## ECE 330 POWER CIRCUITS AND ELECTROMECHANICS

# LECTURE 15 ELECTROMECHANICAL SYSTEMS (1)

Acknowledgment-These handouts and lecture notes given in class are based on material from Prof. Peter Sauer's ECE 330 lecture notes. Some slides are taken from Ali Bazi's presentations

Disclaimer- These handouts only provide highlights and should not be used to replace the course textbook.

- Focus on magnetic circuits in which there is a moving member.
- It can be either a translational or rotational system.



Source: clrwtr.com

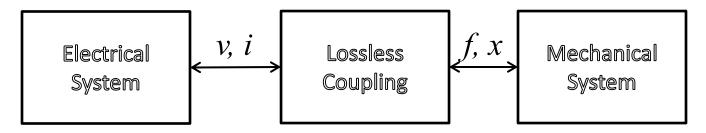


Source: stackexchange.com

The purpose is to present techniques of systematic derivation of mathematical models (differential equations) for lumped-parameter electromechanical systems. The theory is outlined for N electrical coils and M mechanical variables.

- A set of electrical coils that interact to produce the force or torque on the mechanical system.
- The force or torque of electric origin will be derived from fundamental principles of stored energy in the magnetic field.
- both the currents in the electrical coils and the force or torque are time varying.

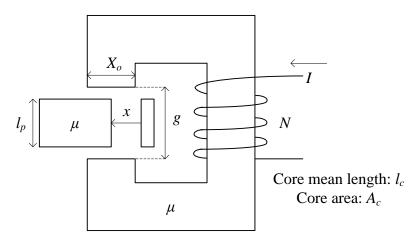
• A basic electromechanical system has an electrical part, a mechanical part, and electromechanical coupling. This coupling is assumed to be lossless.



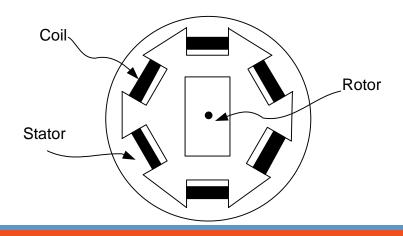
• Systems with linear motion see a force f and a displacement x, while rotational systems see a torque T and a displacement  $\theta$ .

#### **EXAMPLES**

• Linear(translational) motion:



• Rotational motion:



#### FLUX LINKAGE AND VOLTAGE

• The flux linkage  $\lambda$  can be a function of the displacement x and the current i.

• The voltage becomes: 
$$v = \frac{d\lambda}{dt} = \frac{\partial\lambda}{\partial i}\frac{di}{dt} + \frac{\partial\lambda}{\partial x}\frac{dx}{dt}$$
.

• If 
$$\lambda = L(x) i$$
,

• If 
$$\lambda = L(x) i$$
,  $v = \frac{d\lambda}{dt} = L(x) \frac{di}{dt} + i \frac{\partial L(x)}{\partial x} \frac{dx}{dt}$ .

• If 
$$L(x)=L=$$
 constant,  $v=\frac{d\lambda}{dt}=L\frac{di}{dt}$ .

$$v = \frac{d\lambda}{dt} = L\frac{di}{dt}$$
.

#### **MULTI-PORT SYSTEMS**

$$\lambda_{k} = \lambda_{k} (i_{1}, i_{2}, i_{N}, \dots, x_{1}, x_{2}, \dots, x_{N}), \qquad k = 1,\dots, N$$

$$v_{l}, i_{l} \qquad \qquad v_{l}, i_{N} \qquad v_{N}, i_{N} \qquad v_{N} \qquad v_{$$

• The force of electric origin at the mechanical ports can be functions of all the variables.

$$f_{i}^{e} = f_{i}^{e}(i_{1}, i_{2}, .... i_{N}, x_{1}, x_{2}, .... x_{M})$$

#### LINEAR SYSTEM EXAMPLE

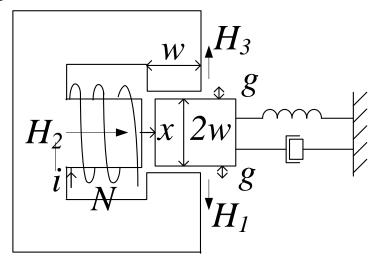
Basic configuration is used for tripping circuit breakers, operating valves, etc. It consists of:

- Fixed structure made of highly permeable material.
- A movable plunger of non magnetic sleeve.

$$B = \mu H$$
 ,  $\mu = \infty$ 

$$H = 0$$
 in the iron

• H exist only in the air gap



#### LINEAR SYSTEM EXAMPLE

### • Ampere's loops:

$$H_{3}g - H_{1}g = 0$$

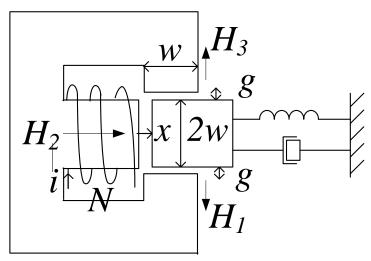
$$H_{2}x + H_{1}g = N i$$

$$\Rightarrow B_{1}A_{1} + B_{3}A_{3} - B_{2}A_{2} = 0$$

$$\Rightarrow 2\mu_{o}H_{1}wd - 2wd \mu_{o}H_{2} = 0$$

$$\Rightarrow H_{1} = H_{2} = \frac{Ni}{g + x}$$

$$\Rightarrow \phi_2 = \mu_o H_2(2wd) = \frac{2wd \,\mu_o Ni}{g + x}$$



d is the depth of the structure

#### LINEAR MOTOR EXAMPLE

$$\lambda = N \phi_2$$

$$\lambda = L i$$

$$\Rightarrow L(x) = \frac{\lambda}{i} = \frac{2w \ d \ \mu_o N^2}{g + x}$$

$$v = L(x)\frac{di}{dt} + i\frac{dL(x)}{dx}\frac{dx}{dt}$$

$$\Rightarrow v = \frac{2w d \mu_o N^2}{g + x} \frac{di}{dt} - i \frac{2w d \mu_o N^2}{(g + x)^2} \frac{dx}{dt}$$

Transformer voltage

Speed voltage

This problem can be easily solved using the magnetic equivalent circuit!

#### READING MATERIAL

• Reading material: Sections 4.1-4.3.

• Next time: Continue Section 4.3.