ECE 330
POWER CIRCUITS AND ELECTROMECHANICS

LECTURE 25
INDUCTION MACHINES (2)

Acknowledgment-These handouts and lecture notes given in class are based on material from Prof. Peter Sauer's ECE 330 lecture notes. Some slides are taken from Ali Bazi's presentations

Disclaimer- These handouts only provide highlights and should not be used to replace the course textbook.

DERAVATION OF AN EQUIVALENT CIRCUIT

For two-phase machine

$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{ar} \\ \lambda_{br} \end{bmatrix} = \begin{bmatrix} L_s & 0 & M\cos\theta & -M\sin\theta \\ 0 & L_s & M\sin\theta & M\cos\theta \\ M\cos\theta & M\sin\theta & L_r & 0 \\ -M\sin\theta & M\cos\theta & 0 & L_r \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ \hat{i}_{ar} \\ \hat{i}_{br} \end{bmatrix}$$

Let the angle between the rotor and stator axes of phase a be

$$\theta = \omega_m t + \gamma$$

Electric torque requires $\omega_r = \omega_s - \omega_m$

Instead of $\omega_m = \omega_s$, $\omega_r = 0$ (synchronous)

DERAVATION OF AN EQUIVALENT CIRCUIT

Define slip

$$s = \frac{\omega_s - \omega_{act}}{\omega_s} \Rightarrow \omega_m = \omega_s - s\,\omega_s \Rightarrow \omega_m = (1 - s)\omega_s$$

So

$$\omega_r = s\,\omega_s \qquad slip\ \ frequency$$

Assume stator and rotor currents as

$$i_{as} = I_{ms}\cos\omega_s t \qquad\qquad i_{ar} = I_{mr}\cos(s\,\omega_s t + \beta)$$

$$i_{bs} = I_{ms}\sin\omega_s t \qquad\qquad i_{br} = I_{mr}\sin(s\,\omega_s t + \beta)$$

$$\theta = \omega_m t + \gamma = (1 - s)\omega_s + \gamma$$

DERAVATION OF AN EQUIVALENT CIRCUIT

Stator circuit

$$v_{as} = R_s \hat{i}_{as} + \frac{d\lambda_{as}}{dt}$$

$$v_{as} = R_s I_{ms} \cos\omega_s t - \omega_s L_s I_{ms} \sin\omega_s t$$

$$+\frac{d}{dt}[M \cos((1-s)\omega_s t + \gamma) I_{mr} \cos(s\omega_s t + \beta) - M \sin((1-s)\omega_s t + \gamma) I_{mr} \sin(s\omega_s t + \beta)]$$

$$v_{as} = R_s I_{ms} \cos\omega_s t - \omega_s L_s I_{ms} \sin\omega_s t + \frac{d}{dt}[M I_{mr} \cos((1-s)\omega_s t + \gamma + s\omega_s t + \beta)]$$

$$v_{as} = R_s I_{ms} \cos\omega_s t - \omega_s L_s I_{ms} \sin\omega_s t - \omega_s M I_{mr} \sin(\omega_s t + \gamma + \beta)$$

$$v_{as} = V_{ms} \cos(\omega_s t + \phi_{PF})$$

DERAVATION OF AN EQUIVALENT CIRCUIT

Solving for rotor circuit:

$$v_{ar} = R_r \hat{i}_{ar} + \frac{d\lambda_{ar}}{dt}$$

$$v_{ar} = R_r I_{mr} \cos(\omega_s t + \beta) - s\omega_s L_r I_{mr} \sin(s\omega_s t + \beta)$$
$$+ \frac{d}{dt}[M \cos((1-s)\omega_s t + \gamma) I_{ms} \cos\omega_s t + M\ I_{ms} \sin((1-s)\omega_s t + \gamma) I_{ms} \sin\omega_s t]$$

