ECE329: Tutorial Session 6

October 8th, 2024

Share any thoughts on anything!

Last week: Statics

Electrostatics	Magnetostatics
$\oint\!\!\!\oint_S \vec{D} \cdot d\vec{S} = \iiint_V \rho dV$ $\nabla \cdot \vec{D} = \rho$	$\oint\!\!\!\oint_S \vec{B} \cdot d\vec{S} = 0$ $\nabla \cdot \vec{B} = 0$
$\oint_C \vec{E} \cdot d\vec{\ell} = 0$ $\nabla \times \vec{E} = 0$	$\oint_C \vec{H} \cdot d\vec{\ell} = \iint_S \vec{J} \cdot d\vec{S}$ $\nabla \times \vec{H} = \vec{\mathrm{J}}$

This week: Dynamics!

$$-\frac{d\Psi}{dt} = -\frac{d}{dt}\iint_s \vec{B}\cdot d\vec{S} = \oint_c \vec{E}\cdot d\vec{l} = \varepsilon$$

Ψ is **magnetic flux** (notice that we integrated magnetic flux density $\vec{B}$ [Wb/m^2] to get magnetic flux). Units: [Wb]

In electrodynamics, $\vec{E}$ is no longer curl-free/path-independent/conservative; integration over a closed loop yields a nonzero number.

ε is **electromotive force** (or emf). Units: [V]

Electromotive 'force'

$$-\frac{d\Psi}{dt} = -\frac{d}{dt}\iint_S \vec{B} \cdot d\vec{S} = \oint_C \vec{E} \cdot d\vec{l} = \varepsilon$$

ε is **electromotive force** (or emf). Units: [V]
It is not a force.

It is the electromagnetic work done to move a unit electric charge once around the closed loop.

$$\varepsilon = \frac{W}{q} = \oint_C \frac{\vec{F}}{q} \cdot d\vec{l}$$

$$\varepsilon = IR$$

Faraday's Law

$$\oint_c \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \iint_s \vec{B} \cdot d\vec{S}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Problem 1

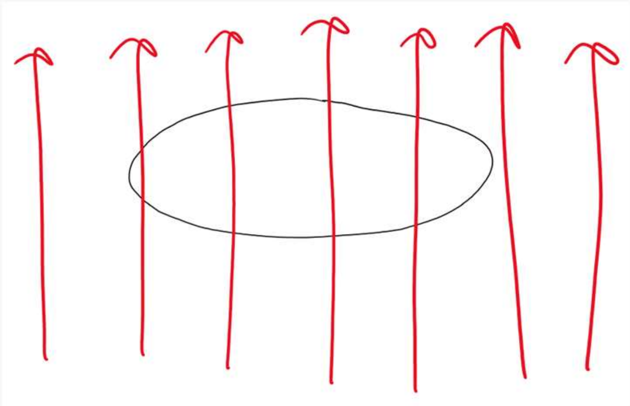
$$-\frac{d\Psi}{dt} = -\frac{d}{dt}\iint_{s} \vec{B} \cdot d\vec{S} = \oint_{c} \vec{E} \cdot d\vec{l} = \varepsilon$$

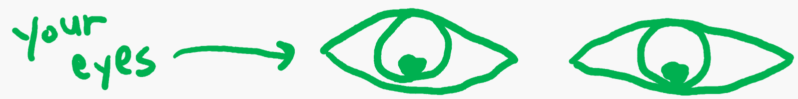
Express the unit [Wb] using [V] and other SI quantities.

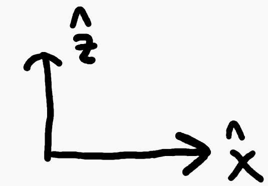
Problem 2

A circular loop is immersed in a uniform magnetic field such that the plane of the loop is perpendicular to the direction of $\vec{B}$. Which of the following will create a nonzero emf in the loop?

1. The coil is rotated about an axis perpendicular to $\vec{B}$ with angular frequency ω.
2. The coil is moved upwards with velocity v.
3. The coil is moved to the left with velocity v.
4. The coil is deformed into a square.
5. The $\vec{B}$ field intensity is increased.

