ECE329: Tutorial Session 8

October 22nd, 2024

Share any thoughts on anything!

Maxwell's Wave Equation

Assumptions: $\rho = 0, \sigma = 0, \vec{J} = 0$, i.e. region is a source-free perfect dielectric.

Wave equation: $\nabla^2 \vec{E} = \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$. Solutions are 'TEM waves.'

TEM = Transverse ElectroMagnetic.

$\vec{E}$ and $\vec{H}$ point **perpendicular** to the direction of travel.

- Why can't they point in the direction of travel?

General form of cosinusoidal solution:

Wave Formulae

$$v = \frac{\omega}{\beta} = \lambda f = \frac{1}{\sqrt{\mu\epsilon}}$$

$$\beta = \omega\sqrt{\mu\epsilon}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$\eta = \frac{\sqrt{\mu}}{\sqrt{\epsilon}}$$

Getting E from H and vice versa

Poynting Vector & Theorem

$\vec{S} = \vec{E} \times \vec{H}$. Units: W/m^2. "Instantaneous" power per unit area passing through surface in direction of $\vec{S}$

Energy unit volume balance equation:

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon\vec{E}\cdot\vec{E}+\frac{1}{2}\mu\vec{H}\cdot\vec{H}\right)+\nabla\cdot\vec{S}+\vec{J}\cdot\vec{E}=0$$

Phasor Notation

$$\vec{E}(x,t) = A\cos(\omega t - \beta x)\hat{z} = \mathrm{Re}\left\{Ae^{j\omega t}e^{-j\beta x}\right\} \leftrightarrow Ae^{-j\beta x} = \tilde{E}(x)$$

Time domain

Phasor

$$\sin(x) = \cos\left(x - \frac{\pi}{2}\right)$$

Time-Averaging

If TEM wave $\vec{E}$ is cosinusoidal,

- Then corresponding $\vec{H}$ is cosinusoidal
- Then phasors $\tilde{E}$ and $\widetilde{H}$ exist
- Also, $\vec{S} = \vec{E} \times \vec{H}$ (instantaneous power per unit area) is cosinusoidal squared
- We can write $\vec{S}$ in phasor form too: $\tilde{S} = \tilde{E} \times \widetilde{H}^*$
- Time-average of cosinusoidal squared is ½ of the magnitude of the cosinusoidal:

$$< \vec{S} > = \frac{1}{2}\mathrm{Re}\{\tilde{E} \times \widetilde{H}^*\}$$

Problem 1

What is the time derivative of a phasor?
What are Maxwell's Equations in phasor form?

Problem 2

Given a uniform TEM wave with $\vec{E}(x,t) = -3\cos(\omega t + \beta x)\hat{z}$ V/m with $f = 100\text{MHz}$ and $v = \frac{2}{3}c$ propagating through a homogenous medium with $\epsilon = 1.5\epsilon_0$, find $\widetilde{\text{E}}(x), \mu_r, \lambda, \beta, \vec{H}(x,t), \widetilde{H}(x)$, instantaneous power per unit area crossing $x = 1$ in $+\hat{x}$ direction, and time-averaged power per unit area crossing $x = 1$ in $+\hat{x}$ direction.

Radiation from Current Sheets

Static $\vec{J}_s$ on a current sheet produces static $\vec{H}$ and no $\vec{E}$, as seen before.

Time-varying $\vec{J}_s$ produces time-varying $\vec{H}$, with changes in $\vec{J}_s$ causing changes in $\vec{H}$ propagating out of the current sheet.
To satisfy Faraday's Law, time-varying $\vec{H}$ must produce accompanying time-varying $\vec{E}$.

Time-varying current sheets produce TEM waves propagating outward!

Problem 3

On the $x = 0$ plane a current sheet exists carrying $\vec{J}_s = 2t\big(u(t) - u(t-2)\big)\hat{z}$ A/m. Regions adjacent to the sheet are vacuum.

1. Plot $\vec{J}_s(t)$.
2. Determine and plot nonzero components of $\vec{E}$ and $\vec{H}$ against t at $z = 9 * 10^8$m.
3. Determine and plot nonzero components of $\vec{E}$ and $\vec{H}$ against t at $z = -9 * 10^8$m.
4. Determine and plot nonzero components of $\vec{E}$ and $\vec{H}$ against z at $t = 4$s.
5. Determine and plot nonzero components of $\vec{E}$ and $\vec{H}$ against z at $t = 1$s.

