac{(N_1/N_2)v_2(t)}{-(N_2/N_1)i_2(t)} = -\left(\frac{N_1}{N_2}\right)^2 \frac{v_2(t)}{i_2(t)} = a^2 R$$

Apparent circuit



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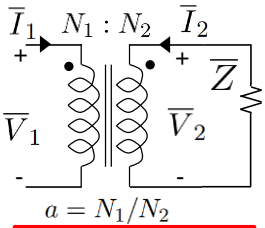
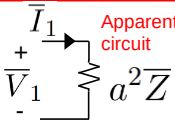
Ideal transformer with impedance



$a = N_1/N_2$

Apparent circuit



Transformation equations

$$\frac{v_1(t)}{v_2(t)} = a \qquad \frac{i_1(t)}{i_2(t)} = -\frac{1}{a}$$

$$\frac{\overline{V}_2}{\overline{I}_2} = -\overline{Z}$$

$$\frac{\overline{V}_1}{\overline{I}_1} = \frac{(N_1/N_2)\overline{V}_2}{-(N_2/N_1)\overline{I}_2} = -\left(\frac{N_1}{N_2}\right)^2 \frac{\overline{V}_2}{\overline{I}_2} = a^2\overline{Z}$$

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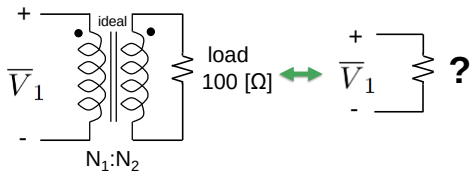
Example 2. Warm-up

Hint: $a = N_1/N_2$



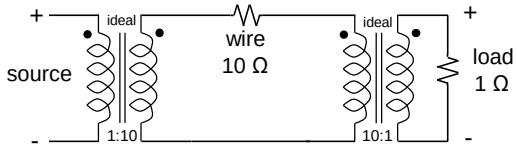
$N_1 = 100$, $N_2 = 10$

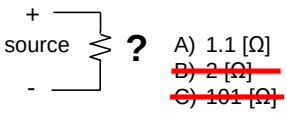
- ~~A) 1 [Ω]~~
- ~~B) 10 [Ω]~~
- ~~C) 100 [Ω]~~
- ~~D) 1,000 [Ω]~~
- E) 10,000 [Ω]

$N_1 = 1$, $N_2 = 10$

- A) 1 [Ω]
- ~~B) 10 [Ω]~~
- ~~C) 100 [Ω]~~
- ~~D) 1,000 [Ω]~~
- ~~E) 10,000 [Ω]~~

Example 3. High voltage transmission

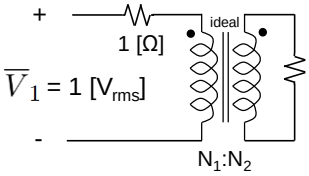




- A) 1.1 [Ω]
- ~~B) 2 [Ω]~~
- ~~C) 101 [Ω]~~
- ~~D) 110 [Ω]~~
- ~~E) 200 [Ω]~~

$$\left(\frac{1}{10}\right)^2 \left[10 + \left(\frac{10}{1}\right)^2 1\right]$$

Example 4. Power transfer



Apparent resistance

$$R' = \left(\frac{N_1}{N_2}\right)^2 R$$

$N_1 = 1$, $N_2 = 2$

- ~~A) 1/25~~
- ~~B) 1/5~~
- C) 1/4
- ~~D) 4/5~~
- ~~E) 4/25~~

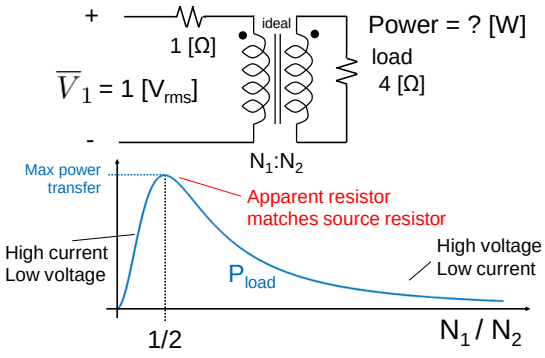
Power equal to power at apparent resistor

$$P = |\overline{I}_2|^2 R = \left(\frac{N_1}{N_2}\right)^2 |\overline{I}_1|^2 R = |\overline{I}_1|^2 R'$$

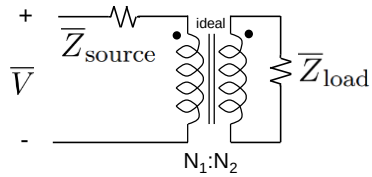
Compute power at apparent resistor

$$\overline{I}_1 = \frac{1}{1+R'}, \qquad P = \frac{R'}{(1+R')^2}$$

Maximum power transfer



Impedance matching



Theorem. Maximum power transfer if ratio N_1:N_2 satisfies

$$|\overline{Z}_{source}| = \left(\frac{N_1}{N_2}\right)^2 |\overline{Z}_{load}|$$