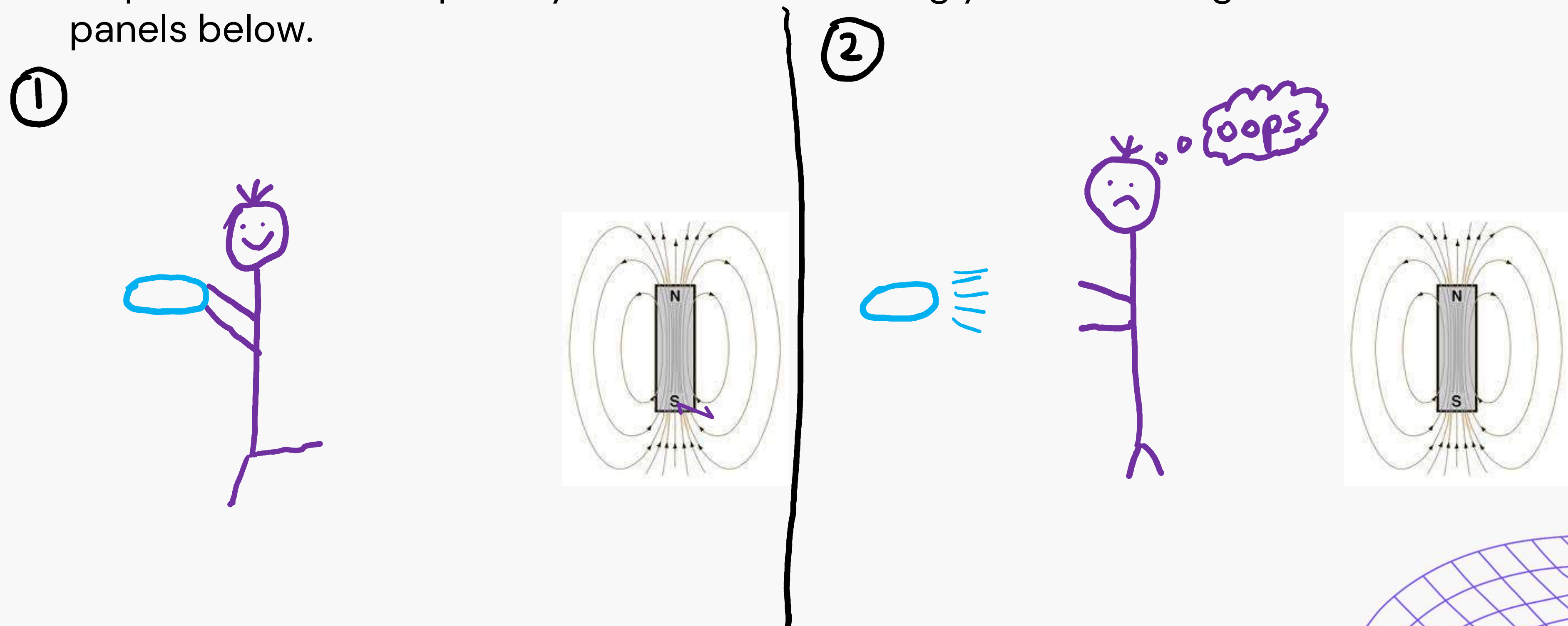


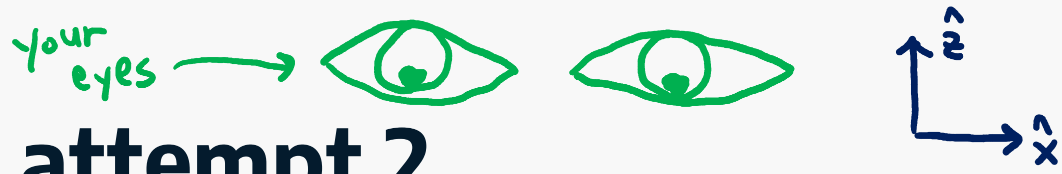




Problem 3, attempt 1

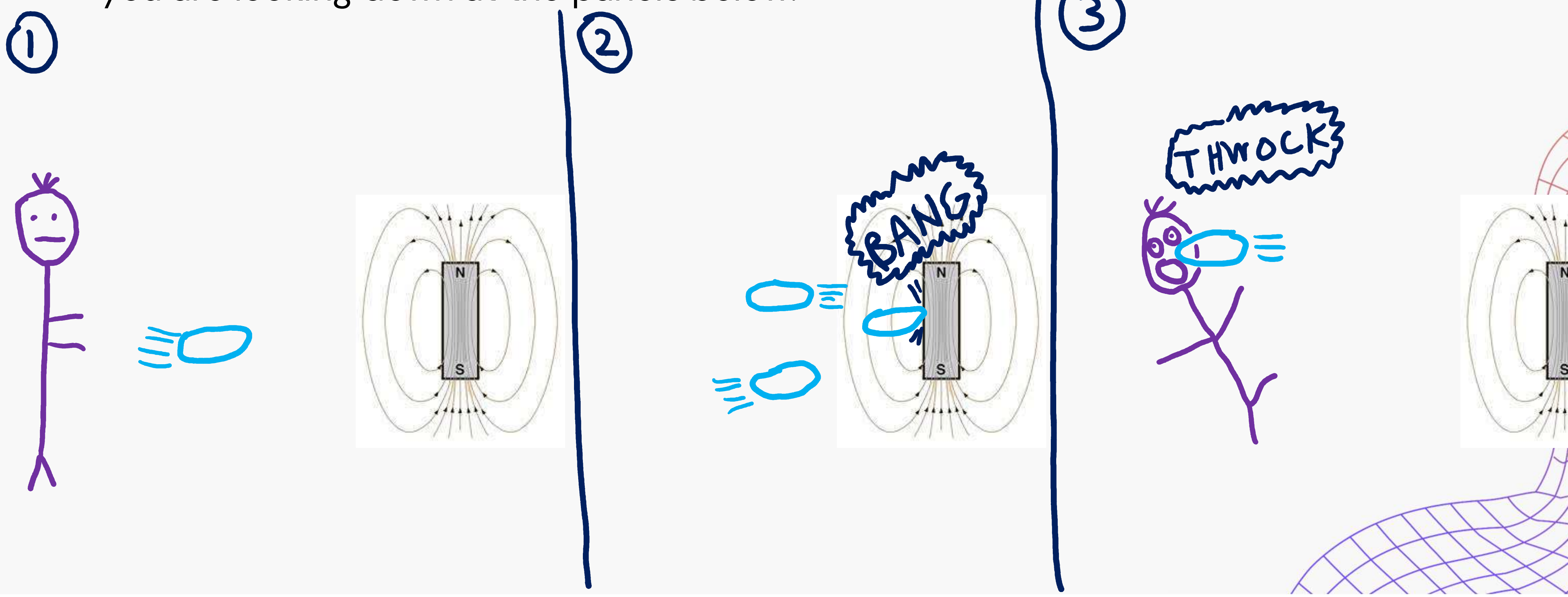
Assume the magnetic field at my location is negligible and the loop is parallel to the xy-plane. What is the direction of current generated in the conducting loop as it travels? Express your answer assuming you are looking down at the panels below.

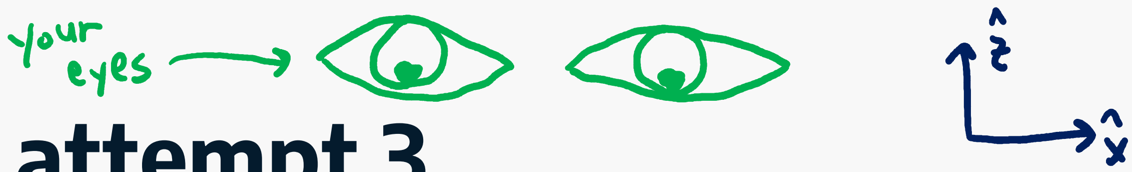




Problem 3, attempt 2

Assume the loop is parallel to the xy-plane. What is the direction of current generated in the conducting loop as it travels? Express your answer assuming you are looking down at the panels below.