DERAVATION OF AN EQUIVALENT CIRCUIT

$$v_{ar} = R_r I_{mr} \cos(s\omega_s t + \beta) - s\omega_s L_r I_{mr} \sin(s\omega_s t + \beta)$$

$$+ \frac{d}{dt}[M \; I_{ms} \cos(s\omega_s t - \gamma)]$$

$$v_{ar} = R_r I_{mr} \cos(s\omega_s t + \beta) - s\omega_s L_r I_{mr} \sin(s\omega_s t + \beta)$$

$$-M \; I_{ms} \sin(s\omega_s t - \gamma)$$

$$v_{ar} = V_{mr} \cos(s\omega_s t + \theta_{v_{ar}})$$

PER-PHASE EQUIVALENT CIRCUIT

Phasors:

$$v_{as} = V_{ms}\cos(\omega_s t + \phi_{PF})$$

$$v_{as} = \frac{V_{ms}}{\sqrt{2}}\angle\phi_{PF} = R_s\frac{I_{ms}}{\sqrt{2}}\angle 0^o + j\,\omega_s L_s\frac{I_{ms}}{\sqrt{2}}\angle 0^o + j\,\omega_s M\frac{I_{mr}}{\sqrt{2}}\angle\gamma + \beta$$

$$v_{ar} = V_{mr}\cos(s\,\omega_s t + \theta_{v_{ar}})$$

$$v_{ar} = \frac{V_{mr}}{\sqrt{2}}\angle\theta_{vr} = R_r\frac{I_{mr}}{\sqrt{2}}\angle\beta + js\,\omega_s L_r\frac{I_{mr}}{\sqrt{2}}\angle\beta + js\,\omega_s M\frac{I_{ms}}{\sqrt{2}}\angle -\gamma$$

Rotate all vectors by γ

PER-PHASE EQUIVALENT CIRCUIT

$$v_{ar} = \frac{V_{mr}}{s\sqrt{2}}\angle\theta_{vr} + \gamma = \frac{R_r}{s}\frac{I_{mr}}{\sqrt{2}}\angle\gamma + \beta + js\omega_s L_r \frac{I_{mr}}{\sqrt{2}}\angle\gamma + \beta + js\omega_s M \frac{I_{ms}}{\sqrt{2}}\angle 0$$

Recall: $L_s = L_{\ell s} + \frac{N_s}{N_r} M$ $\quad\quad$ $L_r = L_{\ell r} + \frac{N_r}{N_s} M$

Define: $I'_{mr} = I_{mr}(\frac{N_r}{N_s})$, $\quad\quad$ $V'_{mr} = V_{mr}(\frac{N_s}{N_r})$

$$R'_r = R_r(\frac{N_s}{N_r})^2 \quad , \quad L'_{\ell r} = L_{\ell r}(\frac{N_s}{N_r})^2$$

$'$ Means referred to stator

PER-PHASE EQUIVALENT CIRCUIT

$$\frac{V_{ms}}{\sqrt{2}}\angle\phi_{PF} = R_s\frac{I_{ms}}{\sqrt{2}}\angle 0^o + j\,\omega_s\,(L_{\ell s} + \frac{N_s}{N_r}M\,)\frac{I_{ms}}{\sqrt{2}}\angle 0^o + j\,\omega_s M\,\frac{I_{mr}}{\sqrt{2}}\angle\gamma+\beta$$

$$= R_s\frac{I_{ms}}{\sqrt{2}}\angle 0^o + j\,\omega_s L_{\ell s}\frac{I_{ms}}{\sqrt{2}}\angle 0^o + j\,\omega_s\frac{N_s}{N_r}M\,\frac{I_{ms}}{\sqrt{2}}\angle 0^o + j\,\omega_s M\,\frac{I_{mr}}{\sqrt{2}}\angle\gamma+\beta$$

$$\frac{V'_{mr}}{s\sqrt{2}}\angle\theta_{vr}+\gamma\quad\frac{N_r}{N_s} = \frac{R'_r}{s}\left(\frac{N_r}{N_s}\right)\frac{I'_{mr}}{\sqrt{2}}\left(\frac{N_s}{N_r}\right)\angle\gamma+\beta$$

