ECE329: Tutorial Session 9

October 29th, 2024

Maxwell’s Wave Equation

Assumptions: $\rho = 0, \sigma = 0, \vec{J} = 0$, i.e. region is a source-free perfect dielectric.

Wave equation: $\nabla^2 \vec{E} = \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$ OR $\nabla^2 \tilde{E} = -\omega^2 \mu\epsilon \tilde{E}$. Solutions are TEM waves.

$\vec{E}$ and $\vec{H}$ point **perpendicular** to the direction of travel.

General form of cosinusoidal solution:

Wave Equation in Material Media

Assumptions: $\rho = 0$, $\vec{J} = 0$, i.e. region is a source-free. $\sigma \neq 0$ now!

Wave equation: $\nabla^2 \tilde{E} = (j\omega\mu)(\sigma + j\omega\epsilon)\tilde{E}$. Solutions are TEM waves.
$\vec{E}$ and $\vec{H}$ point **perpendicular** to the direction of travel.

General form of cosinusoidal solution:

The Big Table

	Condition	β	α	$\lvert\eta\rvert$	τ	$\lambda = \frac{2\pi}{\beta}$	$\delta = \frac{1}{\alpha}$
Perfect dielectric	$\sigma = 0$	$\omega\sqrt{\epsilon\mu}$	0	$\sqrt{\frac{\mu}{\epsilon}}$	0	$\frac{2\pi}{\omega\sqrt{\epsilon\mu}}$	∞
Imperfect dielectric	$\frac{\sigma}{\omega\epsilon} \ll 1$	$\sim \omega\sqrt{\epsilon\mu}$	$\beta\frac{1}{2}\frac{\sigma}{\omega\epsilon} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$	$\sim \sqrt{\frac{\mu}{\epsilon}}$	$\sim \frac{\sigma}{2\omega\epsilon}$	$\sim \frac{2\pi}{\omega\sqrt{\epsilon\mu}}$	$\frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}$
Good conductor	$\frac{\sigma}{\omega\epsilon} \gg 1$	$\sim \sqrt{\pi f\mu\sigma}$	$\sim \sqrt{\pi f\mu\sigma}$	$\sqrt{\frac{\omega\mu}{\sigma}}$	$45°$	$\sim \frac{2\pi}{\sqrt{\pi f\mu\sigma}}$	$\sim \frac{1}{\sqrt{\pi f\mu\sigma}}$
Perfect conductor	$\sigma = \infty$	∞	∞	0	-	0	0

Problem 1

Which of the following are true?

1. For a wave propagating in a good conductor, the wave amplitude decays to less than 1% of its initial value after traveling only a few wavelengths.
2. Wave propagation velocity never depends on frequency.
3. Skin depth is infinite for perfect conductors.
4. Low frequencies are preferable to high frequencies in applications requiring propagating through conductors.
5. Perfect dielectric is an example of a lossy media.
6. E and H are out of phase when propagating through imperfect dielectrics.

Problem 2

A wave is propagating through a material with $\sigma = 1, \mu = \mu_0, \epsilon = \epsilon_0$.

- For what (angular) frequencies does the material act like an imperfect dielectric? For what (angular) frequencies does the material act like a good conductor?
- It is discovered that $\vec{E}(0,t) = 4\cos(2 * 10^8 t)\,\hat{x}$ and that the wave is travelling in the y-direction. Find: $\vec{E}(x,y,z,t)$, $\vec{H}(x,y,z,t)$, $\tilde{E}(x,y,z)$, $\tilde{H}(x,y,z)$, λ, v_p, $< \vec{S} >$
- Find the distance it takes for the wave's amplitude to decay by 95%. $e^{-3} \approx 0.05$.

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Wave Polarization

Polarization is how the tip of E varies over time.