Problem 3, attempt 3

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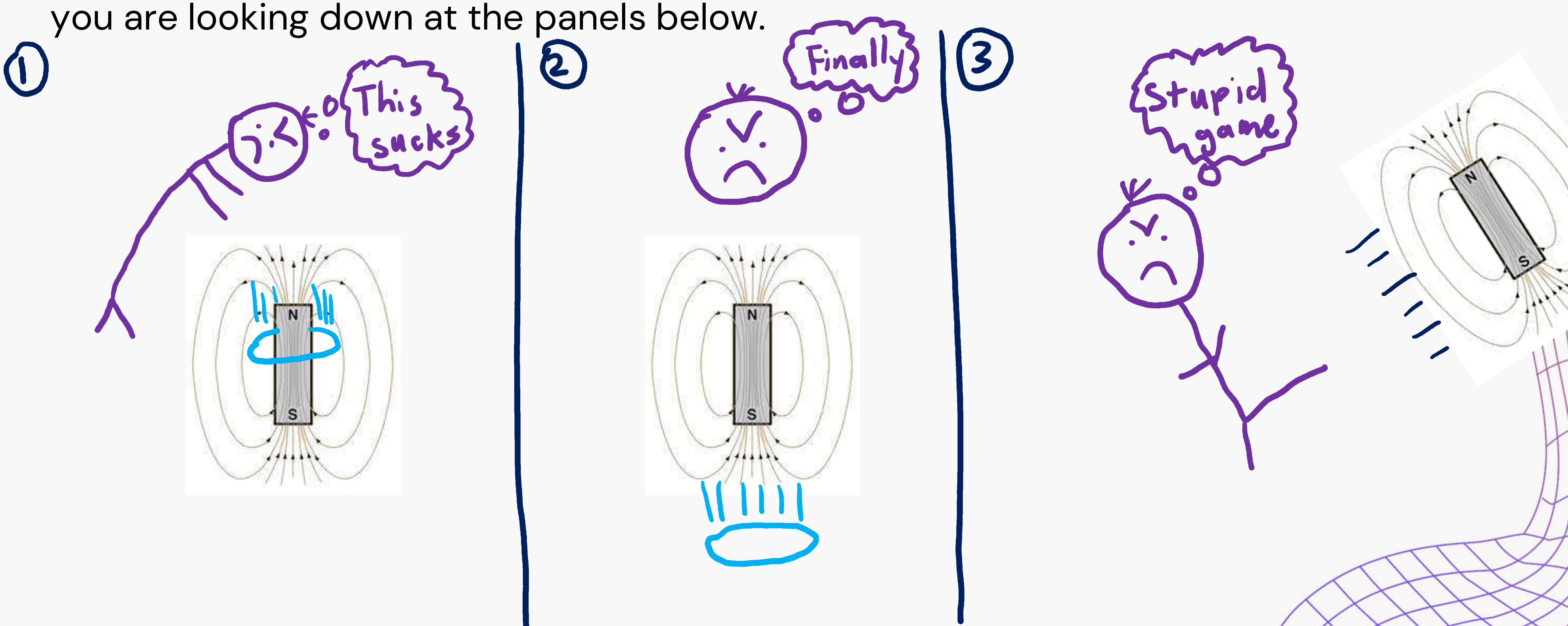

Problem 4

Consider two infinite line currents $I_x\hat{x}$ [A] and $I_y\hat{y}$ [A] along the x and y axis and crossing at the origin but not interfering with each other.

1. Find the $\vec{B}$ field in the region $x > 0$, $y > 0$ on the xy-plane.
2. Find the emf generated in a square wire loop (sidelength ℓ) moving with a velocity $\vec{v} = v_x\hat{x} + v_y\hat{y}$ [m/s] in the region $x > 0$, $y > 0$ on the xy-plane. Assume the wire loop is very small such that $\vec{B}$ can be assumed to be uniform across the loop.