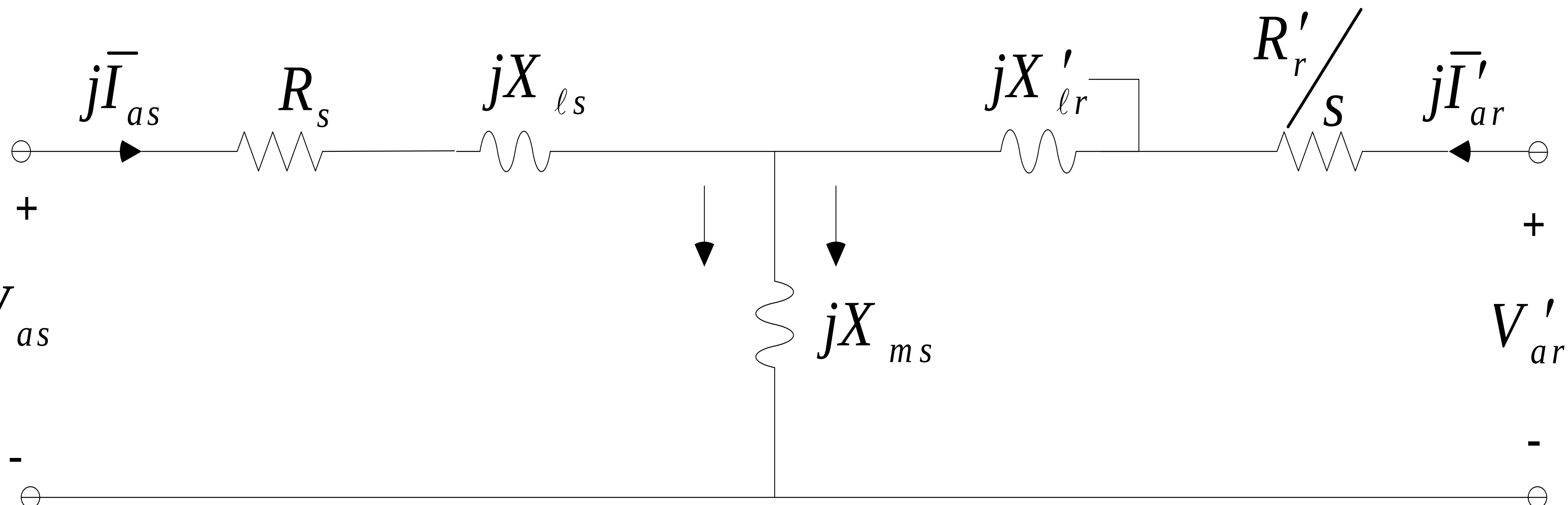
$$+\,j\,\omega_s L'_r\left(\frac{N_r}{N_s}\right)^2\frac{I'_{mr}}{\sqrt{2}}\left(\frac{N_s}{N_r}\right)\angle\gamma+\beta + j\,\omega_s M\,\frac{I_{ms}}{\sqrt{2}}\angle 0$$

PER-PHASE EQUIVALENT CIRCUIT

Multiply by $\frac{N_s}{N_r}$

$$\frac{V'_{mr}}{s\sqrt{2}}\angle\theta_{vr}+\gamma = \frac{R'_r}{s}\cdot\frac{I'_{mr}}{\sqrt{2}}\angle\gamma+\beta + j\omega_s\left(L'_{\ell r}+\frac{N_s}{N_r}\mathrm{M}\right)\frac{I'_{mr}}{\sqrt{2}}\angle\gamma+\beta$$

$$+j\omega_s\frac{N_s}{N_r}M\cdot\frac{I_{ms}}{\sqrt{2}}\angle 0$$



PER-PHASE EQUIVALENT CIRCUIT

Types of Induction Motors:

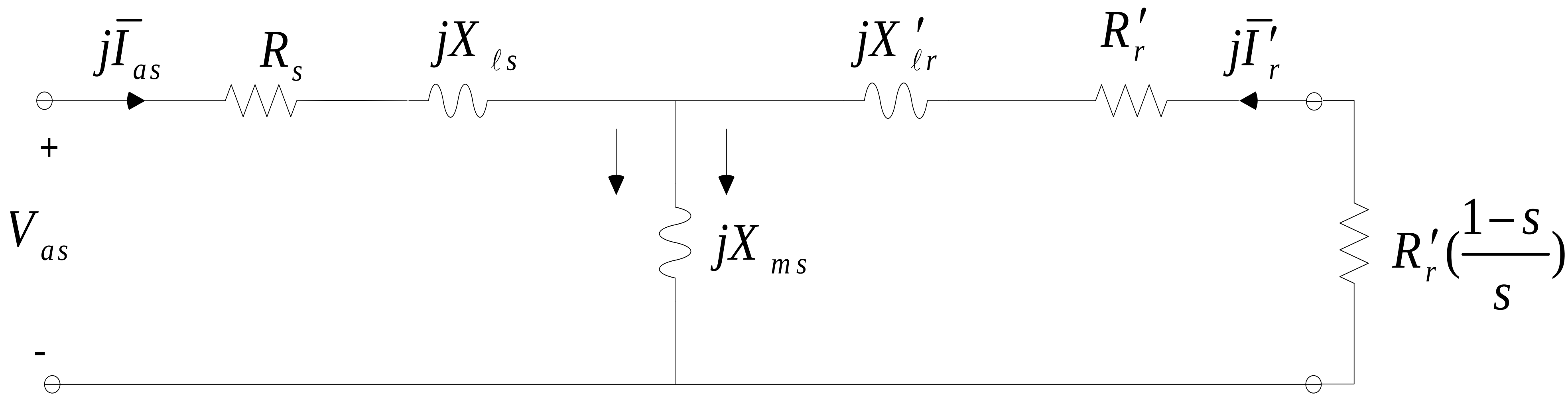
Wound rotor – Could connect something at V'_{ar}

