ECE329: Tutorial Session 7

October 15th, 2024

Share any thoughts on anything!

Last week: Electrodynamics

D and E	B and H
$\oiint_S \vec{D} \cdot d\vec{S} = \iiint_V \rho dV$ $\nabla \cdot \vec{D} = \rho$	$\oiint_S \vec{B} \cdot d\vec{S} = 0$ $\nabla \cdot \vec{B} = 0$
$\oint_C \vec{E} \cdot d\vec{\ell} = -\frac{\partial}{\partial t}\iint_S \vec{B} \cdot d\vec{S}$ $\nabla \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}$	$\oint_C \vec{H} \cdot d\vec{\ell} = \iint_S \vec{J} \cdot d\vec{S}$ $\nabla \times \vec{H} = \vec{\mathrm{J}}$

Maxwell's Correction

$$\nabla \times \vec{H} = \vec{\mathrm{J}}$$

Bound Current & Magnetization Field

Just like we had bound charges, we also have bound currents.
- Created by electrons 'orbiting' a nucleus
- Created by intrinsic spin

Just like we had polarization field, we also have magnetization field.
Magnetization field is divergence-free, much like B-fields.

Just like we had electric susceptibility χ_e, we have magnetic susceptibility χ_m.

What do you need to know?

Bound Current & Magnetization Field

$$\vec{J}_b = \frac{\partial \vec{P}}{\partial t} + \nabla \times \vec{M}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\vec{M} = \chi_m \vec{H}$$

$$\vec{B} = \mu_0(1 + \chi_m)\vec{H} = \mu_0 \mu_r \vec{H} = \mu \vec{H}$$

Material Characterizations

1. Diamagnetic: $\chi_m < 0$.
No permanent magnetic field.
Any applied $\vec{B}$ induces a small $\vec{M}$ pointing against $\vec{B}$.

2. Paramagnetic: $\chi_m > 0$.
Any applied $\vec{B}$ induces a small $\vec{M}$ that reinforces $\vec{B}$.

3. Ferromagnetic: $\chi_m \gg 1$.
Permanent magnets

Maxwell's Equations

Free space

Material Media

Maxwell’s Equations

Everywhere and always

Boundary Conditions

$$\hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s$$

$$\hat{n} \cdot (\vec{B}_1 - \vec{B}_2) = 0$$

$$\hat{n} \cdot (\vec{P}_1 - \vec{P}_2) = -\rho_{b,s}$$

$$\hat{n} \times (\vec{E}_1 - \vec{E}_2) = 0$$

$$\hat{n} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s$$

$$\hat{n} \times (\vec{M}_1 - \vec{M}_2) = \vec{J}_{b,s}$$

Problem 1

Eric sits in a soup with $\mu = 2\mu_0, \epsilon = 8\epsilon_0$ in the $+x$ volume and radiates, with his beautiful vocal cords, $\vec{E} = 4\hat{x} + 4\hat{y} + 4\hat{z}$ V/m and $\vec{B} = 2\hat{x} + 2\hat{y} + 2\hat{z}$ Wb/m^2. Sasha sits in free space in the $-x$ volume and stares at Eric through a plasma TV screen carrying a surface charge $5\epsilon_0$ C/m^2 and surface current $\frac{5}{\mu_0}\hat{y}$ A/m. Find $\vec{E}$ and $\vec{B}$ in the $-x$ volume and find $\rho_{b,s}$ and $\vec{J}_{b,s}$ on the TV screen.

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Maxwell’s Wave Equation

Assumptions: $\rho = 0, \sigma = 0, \vec{J} = 0$, i.e. region is a source-free perfect dielectric.

Wave equation: $\nabla^2 \vec{E} = \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$. Solutions are ‘EM waves.’

General form of cosinusoidal solution:

Wave Formulae

$$v = \frac{\omega}{\beta} = \lambda f = \frac{1}{\sqrt{\mu\epsilon}}$$

$$\beta = \omega\sqrt{\mu\epsilon}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$\eta = \frac{\sqrt{\mu}}{\sqrt{\epsilon}}$$

Problem 2: Two imposters among us...

Which two of the following are not a valid solution to the wave equation?

1. $\vec{E} = 5\cos(3t + 4z)\,\hat{x}$
2. $\vec{E} = 5\cos(3t + 4x)\,\hat{x}$
3. $\vec{E} = 3\hat{x}$
4. $\vec{E} = \sin(5t + 4r)\,\hat{y}$
5. $\vec{E} = \sin(5t + 3z)\,\hat{y}$
6. $\vec{E} = \sin(5t - 3x)\,\hat{z} + \sin(4t - 3y)\,\hat{x}$

TEM Waves

TEM = Transverse electromagnetic.

$\vec{E}$ and $\vec{H}$ point **perpendicular** to the direction of travel.

- Why can't they point in the direction of travel?