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Inductance

Inductance: the tendency of an electrical conductor to oppose a change in electric current flowing through it.

Inductance exists only in conductors!

Self inductance: change in current flowing through a conductor induces an emf in the conductor itself.
Mutual inductance: change in current flowing through a conductor induces an emf in any nearby conductors.

$$\Psi = LI \qquad\qquad Q = CV$$

Pick your poison: solenoid vs. coax

Choose one:

1. Inductance of a solenoid.
2. Inductance of a shorted coaxial cable.
3. None of the above.

Problem 5: Solenoid

Given n, the number of turns in a solenoid per unit length, d, the length of the solenoid, R, the radius of the solenoid, and I, the current each turn carries, find inductance L of the solenoid.

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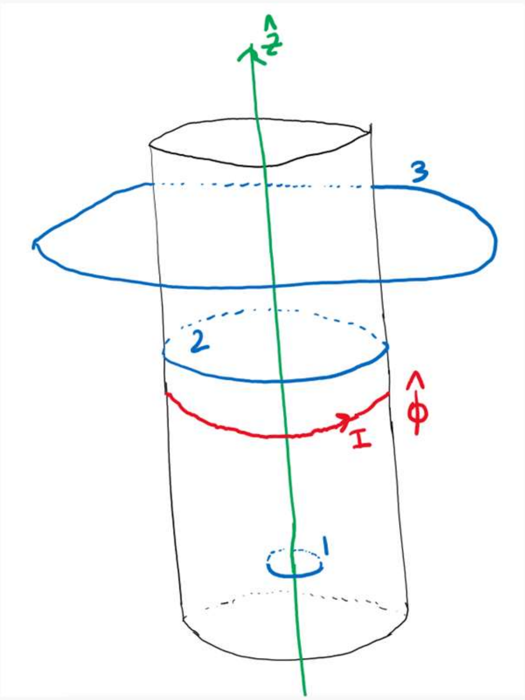

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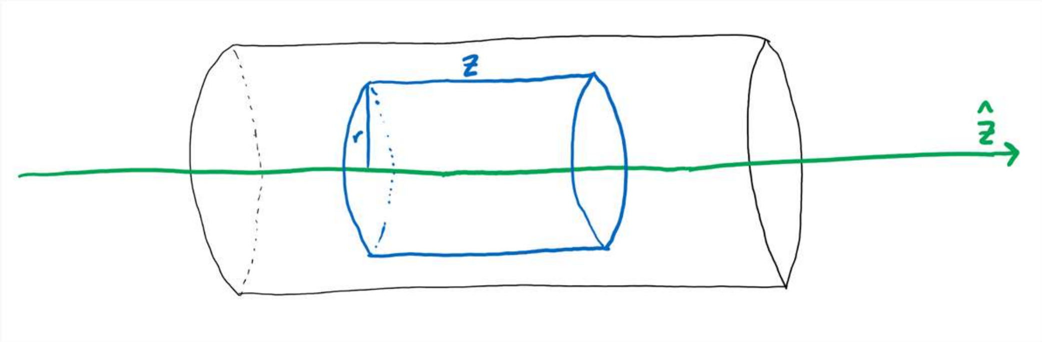

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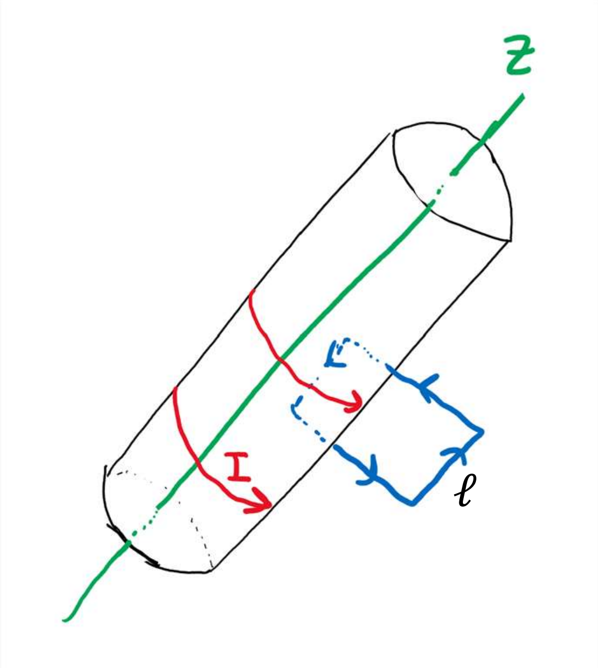