Squirrel cage $V'_{ar} = 0$

Let $V'_{ar} = 0$

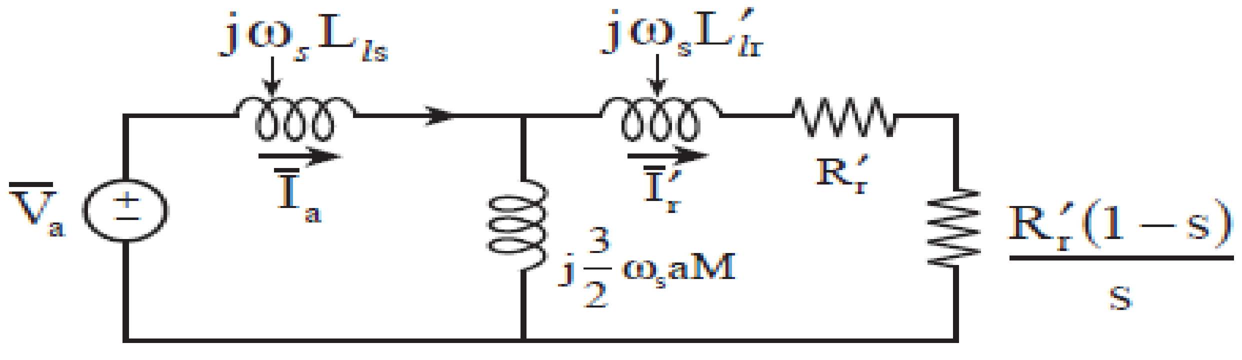
Split R'_r/s into $R'_r + R'_r(\frac{1-s}{s})$

PER-PHASE EQUIVALENT CIRCUIT



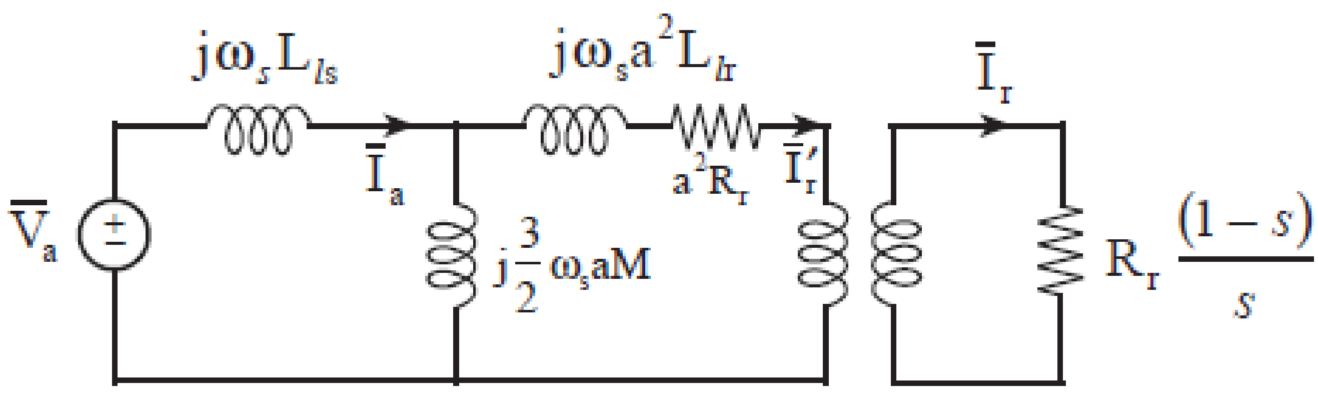
We could add R_c for eddy current and core losses in parallel with jX_{ms} (life transformer)

