ECE329: Tutorial Session 3

September 17th, 2024

Share any thoughts on anything

Exam 1 Content

- Vector Calculus
- Coulomb's Law and Lorentz Force
- Gauss's Law & electric flux
- Charge flux
- Boundary Conditions
- Conductors
- Dielectrics

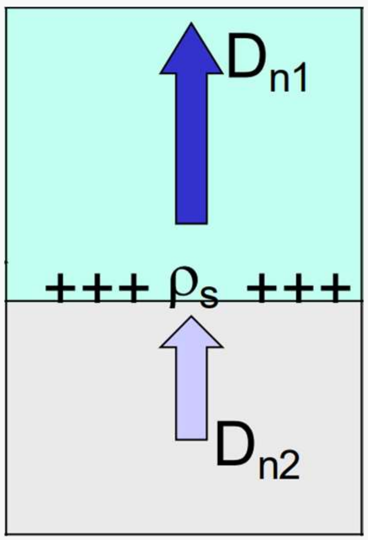
Exam 1 Prep

- Friday’s lecture will be in-class review for all sections.
- HKN will host a review session on Saturday 9/21, 3:00-5:30PM in ECEB 1013.

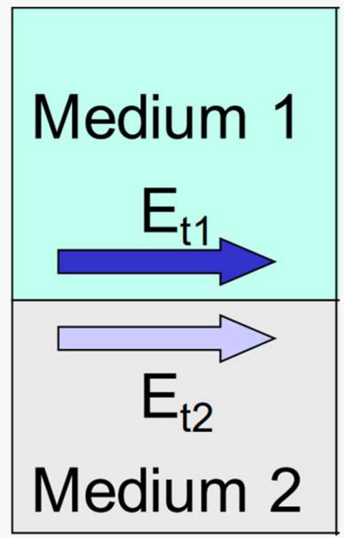
- Make your own 4”x6” notecard. Do not blindly copy other people’s notecards. Make sure you understand what you are writing on your notecard.
- Review HWs 1-3.
- Do the practice exams on course website.
- Read over Professor Kudeki’s notes if you have time. They are a bit dense.

Boundary Conditions

$$\hat{n} \cdot \left(\vec{D}_1 - \vec{D}_2\right) = \rho_s$$



$$\hat{n} \times \left(\vec{E}_1 - \vec{E}_2\right) = 0$$



Problem 1

Suppose the yz-plane in free space holds a surface charge density of $\rho_s = 5$ C/m^2. The electric displacement field on the $-x$ side is given as $\vec{D} = \hat{x} + \hat{y} + \hat{z}$. Find the electric displacement field on the $+x$ side using boundary conditions.

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Conductors: The intuition

Conductors: The math

Described by σ, aka conductivity (units: Siemens/meter)

What to know:

- $\vec{J} = \sigma\vec{E}$ (Ohm's Law)

Assumption: We are dealing with electrostatics.

- If $\sigma \neq 0$, material is an equipotential with zero internal fields and finite surface charge densities.

P fields: The intuition

P fields: The math

$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$: The defining equation.

Assumption: Dielectric is 'isotropic', so $\vec{P}$ is collinear to $\vec{E}$. Then:

- $\vec{P} = \epsilon_0 \chi_e \vec{E}$ with electric susceptibility $\chi_e \geq 0$ nearly always in this class.
- Let electric permittivity be $\epsilon = \epsilon_0(1 + \chi_e)$.
- Let relative electric permittivity be $\epsilon_r = 1 + \chi_e$
- $\vec{D} = \epsilon \vec{E}$

P fields: The math

Assumption: Dielectric is 'isotropic', so $\vec{P}$ is collinear to $\vec{E}$. Then:

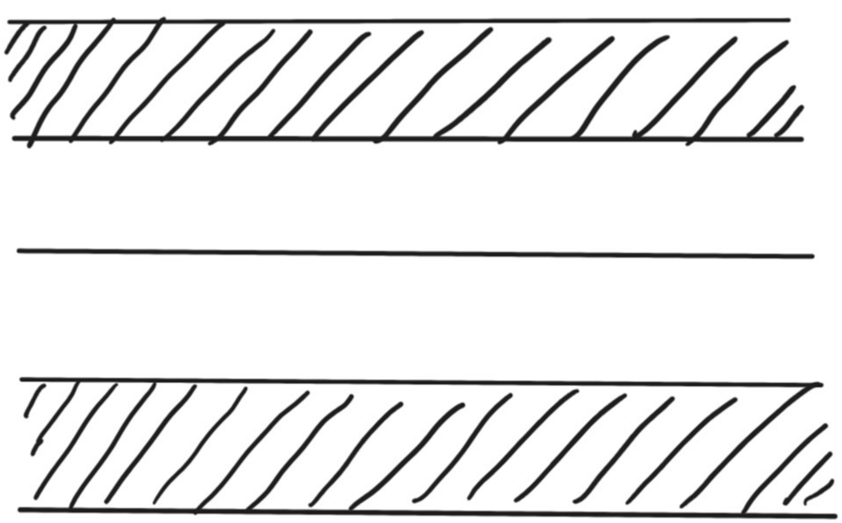
Divergences:

- Gauss's Law: $\nabla \cdot \epsilon_0 \vec{E} = \rho_f + \rho_b$
- Gauss's Law: $\nabla \cdot \vec{D} = \rho_f = \rho$

- Therefore, $\rho_b = -\nabla \cdot \vec{P}$

Problem 2

Find $\vec{D}$, $\vec{E}$, and $\vec{P}$, in all volumes. Find ρ_s at each material boundary. What is the voltage drop from $z = 0$ to $z = 4$?



$z = 4$

$\epsilon = 2\epsilon_0, \sigma = 10^6$

$z = 3$

$\epsilon = 5\epsilon_0, \sigma = 0$

$z = 2$

$\epsilon = 3\epsilon_0, \sigma = 0$

$z = 1$

$\rho_s = 6$ C/m^2

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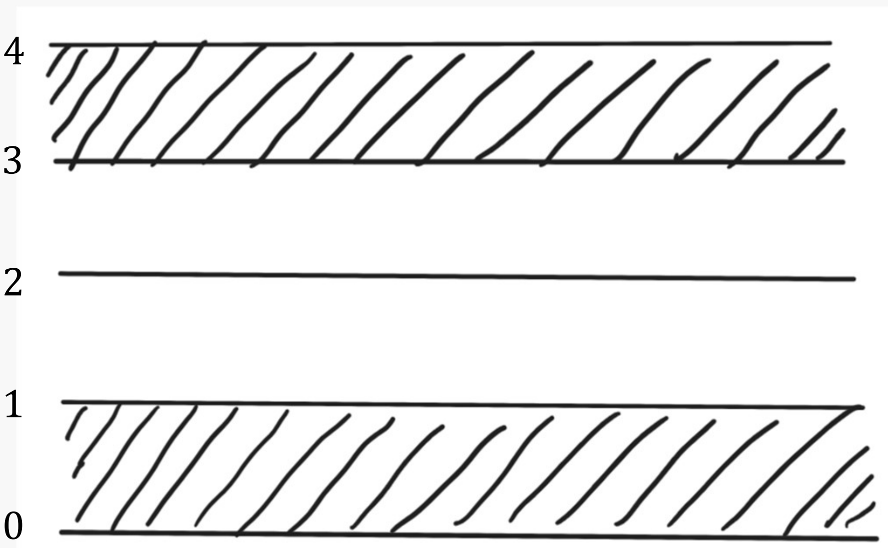

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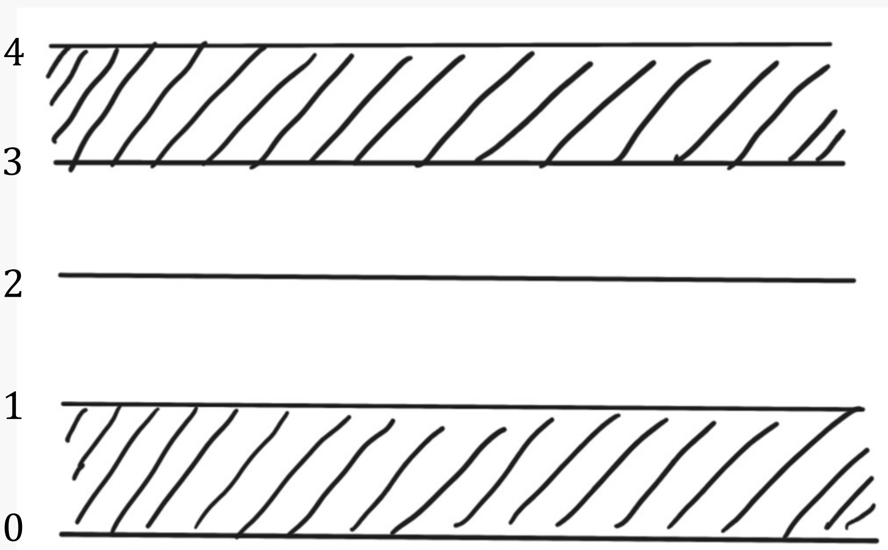