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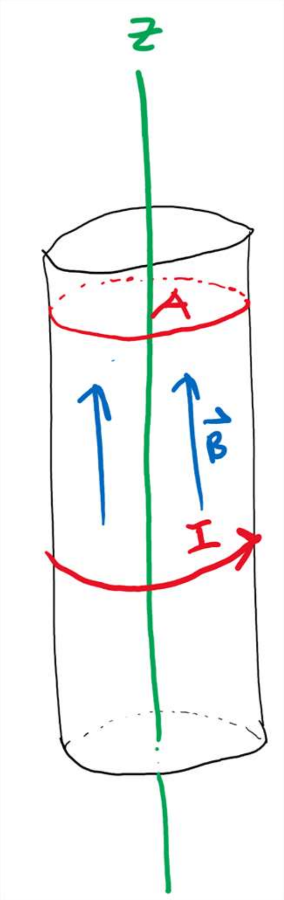
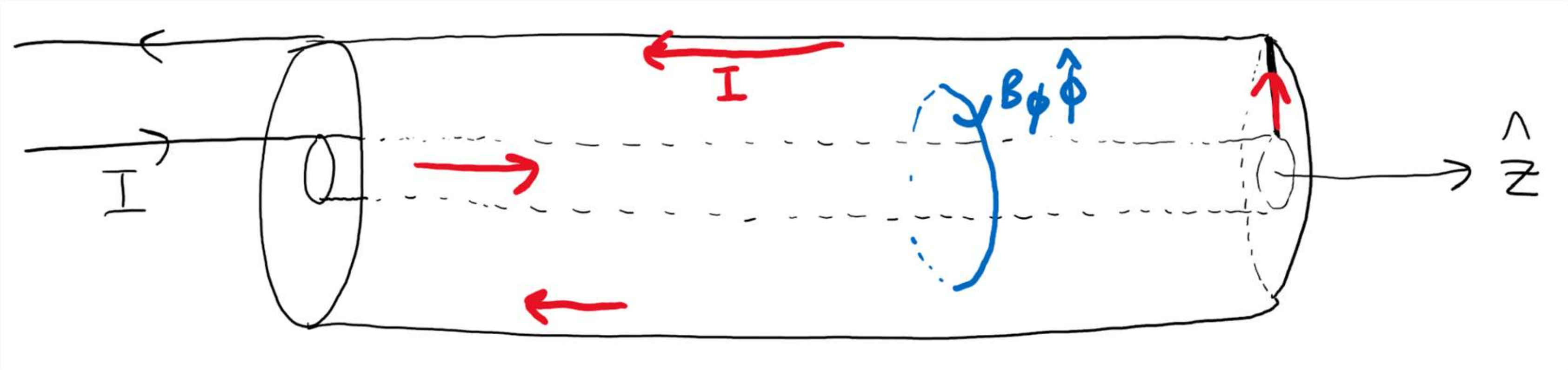


Problem 6: Shorted coaxial cable

Given a shorted coaxial cable with inner radius a, outer radius b and length ℓ, carrying current I on the inner conductor, find the inductance L of the shorted coax cable.