PER-PHASE EQUIVALENT CIRCUIT FOR TWO POLES THREE-PHASE INDUCTION MACHINE



$$\frac{R_r'}{s} = R_r' + \frac{R_r'(1-s)}{s}$$

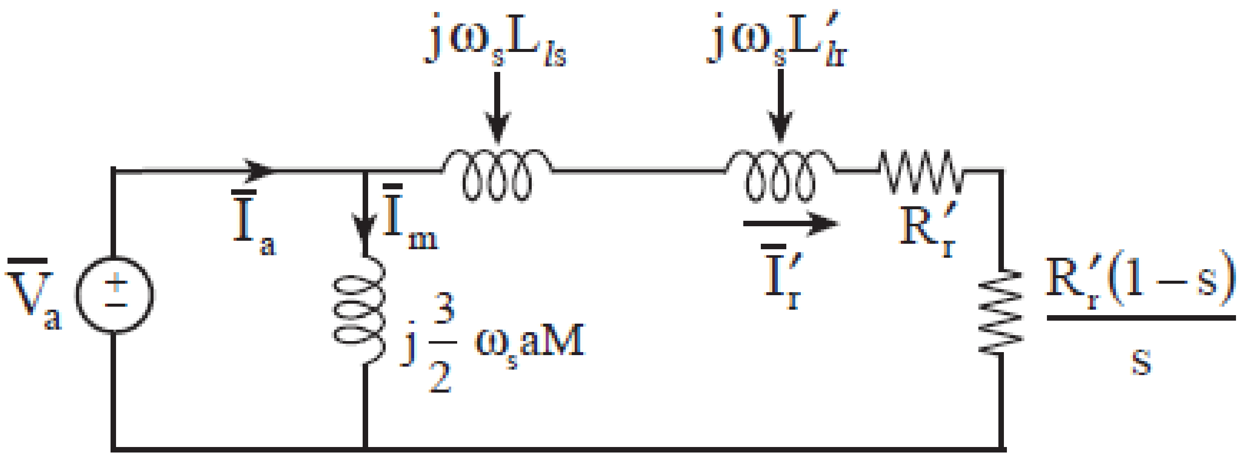
(a)

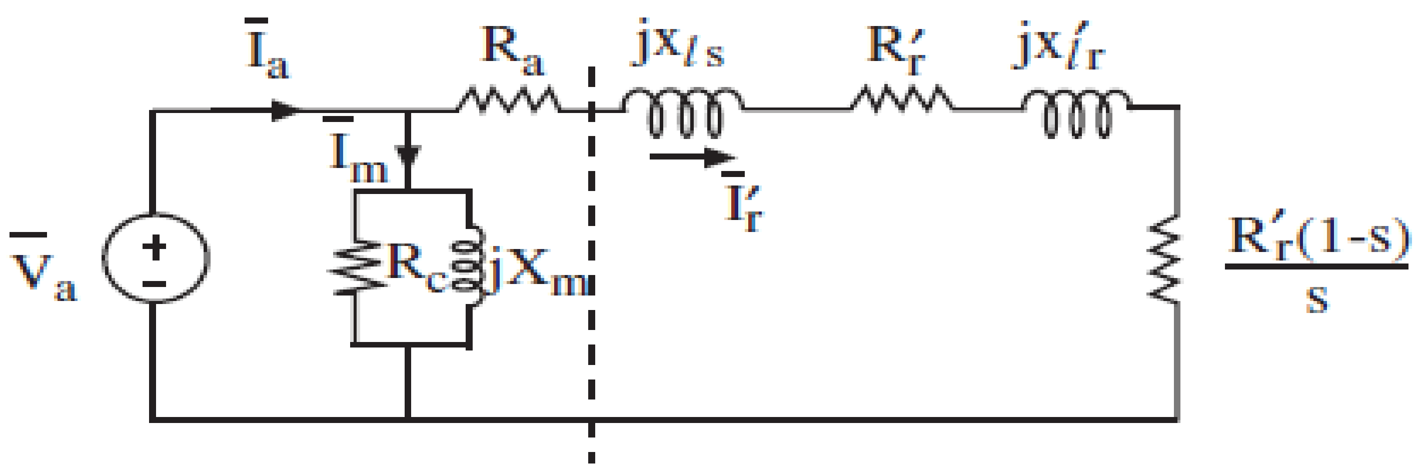


(b)