Linear: In phase
Circular: 90 degrees out-of-phase with equal magnitude
- Right-Handed
- Left-Handed

Elliptical: Anything else

Wave Polarization

Example of linear polarization: $\vec{\mathrm{E}} = 3\cos(\omega t - \beta z)\hat{x}$

Example of right-handed circular polarization (RHCP):

$$\vec{\mathrm{E}} = 3\cos(\omega t - \beta z)\,\hat{x} + 3\cos\left(\omega t - \beta z - \frac{\pi}{2}\right)\hat{y}$$

Example of left-handed circular polarization (LHCP):

$$\vec{\mathrm{E}} = 3\cos(\omega t - \beta z)\,\hat{x} - 3\cos\left(\omega t - \beta z - \frac{\pi}{2}\right)\hat{y}$$

Wave Polarization

Example of elliptical polarization:

$$\vec{\mathrm{E}} = 3\cos(\omega t - \beta z)\,\hat{x} + \cos\left(\omega t - \beta z - \frac{\pi}{2}\right)\hat{y}$$

Example of elliptical polarization:

$$\vec{\mathrm{E}} = 3\cos(\omega t - \beta z)\,\hat{x} + 3\cos\left(\omega t - \beta z - \frac{\pi}{3}\right)\hat{y}$$

What about this?

$$\vec{\mathrm{E}} = 2\cos(\omega t - \beta z)\,\hat{x} + 4\cos(\omega t - \beta z)\hat{y}$$

Problem 3

We have a wave propagating in the z-direction such that the x-direction and y-direction waves are 270 degrees out of phase. What polarization do they have?

We have a wave propagating in the z-direction such that the x-direction and the y-direction waves have the same amplitude. What polarization do they have?

We have a wave propagating in the z-direction such that the x-direction and the y-direction waves have unequal amplitude but equal phase. What's the polarization?

Problem 4

Assuming free space conditions, determine:

- The phasor electric field
- The phasor magnetic field
- The electric field wave polarization
- The magnetic field wave polarization

for the following electric field wave: $\vec{E} = 5\cos(\omega t - \beta y)\,\hat{x} + 5\sin(\omega t - \beta y)\hat{z}$ V/m.

Problem 4: Blank slide

Assuming free space conditions, determine:

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- The phasor magnetic field
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Midterm 1 equations, in one place

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{F} = q_1\vec{E} + q_1(\vec{v}_1 \times \vec{B})$$

$$\vec{E} = \frac{q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\epsilon \oiint \vec{E}\cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\oiint \vec{D}\cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\iiint \rho dV = Q_{\text{enclosed}}$$

$$\oiint \vec{B}\cdot d\vec{S} = 0$$

$$I = \oiint \vec{J}\cdot d\vec{S} = -\frac{\partial Q_{\text{enclosed}}}{\partial t}$$

$$\nabla \times \vec{E} = 0$$

$$\vec{E} = -\nabla V$$

$$\oint \vec{E}\cdot d\vec{l} = 0$$

$$V_{ab} = V(b) - V(a) = -\int_a^b \vec{E}\cdot d\vec{l}$$

$$\hat{n}\cdot(\vec{D}_1 - \vec{D}_2) = \rho_s$$

$$\hat{n}\times(\vec{E}_1 - \vec{E}_2) = 0$$

$$\hat{n}\cdot(\vec{P}_1 - \vec{P}_2) = -\rho_{b,s}$$

$$\epsilon = \epsilon_0(1 + \chi_e)$$

$$\vec{P} = \epsilon_0\chi_e\vec{E}$$

$$\vec{D} = \epsilon_0\vec{E} + \vec{P} = \epsilon\vec{E}$$

$$\vec{J} = \sigma\vec{E}$$

$$\rho_b = -\nabla\cdot\vec{P}$$

$$\nabla\cdot\epsilon_0\vec{E} = \rho_f + \rho_b$$

$$\nabla\cdot\vec{D} = \rho$$

$$\nabla\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$$

$$-\nabla^2 V = \frac{\rho}{\epsilon}$$

$$\oiint \vec{D}\cdot d\vec{S} = \iiint \nabla\cdot\vec{D}dV$$

$$\oint \vec{E}\cdot d\vec{l} = \iint (\nabla\times\vec{E})\cdot d\vec{S}$$

$$\int_a^b \nabla V\cdot d\vec{l} = V(b) - V(a)$$

Midterm 2 equations, in one place

$$\vec{B} = \frac{\mu I}{2\pi r}\hat{\phi}$$

$$d\vec{B} = \frac{\mu I d\vec{\ell} \times \hat{r}}{4\pi r^2}$$

$$\oint_C \vec{H} \cdot d\vec{\ell} = \iint_S \vec{J} \cdot d\vec{S}$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu I_{\text{encl}}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\Psi = \iint_s \vec{B} \cdot d\vec{S}$$