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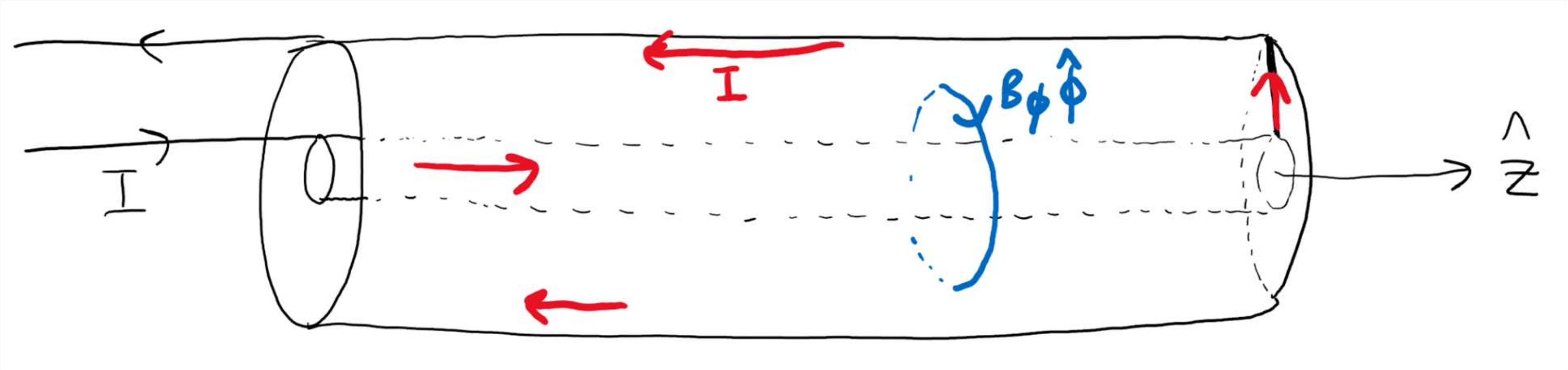

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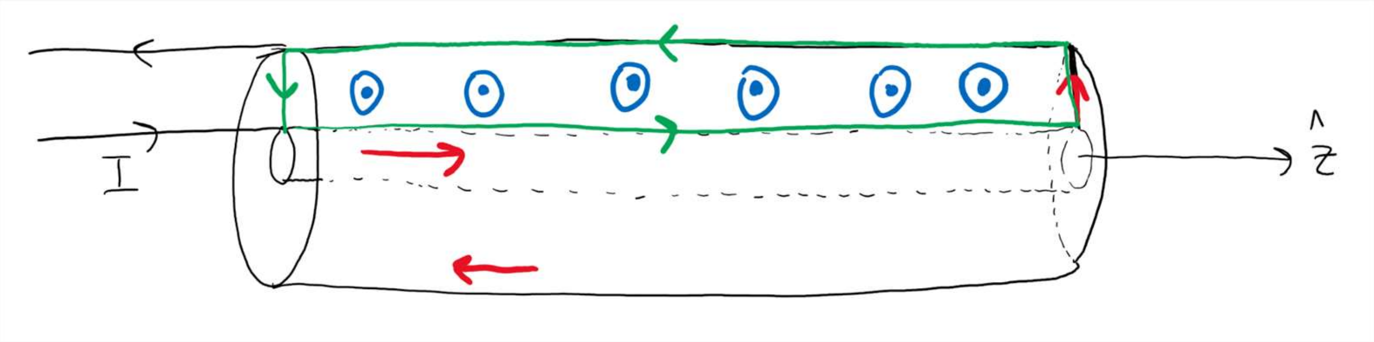

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Problem 7

$$\Psi = LI$$

Express the unit [H] using the SI quantities [kg], [m], [s], and [C].

Problem 8 (Homework Hint): Rotations!

A square loop of sidelength 2m sits on the xy-plane at $t = 0$ and begins to rotate about the x-axis clockwise with angular frequency ω when viewed at $x = 10$ looking at the origin.

1. Express the angle between the plane of the current loop at the xy-plane as a function of t.
2. Suppose $d\vec{S}$ points in the $+\hat{z}$ direction at $t = 0$. Find the direction of $d\vec{S}$ as a function of t.
3. Suppose $\vec{B} = 3\hat{z} + 6\hat{y}$ [Wb/m^2]. Find $\Psi(t)$.