PER-PHASE EQUIVALENT CIRCUIT FOR THREE-PHASE INDUCTION MACHINE

Approximate equivalent circuit:





POWER RELATIONSHIP

Power relationships:

The total input power is given by $P_{IN} = \mathrm{Re}al\,(3\bar{V}_a\ \bar{I}_a^{*})$

$$= P_{ag} + P_{sc\ell} + P_c$$

Where

$$P_c = core\ \ loss\ = 3V_a^{\,2}/R_c$$

$$P_{sc\ell} = stator\ \ copper\ \ loss\ \ = 3(I_r')^2 R_a$$

$$P_{ag} = power\ \ across\ \ the\,air\ \ gap\ \ = 3I_r'^2 R_r'/s$$

POWER RELATIONSHIP

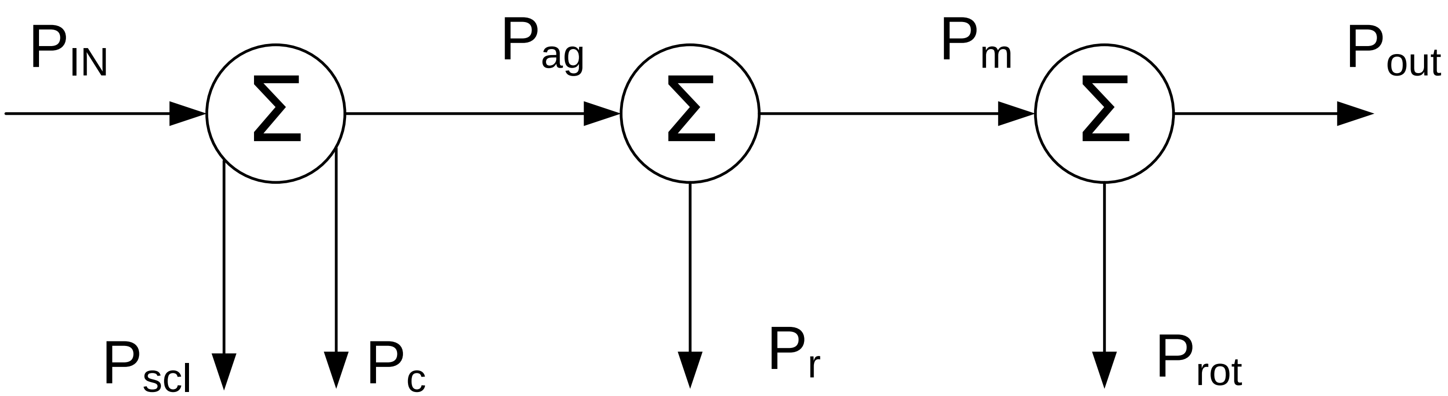
The power transformed across the air gap is the sum of the mechanical power P_m developed and the rotor loss

$$P_r = 3I_r'^2 R_r' = sP_{ag}$$

$$P_m = 3I_r'^2 R_r' \left(\frac{1-s}{s}\right) = P_{ag}(1-s)$$

$$P_{ag} = P_m + P_r$$

$$P_{shaft} = P_m - P_{rot}$$



POWER AND TERQUE RELATIONSHIP

Rotor copper loss in terms of P_{ag}

$$P_r = 3\left|\bar{I}_r'\right|^2 R_r' = s\,P_{ag}$$

$$For \quad P \neq 2 \qquad \omega_m = (1-s)\omega_s \frac{2}{P}$$

$$T^e = \frac{P_m}{\omega_m} = \frac{P_{ag}(1-s)}{(1-s)\omega_s \frac{2}{P}} = \frac{P_{ag}}{\frac{2}{P}\omega_s}$$