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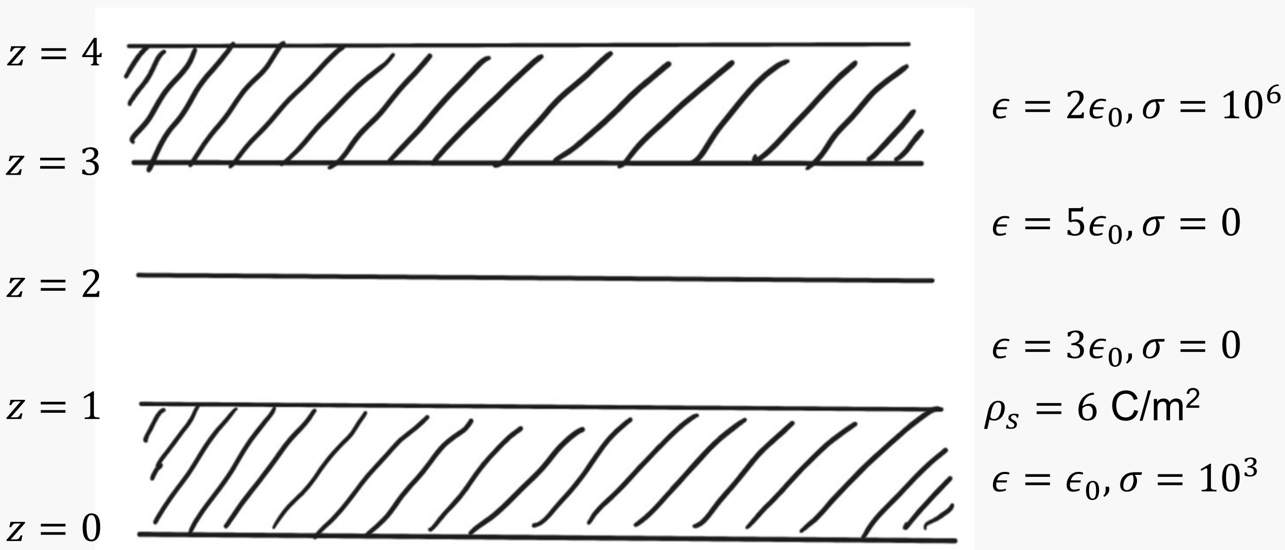

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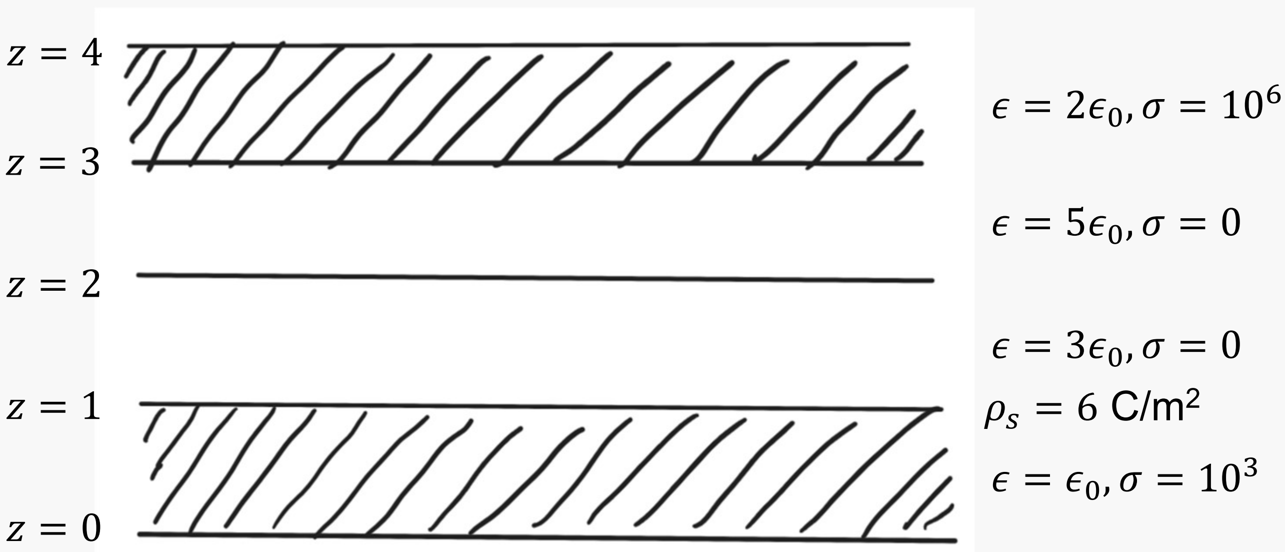