Midterm 1 equations, in one place

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{F} = q_1\vec{E} + q_1(\vec{v}_1 \times \vec{B})$$

$$\vec{E} = \frac{q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\epsilon \oiint \vec{E} \cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\oiint \vec{D} \cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\iiint \rho dV = Q_{\text{enclosed}}$$

$$\oiint \vec{B} \cdot d\vec{S} = 0$$

$$I = \oiint \vec{J} \cdot d\vec{S} = -\frac{\partial Q_{\text{enclosed}}}{\partial t}$$

$$\nabla \times \vec{E} = 0$$

$$\vec{E} = -\nabla V$$

$$\oint \vec{E} \cdot d\vec{l} = 0$$

$$V_{ab} = V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{l}$$

$$\hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s$$

$$\hat{n} \times (\vec{E}_1 - \vec{E}_2) = 0$$

$$\hat{n} \cdot (\vec{P}_1 - \vec{P}_2) = -\rho_{b,s}$$

$$\epsilon = \epsilon_0(1 + \chi_e)$$

$$\vec{P} = \epsilon_0 \chi_e \vec{E}$$

$$\vec{D} = \epsilon_0\vec{E} + \vec{P} = \epsilon\vec{E}$$

$$\vec{J} = \sigma\vec{E}$$

$$\rho_b = -\nabla \cdot \vec{P}$$

$$\nabla \cdot \epsilon_0\vec{E} = \rho_f + \rho_b$$

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

$$-\nabla^2 V = \frac{\rho}{\epsilon}$$

$$\oiint \vec{D} \cdot d\vec{S} = \iiint \nabla \cdot \vec{D} dV$$

$$\oint \vec{E} \cdot d\vec{l} = \iint (\nabla \times \vec{E}) \cdot d\vec{S}$$

$$\int_a^b \nabla V \cdot d\vec{l} = V(b) - V(a)$$

Midterm 2 equations, in one place

$$\vec{B} = \frac{\mu I}{2\pi r}\hat{\phi}$$

$$d\vec{B} = \frac{\mu I d\vec{\ell} \times \hat{r}}{4\pi r^2}$$

$$\oint_C \vec{H} \cdot d\vec{\ell} = \iint_S \vec{J} \cdot d\vec{S}$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu I_{\text{encl}}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\Psi = \iint_s \vec{B} \cdot d\vec{S}$$

$$-\frac{d}{dt}\iint_s \vec{B} \cdot d\vec{S} = \oint_c \vec{E} \cdot d\vec{l}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint_c \vec{E} \cdot d\vec{l} = \varepsilon$$

$$\varepsilon = \frac{W}{q} = \oint_C \frac{\vec{F}}{q} \cdot d\vec{l}$$

$$\Psi = LI$$

$$\varepsilon = IR$$

$$v = \frac{\omega}{\beta} = \lambda f = \frac{1}{\sqrt{\mu\epsilon}}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$\eta = \frac{\sqrt{\mu}}{\sqrt{\epsilon}}$$

$$\beta = \omega\sqrt{\mu\epsilon}$$

$$\nabla^2 \vec{E} = \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\vec{S} = \vec{E} \times \vec{H}$$

$$\tilde{S} = \tilde{E} \times \tilde{H}^*$$

$$< \vec{S} > = \frac{1}{2}\text{Re}\{\tilde{E} \times \tilde{H}^*\}$$

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon\vec{E} \cdot \vec{E} + \frac{1}{2}\mu\vec{H} \cdot \vec{H}\right) + \nabla \cdot \vec{S} + \vec{J} \cdot \vec{E} = 0$$

$$Q = CV$$

$$G = \frac{\sigma}{\epsilon}C \qquad R = \frac{1}{G}$$

$$\vec{J}_b = \frac{\partial \vec{P}}{\partial t} + \nabla \times \vec{M}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\vec{M} = \chi_m \vec{H}$$

$$\vec{B} = \mu_0 \mu_r \vec{H} = \mu \vec{H}$$

$$A\cos(\omega t - \beta x)\hat{z} \leftrightarrow Ae^{-j\beta x}$$

$$\hat{n} \cdot (\vec{B}_1 - \vec{B}_2) = 0$$

$$\hat{n} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s$$

$$\hat{n} \times (\vec{M}_1 - \vec{M}_2) = \vec{J}_{b,s}$$

Units

Charge Q: C
Current I: A
Electric field strength $\vec{E}$: N/C or V/m
Electric flux density $\vec{D}$: C/m^2
Polarization field $\vec{P}$: C/m^2
Electric potential V: V
Capacitance C: F
Magnetic flux density $\vec{B}$: T or Wb/m^2
Magnetic field strength $\vec{H}$: A/m
Magnetic flux Ψ: Wb
Electromotive force ε: V
Inductance L: H

Electric permittivity ϵ: F/m
Magnetic permeability μ: H/m
Conductivity σ: Si/m

Charge density ρ: C/m^3
Surface charge density ρ_s: C/m^2
Current density $\vec{J}$: A/m^2

Intrinsic impedance η: Ohm
Wave number β: rad/m

Office Hours

Any questions?

Share any thoughts on anything!