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Midterm 1 equations, in one place

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{F} = q_1\vec{E} + q_1(\vec{v}_1 \times \vec{B})$$

$$\vec{E} = \frac{q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\epsilon \oiint \vec{E}\cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\oiint \vec{D}\cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\iiint \rho dV = Q_{\text{enclosed}}$$

$$\oiint \vec{B}\cdot d\vec{S} = 0$$

$$I = \oiint \vec{J}\cdot d\vec{S} = -\frac{\partial Q_{\text{enclosed}}}{\partial t}$$

$$\nabla \times \vec{E} = 0$$

$$\vec{E} = -\nabla V$$

$$\oint \vec{E}\cdot d\vec{l} = 0$$

$$V_{ab} = V(b) - V(a) = -\int_a^b \vec{E}\cdot d\vec{l}$$

$$\hat{n}\cdot\left(\vec{D}_1 - \vec{D}_2\right) = \rho_s$$

$$\hat{n}\times\left(\vec{E}_1 - \vec{E}_2\right) = 0$$

$$\hat{n}\cdot\left(\vec{P}_1 - \vec{P}_2\right) = -\rho_{b,s}$$

$$\epsilon = \epsilon_0(1 + \chi_e)$$

$$\vec{P} = \epsilon_0\chi_e\vec{E}$$

$$\vec{D} = \epsilon_0\vec{E} + \vec{P} = \epsilon\vec{E}$$

$$\vec{J} = \sigma\vec{E}$$

$$\rho_b = -\nabla\cdot\vec{P}$$

$$\nabla\cdot\epsilon_0\vec{E} = \rho_f + \rho_b$$

$$\nabla\cdot\vec{D} = \rho$$

$$\nabla\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$$

$$-\nabla^2 V = \frac{\rho}{\epsilon}$$

$$\oiint \vec{D}\cdot d\vec{S} = \iiint \nabla\cdot\vec{D}dV$$

$$\oint \vec{E}\cdot d\vec{l} = \iint\left(\nabla\times\vec{E}\right)\cdot d\vec{S}$$

$$\int_a^b \nabla V\cdot d\vec{l} = V(b) - V(a)$$

Midterm 2 equations, in one place

$$\vec{B} = \frac{\mu I}{2\pi r}\hat{\phi}$$

$$d\vec{B} = \frac{\mu I d\vec{\ell} \times \hat{r}}{4\pi r^2}$$

$$\oint_C \vec{H} \cdot d\vec{\ell} = \iint_S \vec{J} \cdot d\vec{S}$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu I_{\text{encl}}$$

$$\nabla \times \vec{H} = \vec{J}$$

$$\nabla \cdot \vec{B} = 0$$

$$\Psi = \iint_s \vec{B} \cdot d\vec{S}$$

$$-\frac{d}{dt}\iint_s \vec{B} \cdot d\vec{S} = \oint_c \vec{E} \cdot d\vec{l}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint_c \vec{E} \cdot d\vec{l} = \varepsilon$$

$$\varepsilon = \frac{W}{q} = \oint_C \frac{\vec{F}}{q} \cdot d\vec{l}$$

$$\Psi = LI$$

$$\varepsilon = IR$$

$$Q = CV$$

$$G = \frac{\sigma}{\epsilon} C \qquad R = \frac{1}{G}$$

Units

Charge Q: C
Current I: A
Electric field strength $\vec{E}$: N/C or V/m
Electric flux density $\vec{D}$: C/m^2
Polarization field $\vec{P}$: C/m^2
Electric potential V: V
Capacitance C: F
Magnetic flux density $\vec{B}$: T or Wb/m^2
Magnetic field strength $\vec{H}$: A/m
Magnetic flux Ψ: Wb
Electromotive force ε: V
Inductance L: H

Electric permittivity ϵ: F/m
Magnetic permeability μ: H/m
Conductivity σ: Si/m

Charge density ρ: C/m^3
Surface charge density ρ_s: C/m^2
Current density $\vec{J}$: A/m^2

Office Hours

Any questions?

Share any thoughts on anything!