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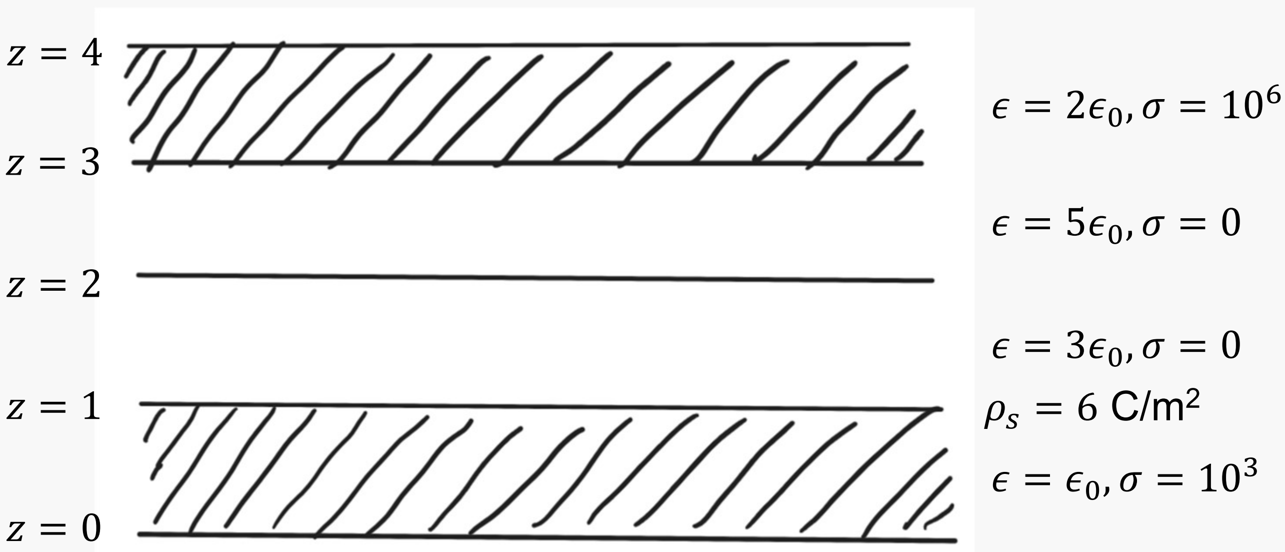

Problem 3

Let $\rho = 6\epsilon_0\delta(z) + \rho_s\delta(z-4)$ C/m^3. The displacement field in the $0 < z < 4$ region is given as $\vec{D} = \epsilon_0\hat{x} + 3\epsilon_0\hat{z}$ and the electric permittivity is known to be $\epsilon_2 = 4\epsilon_0$. It is known that $D_z = 2\epsilon_0$ and $\epsilon_3 = 2\epsilon_0$ for the $z > 4$ region, while $\epsilon_1 = \epsilon_3$ for the $z < 0$ region.
Find ρ_s. Find $\vec{D}$ and $\vec{E}$, in all volumes. Is the plane at $z = 4$ an equipotential surface?