Apparent load impedance

Today

- ~~Ideal transformer model~~
- ~~Apparent impedance~~
- Practical transformer model

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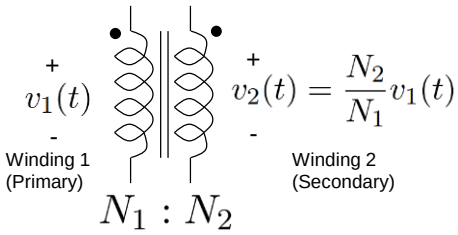
So far: Ideal transformers



Real transformers far from ideal
Must use practical model

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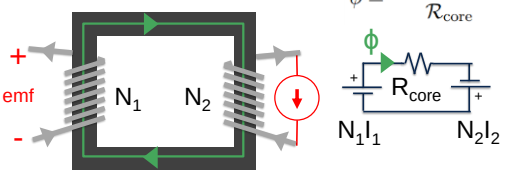
Mutual inductance



Flux linkage at winding 1 $\lambda_1 = N_1\phi = \frac{N_1^2}{\mathcal{R}_{core}} I_1 + \frac{N_1 N_2}{\mathcal{R}_{core}} I_2$

$$-\text{emf}_1 = \underbrace{\frac{N_1^2}{\mathcal{R}_{core}}}_{L_1} \frac{di_1}{dt} + \underbrace{\frac{N_1 N_2}{\mathcal{R}_{core}}}_{M} \frac{di_2}{dt}$$

Self Inductance Mutual Inductance

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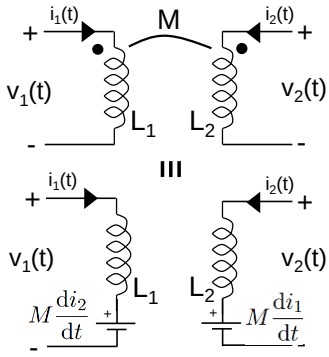
Circuit models of mutual inductance



Polarity (Important!)
v pos. terminal at dot
i flows into dot

$$\lambda_1 = L_1 i_1 + M i_2$$
$$\lambda_2 = M i_1 + L_2 i_2$$

$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$
$$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

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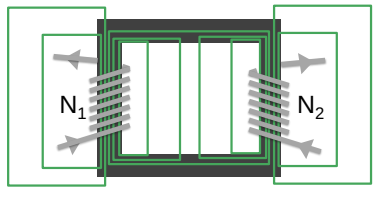
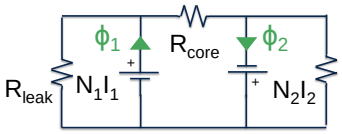
Imperfect coupling (k < 1)

Coefficient of coupling

$$k = \frac{M}{\sqrt{L_1 L_2}}$$



leakage flux path



Flux from coil 1 partially received by coil 2, and partially leaked out

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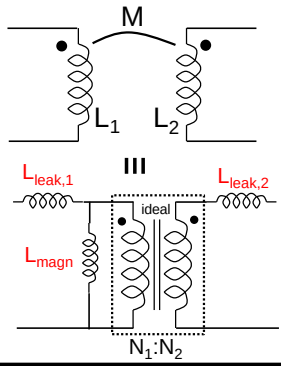
Lossless linear transformer model



Leakage inductance from leakage flux

Magnetizing inductance from coupled flux

Unique solution for each fixed N_1:N_2

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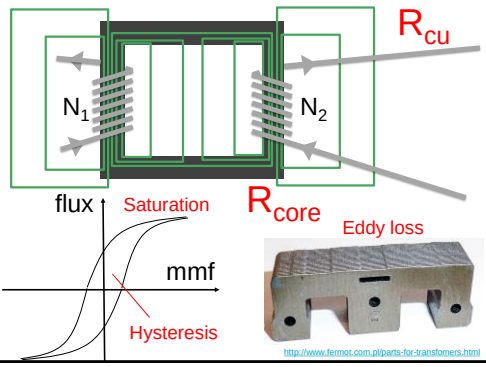
Winding and core loss



http://www.fermot.com.pl/parts-for-transfomers.html

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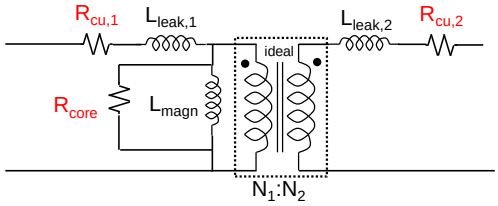
Lossy linear transformer model



Design considerations

- Cost
- Size
- Weight
- Temperature
- Insulation rating
- Load current
- Short circuit current
- Load factor
- Target efficiency
- Target service life

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