ECE 330 POWER CIRCUITS AND ELECTROMECHANICS

## LECTURE 15 <br> ELECTROMECHANICAL SYSTEMS (1)

Acknowledgment-These handouts and lecture notes given in class are based on material from Prof. Peter Sauer's ECE 330 lecture notes. Some slides are taken from Ali Bazi's presentations

Disclaimer- These handouts only provide highlights and should not be used to replace the course textbook.

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## ELECTROMECHANICAL SYSTEMS

- Focus on magnetic circuits in which there is a moving member.
- It can be either a translational or rotational system.

- Source: clrwtr.com



## ELECTROMECHANICAL SYSTEMS

The purpose is to present techniques of systematic derivation of mathematical models (differential equations) for lumped-parameter electromechanical systems. The theory is outlined for N electrical coils and M mechanical variables.

## ELECTROMECHANICAL SYSTEMS

- A set of electrical coils that interact to produce the force or torque on the mechanical system.
- The force or torque of electric origin will be derived from fundamental principles of stored energy in the magnetic field.
- both the currents in the electrical coils and the force or torque are time varying.


## ELECTROMECHANICAL SYSTEMS

- A basic electromechanical system has an electrical part, a mechanical part, and electromechanical coupling. This coupling is assumed to be lossless.

- Systems with linear motion see a force $f$ and a displacement $x$, while rotational systems see a torque $T$ and a displacement $\theta$.


## EXAMPLES

- Linear(translational) motion:
- Rotational motion:



## FLUX LINKAGE AND VOLTAGE

- The flux linkage $\lambda$ can be a function of the displacement $x$ and the current $i$.
- The voltage becomes: $v=\frac{d \lambda}{d t}=\frac{\partial \lambda}{\partial i} \frac{d i}{d t}+\frac{\partial \lambda}{\partial x} \frac{d x}{d t}$.
- If $\lambda=L(x) i, \quad v=\frac{d \lambda}{d t}=L(x) \frac{d i}{d t}+i \frac{\partial L(x)}{\partial x} \frac{d x}{d t}$.
- If $L(x)=L=$ constant,$\quad v=\frac{d \lambda}{d t}=L \frac{d i}{d t}$.


## MULTI-PORT SYSTEMS

$$
\lambda_{k}=\lambda_{k}\left(i_{1}, i_{2}, i_{N}, \ldots . x_{1}, x_{2}, \ldots x_{N}\right), \quad k=1 \ldots . . N
$$


$v_{k}=\frac{d \lambda_{k}}{d t}=\sum_{j=1}^{N} \frac{\partial \lambda_{k}}{\partial i_{j}} \frac{d i_{j}}{d t}+\sum_{j=1}^{M} \frac{\partial \lambda_{k}}{\partial x_{j}} \frac{d x_{j}}{d t}$,
$k=1, \ldots . \mathrm{N}$

- The force of electric origin at the mechanical ports can be functions of all the variables.

$$
f_{i}^{e}=f_{i}^{e}\left(i_{1}, i_{2}, \ldots . i_{N}, x_{1}, x_{2}, \ldots . x_{M}\right)
$$

## LINEAR SYSTEM EXAMPLE

Basic configuration is used for tripping circuit breakers, operating valves, etc. It consists of:

- Fixed structure made of highly permeable material.
- A movable plunger of non magnetic sleeve.

$$
\begin{aligned}
& B=\mu H \quad, \mu=\infty \\
& H=0 \text { in the iron }
\end{aligned}
$$

- H exist only in the air gap



## LINEAR SYSTEM EXAMPLE

- Ampere's loops:

$$
\begin{aligned}
& H_{3} g-H_{1} g=0 \\
& H_{2} x+H_{1} g=N i \\
& \Rightarrow B_{1} A_{1}+B_{3} A_{3}-B_{2} A_{2}=0 \\
& \Rightarrow 2 \mu_{o} H_{1} w d-2 w d \mu_{o} H_{2}=0 \\
& \Rightarrow H_{1}=H_{2}=\frac{N i}{g+x}
\end{aligned}
$$


$d$ is the depth of the structure

$$
\Rightarrow \phi_{2}=\mu_{o} H_{2}(2 w d)=\frac{2 w d \mu_{o} N i}{g+x}
$$

## LINEAR MOTOR EXAMPLE

$$
\begin{aligned}
& \lambda=N \phi_{2} \\
& \lambda=L i \\
& \Rightarrow L(x)=\frac{\lambda}{i}=\frac{2 w d \mu_{o} N^{2}}{g+x} \\
& \nu=L(x) \frac{d i}{d t}+i \frac{d L(x)}{d x} \frac{d x}{d t} \\
& \Rightarrow v=\frac{2 w d \mu_{0} N^{2}}{g+x} \frac{d i}{d t}-i \underbrace{2 w d \mu_{o} N^{2}}_{\begin{array}{l}
\text { Transformer } \\
\text { voltage }
\end{array}} \frac{d x}{(g+x)^{2}} \frac{d x}{d t}
\end{aligned}
$$

This problem can be easily solved using the magnetic equivalent circuit !

## READING MATERIAL

- Reading material: Sections 4.1-4.3.
- Next time: Continue Section 4.3.