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$z = 4$

$\vec{D} = \epsilon_0\hat{x} + 3\epsilon_0\hat{z}$ $\quad$ $\epsilon_2 = 4\epsilon_0$

$z = 0$

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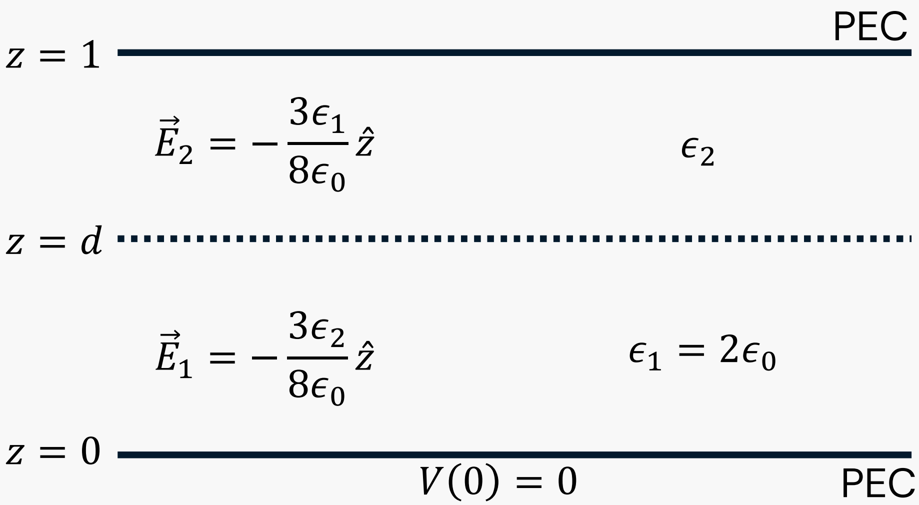
Poisson's Equation

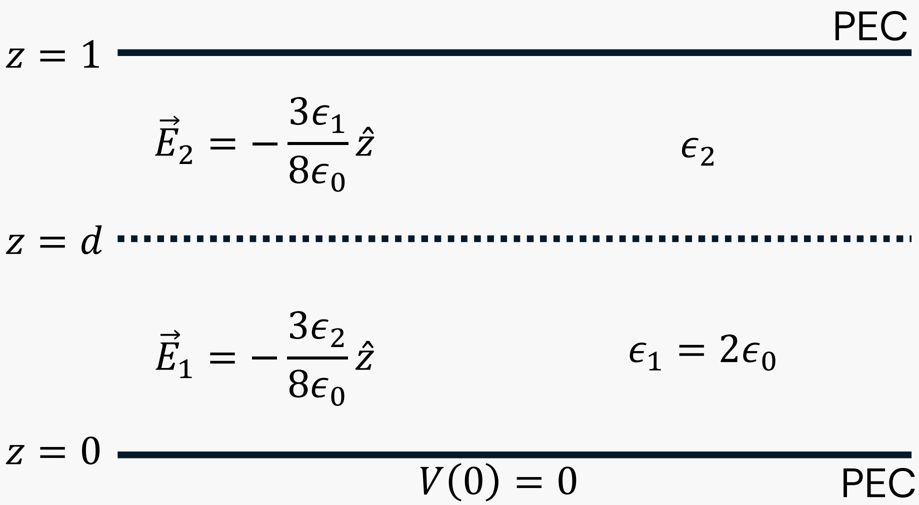
Laplace’s Equation

Problem 4



Verify that the fields given satisfies Maxwell's boundary condition regarding $\vec{D}$ at the boundary between the two dielectric slabs.

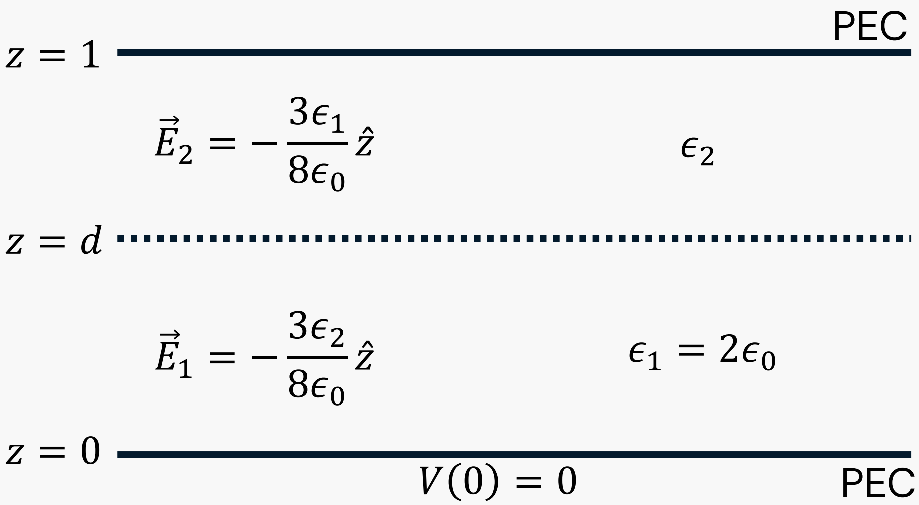
Problem 4



Write the expression for the electrostatic potential V (z) for $0 < z < 1$ in terms of ϵ_1, ϵ_2, and d.