Midterm 1 equations, in one place

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{F} = q_1\vec{E} + q_1(\vec{v}_1 \times \vec{B})$$

$$\vec{E} = \frac{q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\hat{n}\cdot\left(\vec{D}_1 - \vec{D}_2\right) = \rho_s$$

$$\hat{n}\times\left(\vec{E}_1 - \vec{E}_2\right) = 0$$

$$\hat{n}\cdot\left(\vec{P}_1 - \vec{P}_2\right) = -\rho_{b,s}$$

$$\epsilon \oiint \vec{E}\cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\oiint \vec{D}\cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\iiint \rho dV = Q_{\text{enclosed}}$$

$$\oiint \vec{B}\cdot d\vec{S} = 0$$

$$I = \oiint \vec{J}\cdot d\vec{S} = -\frac{\partial Q_{\text{enclosed}}}{\partial t}$$

$$\epsilon = \epsilon_0(1+\chi_e)$$

$$\vec{P} = \epsilon_0\chi_e\vec{E}$$

$$\vec{D} = \epsilon_0\vec{E} + \vec{P} = \epsilon\vec{E}$$

$$\vec{J} = \sigma\vec{E}$$

$$\rho_b = -\nabla\cdot\vec{P}$$

$$\nabla\cdot\epsilon_0\vec{E} = \rho_f + \rho_b$$

$$\nabla\cdot\vec{D} = \rho$$

$$\nabla\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$$

$$-\nabla^2 V = \frac{\rho}{\epsilon}$$

$$\nabla\times\vec{E} = 0$$

$$\vec{E} = -\nabla V$$

$$\oint \vec{E}\cdot d\vec{l} = 0$$

$$V_{ab} = V(b) - V(a) = -\int_a^b \vec{E}\cdot d\vec{l}$$

$$\oiint \vec{D}\cdot d\vec{S} = \iiint \nabla\cdot\vec{D}dV$$

$$\oint \vec{E}\cdot d\vec{l} = \iint \left(\nabla\times\vec{E}\right)\cdot d\vec{S}$$

$$\int_a^b \nabla V\cdot d\vec{l} = V(b) - V(a)$$

Midterm 2 equations, in one place

$$\vec{B} = \frac{\mu I}{2\pi r}\hat{\phi}$$

$$d\vec{B} = \frac{\mu I d\vec{\ell} \times \hat{r}}{4\pi r^2}$$

$$\oint_C \vec{H} \cdot d\vec{\ell} = \iint_S \vec{J} \cdot d\vec{S}$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu I_{\text{encl}}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\Psi = \iint_s \vec{B} \cdot d\vec{S}$$

$$-\frac{d}{dt}\iint_s \vec{B} \cdot d\vec{S} = \oint_c \vec{E} \cdot d\vec{l}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint_c \vec{E} \cdot d\vec{l} = \varepsilon$$

$$\varepsilon = \frac{W}{q} = \oint_C \frac{\vec{F}}{q} \cdot d\vec{l}$$

$$\Psi = LI$$

$$\varepsilon = IR$$

$$v = \frac{\omega}{\beta} = \lambda f = \frac{1}{\sqrt{\mu\epsilon}}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$\eta = \frac{\sqrt{\mu}}{\sqrt{\epsilon}}$$

$$\beta = \omega\sqrt{\mu\epsilon}$$

$$\nabla^2 \vec{E} = \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\hat{n} \cdot (\vec{B}_1 - \vec{B}_2) = 0$$

$$\hat{n} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s$$

$$\hat{n} \times (\vec{M}_1 - \vec{M}_2) = \vec{J}_{b,s}$$

$$Q = CV$$

$$G = \frac{\sigma}{\epsilon}C \qquad R = \frac{1}{G}$$

$$\vec{J}_b = \frac{\partial \vec{P}}{\partial t} + \nabla \times \vec{M}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\vec{M} = \chi_m \vec{H}$$

$$\vec{B} = \mu_0 \mu_r \vec{H} = \mu \vec{H}$$

Units

Charge Q: C
Current I: A
Electric field strength $\vec{E}$: N/C or V/m
Electric flux density $\vec{D}$: C/m^2
Polarization field $\vec{P}$: C/m^2
Electric potential V: V
Capacitance C: F
Magnetic flux density $\vec{B}$: T or Wb/m^2
Magnetic field strength $\vec{H}$: A/m
Magnetic flux Ψ: Wb
Electromotive force ε: V
Inductance L: H

Electric permittivity ϵ: F/m
Magnetic permeability μ: H/m
Conductivity σ: Si/m

Charge density ρ: C/m^3
Surface charge density ρ_s: C/m^2
Current density $\vec{J}$: A/m^2

Intrinsic impedance η: Ohm
Wave number β: rad/m

Office Hours

Any questions?

Share any thoughts on anything!