$$-\frac{d}{dt}\iint_s \vec{B} \cdot d\vec{S} = \oint_c \vec{E} \cdot d\vec{l}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint_c \vec{E} \cdot d\vec{l} = \varepsilon$$

$$\varepsilon = \frac{W}{q} = \oint_C \frac{\vec{F}}{q} \cdot d\vec{l}$$

$$\Psi = LI$$

$$\varepsilon = IR$$

$$v = \frac{\omega}{\beta} = \lambda f = \frac{1}{\sqrt{\mu\epsilon}}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$\eta = \frac{\sqrt{\mu}}{\sqrt{\epsilon}}$$

$$\beta = \omega\sqrt{\mu\epsilon}$$

$$\nabla^2 \vec{E} = \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\vec{S} = \vec{E} \times \vec{H}$$

$$\tilde{S} = \tilde{E} \times \tilde{H}^*$$

$$< \vec{S} > = \frac{1}{2}\text{Re}\{\tilde{E} \times \tilde{H}^*\}$$

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon\vec{E} \cdot \vec{E} + \frac{1}{2}\mu\vec{H} \cdot \vec{H}\right) + \nabla \cdot \vec{S} + \vec{J} \cdot \vec{E} = 0$$

$$Q = CV$$

$$G = \frac{\sigma}{\epsilon}C \qquad R = \frac{1}{G}$$

$$\vec{J}_b = \frac{\partial \vec{P}}{\partial t} + \nabla \times \vec{M}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\vec{M} = \chi_m \vec{H}$$

$$\vec{B} = \mu_0\mu_r\vec{H} = \mu\vec{H}$$

$$A\cos(\omega t - \beta x)\hat{z} \leftrightarrow Ae^{-j\beta x}$$

$$\hat{n} \cdot (\vec{B}_1 - \vec{B}_2) = 0$$

$$\hat{n} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s$$

$$\hat{n} \times (\vec{M}_1 - \vec{M}_2) = \vec{J}_{b,s}$$

Midterm 3 equations, in one place

	Condition	β	α	$\lvert\eta\rvert$	τ	$\lambda = \frac{2\pi}{\beta}$	$\delta = \frac{1}{\alpha}$
Perfect dielectric	$\sigma = 0$	$\omega\sqrt{\epsilon\mu}$	0	$\sqrt{\frac{\mu}{\epsilon}}$	0	$\frac{2\pi}{\omega\sqrt{\epsilon\mu}}$	∞
Imperfect dielectric	$\frac{\sigma}{\omega\epsilon} \ll 1$	$\sim \omega\sqrt{\epsilon\mu}$	$\beta\frac{1}{2}\frac{\sigma}{\omega\epsilon} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$	$\sim \sqrt{\frac{\mu}{\epsilon}}$	$\sim \frac{\sigma}{2\omega\epsilon}$	$\sim \frac{2\pi}{\omega\sqrt{\epsilon\mu}}$	$\frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}$
Good conductor	$\frac{\sigma}{\omega\epsilon} \gg 1$	$\sim \sqrt{\pi f \mu\sigma}$	$\sim \sqrt{\pi f \mu\sigma}$	$\sqrt{\frac{\omega\mu}{\sigma}}$	$45°$	$\sim \frac{2\pi}{\sqrt{\pi f \mu\sigma}}$	$\sim \frac{1}{\sqrt{\pi f \mu\sigma}}$
Perfect conductor	$\sigma = \infty$	∞	∞	0	-	0	0

$$v = \frac{\omega}{\beta} = \lambda f \qquad \nabla^2 \tilde{E} = (j\omega\mu)(\sigma + j\omega\epsilon)\tilde{E}$$

Units

Charge Q: C
Current I: A
Electric field strength $\vec{E}$: N/C or V/m
Electric flux density $\vec{D}$: C/m^2
Polarization field $\vec{P}$: C/m^2
Electric potential V: V
Capacitance C: F
Magnetic flux density $\vec{B}$: T or Wb/m^2
Magnetic field strength $\vec{H}$: A/m
Magnetic flux Ψ: Wb
Electromotive force ε: V
Inductance L: H

Electric permittivity ϵ: F/m
Magnetic permeability μ: H/m
Conductivity σ: Si/m

Charge density ρ: C/m^3
Surface charge density ρ_s: C/m^2
Current density $\vec{J}$: A/m^2

Intrinsic impedance η: Ohm
Wave number β: rad/m

Office Hours

Any questions?