Determine ϵ_2 if the surface charge density on the top plate $\rho_s = 3\epsilon_0$ C/m^2.

Problem 4: Blank slide



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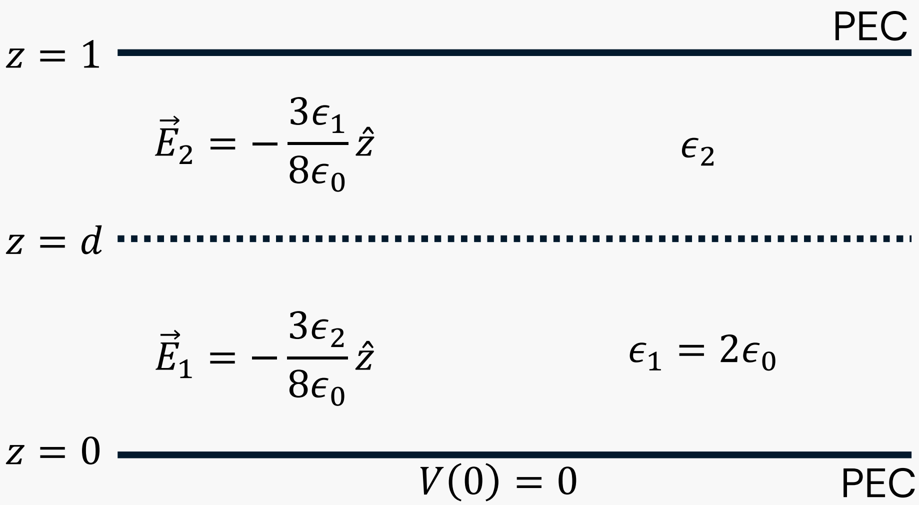

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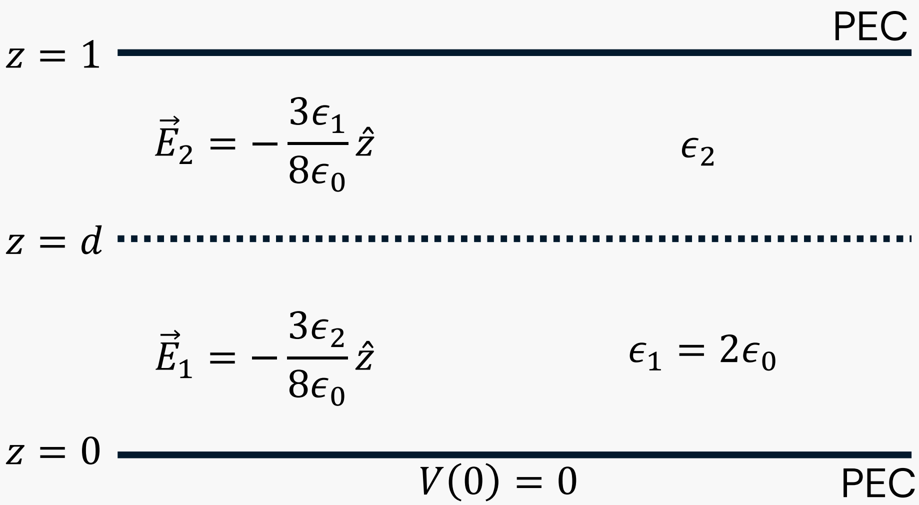