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EXAMPLE

Four pole induction motor, 60 Hz, 120 V, has

$$R_s = 0, \quad X_{\ell s} = 0, \quad X_{ms} = 40\ \Omega$$

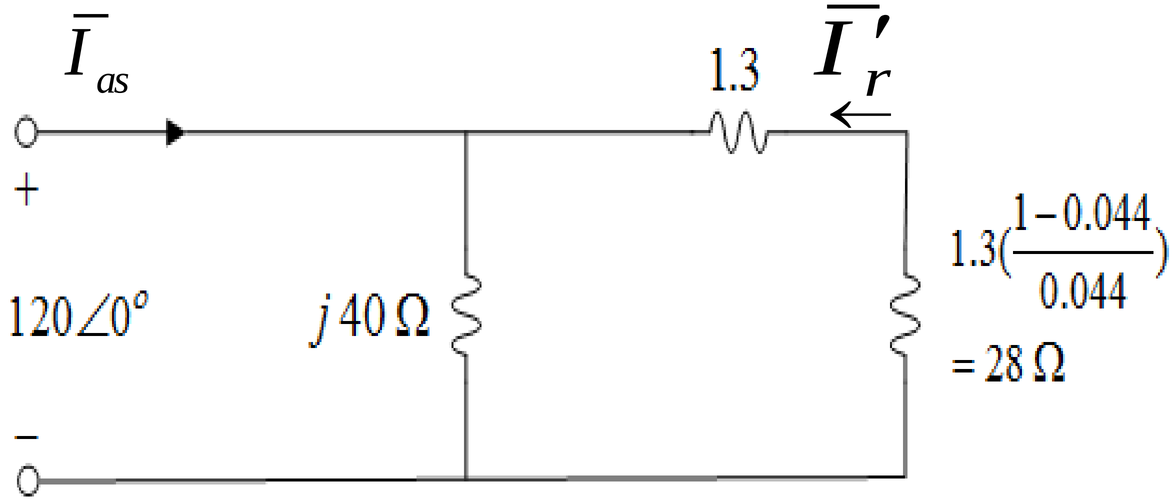
$$X'_{\ell r} = 1\Omega, \quad R'_r = 1.3\ \Omega$$

$$N_{act} = 1720\ rpm. \qquad Neglect\ \ P_c\ \ and\ \ P_{rot}$$

$$Find\ \ s,\ f_r,\ |I'_{ar}|,\ P_m,\ \omega_m,\ T^e,\ P_{ag},\ P_{IN},\ \bar{I}_{as},\ P.F.$$

EXAMPLE

$$N_s = \frac{120 f_s}{P} = \frac{120X\ 60}{4} = 1800$$



$$s = \frac{1800-1720}{1800} = 0.044$$

$$f_r = s\,f_s = 0.044X\ 60 = 2.7\ Hz$$

$$\bar{I}'_r = \frac{-120\angle 0}{29.3 + j\,1} = -4.1\angle 0$$

$$P_m = 3\times 4.1^2 \times 28 = 1412\,W$$

EXAMPLE

$$\omega_{m_{mech}} = 1720 \frac{REV}{MIN}\left(\frac{2\pi R}{60\sec}\right) = 180\ \ R/\sec$$

$$T^{e} = \frac{1420}{180} = 7.8\ \ N-m$$

$$P_{ag} = \frac{1412}{1-0.044} = 1477\,W$$

$$Check\ :\ \ T^{e} = \frac{1477}{2\pi 60(\frac{2}{4})} = 7.8\ \ N-m$$

EXAMPLE

$$P_{IN} = P_{ag} + 3\left|\bar{I}_{as}\right|^2 R_s = 1477 \quad (R_s = 0)$$

$$\bar{I}_{as} = \frac{120\angle 0}{j40} - (-\ 4.1\angle 0) = 4.1 - j3 = 5.1\angle -36^o$$

$$P.F. = \cos(-36^o) = 0.81 \quad \text{lagging}$$

EFFICIENCY

The efficiency is

$$\eta = \frac{P_{out}}{P_{IN}} = \frac{P_m}{P_{IN}} \times 100 = \frac{P_{IN} - (P_{sc\ell} + P_c + P_r)}{P_{IN}} \times 100$$

If we neglect the stator copper loss $P_{sc\ell}$ and the core losses P_c

$$\eta = \frac{mechanical\ powe\ developed}{input\ power}$$

-

$$\eta = \frac{P_m}{P_{ag}} \times 100 = (1-s) \times 00$$

Given the actual speed, it gives a quick idea about the

efficiency of the motor.

EFFICIENCY

If rotational losses P_{rot} are considered, they should be subtracted from P_m to get P_{shaft}

$$P_{shaft} = P_m - P_{rot}$$

The overall efficiency

$$\eta = \frac{P_{shaft}}{P_{IN}} \times 100$$