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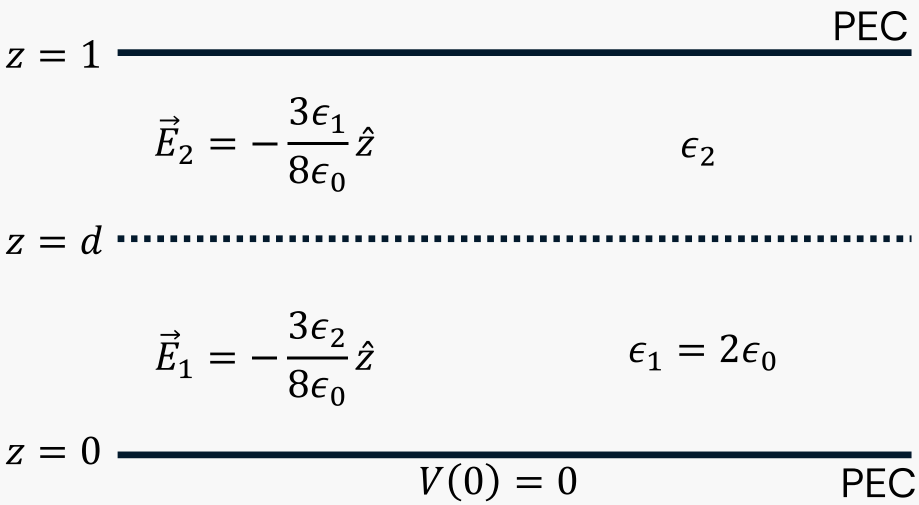

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Jumping Between Quantities

Q

$\rho_f = \rho$

$\vec{D}$

ρ_b

V

$\vec{E}$

$\vec{P}$

Kahoot time

12 conceptual questions.
No calculations needed.

It'll be quick :)

Kahoot: Blank slides

Kahoot: Blank slides

Kahoot: Blank slides

Kahoot: Blank slides

Week 3 equations, in one place

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{r}$$
$$\vec{F} = q_1\vec{E} + q_1(\vec{v}_1 \times \vec{B})$$
$$\vec{E} = \frac{q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\hat{n}\cdot\left(\vec{D}_1 - \vec{D}_2\right) = \rho_s$$
$$\hat{n}\times\left(\vec{E}_1 - \vec{E}_2\right) = 0$$

$$\nabla\cdot\vec{D} = \rho$$
$$\nabla\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$$
$$-\nabla^2 V = \frac{\rho}{\epsilon}$$

$$\epsilon \oiint \vec{E}\cdot d\vec{S} = Q_{\text{enclosed}}$$
$$\oiint \vec{D}\cdot d\vec{S} = Q_{\text{enclosed}}$$
$$\iiint \rho dV = Q_{\text{enclosed}}$$
$$\oiint \vec{B}\cdot d\vec{S} = 0$$
$$I = \oiint \vec{J}\cdot d\vec{S} = -\frac{\partial Q_{\text{enclosed}}}{\partial t}$$

$$\epsilon = \epsilon_0(1+\chi_e)$$
$$\vec{P} = \epsilon_0\chi_e\vec{E}$$
$$\vec{D} = \epsilon_0\vec{E} + \vec{P} = \epsilon\vec{E}$$
$$\vec{J} = \sigma\vec{E}$$
$$\rho_b = -\nabla\cdot\vec{P}$$
$$\nabla\cdot\epsilon_0\vec{E} = \rho_f + \rho_b$$

$$\nabla\times\vec{E} = 0$$
$$\vec{E} = -\nabla V$$
$$\oint \vec{E}\cdot d\vec{l} = 0$$
$$V_{ab} = V(b) - V(a) = -\int_a^b \vec{E}\cdot d\vec{l}$$

$$\oiint \vec{D}\cdot d\vec{S} = \iiint \nabla\cdot\vec{D}dV$$
$$\oint \vec{E}\cdot d\vec{l} = \iint \left(\nabla\times\vec{E}\right)\cdot d\vec{S}$$
$$\int_a^b \nabla V\cdot d\vec{l} = V(b) - V(a)$$

Units

Charge Q: C
Electric field $\vec{E}$: N/C or V/m
Displacement field $\vec{D}$: C/m^2
Polarization field $\vec{P}$: C/m^2
Electric potential V: V
Magnetic field $\vec{B}$: T or Wb/m^2
Charge density ρ: C/m^3
Surface charge density ρ_s: C/m^2
Current density $\vec{J}$: A/m^2

Electric permittivity ϵ: F/m
Magnetic permeability μ: H/m
Conductivity σ: Si/m

Office Hours

Any questions?

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